REFLECTION OF PLANE WAVES IN TRANSVERSELY ISOTROPIC MICROPOLAR VISCOTHERMOELASTIC SOLID

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Abstract. The reflection of plane waves at the free surface of transversely isotropic micropolar viscothermoelastic medium is investigated. It is found that there exist four types of plane waves in transversely isotropic micropolar viscothermoelastic medium, namely quasi-longitudinal displacement (qLD), quasi-transverse displacement (qTD), quasi-transverse microrotational (qTM), quasi-thermal (qT) wave. The amplitude ratios corresponding to reflected waves are obtained numerically when different waves are incident. Numerically simulated results are depicted graphically with respect to the frequency. The effect of viscosity on the amplitude ratios are shown graphically for specific model. Some particular results are also deduced.

1. Introduction
Viscoelastic materials are those for which the relationship between stress and strain depends on time. All materials exhibit some viscoelastic response. In common metals such as steel, aluminum, copper etc. At room temperature and small strain, the behavior does not deviate much from linear elasticity. Synthetic polymer, wood as well as metals at high temperature display significant viscoelastic effects. With the rapid development of polymer science and plastic industry, as well as the wide use of materials under high temperature in modern technology and application of biology and geology in engineering, the theoretical study and applications in viscoelastic materials has become an important task for solid mechanics.

The Kelvin-Voigt (1887) [1] model is one of the macroscopic mechanical models often used to describe the viscoelastic behavior of the material. This model represents the delayed elastic response subjected to stress when the deformation is time dependent but recoverable. The dynamical interaction of thermal and mechanical fields in solids has great practical applications in modern aeronautics, astronautics, nuclear reactors and high energy particle accelerators.


Kumar, Sharma and Garg (2014) [26] studied the effect of two temperatures on reflection coefficient in micropolar thermoelastic with and without energy dissipation media. Kumar, Kaur and Rajvanshi (2014) [27] investigated reflection and transmission of plane wave at an interface between two micropolar thermoelastic half-space with different micropolarity and thermoelastic properties. Kumar and Kaur (2014) [28] discussed the reflection and refraction of waves at the interface of an elastic solid and microstretch thermoelastic solid with microtemperature. Magana and Quintanilla (2014) [29] investigated the uniqueness and analyticity of solutions in micropolar thermoviscoelasticity. The fundamental solution of the system of equations of steady vibrations was constructed by means of elementary functions and its basic properties were established by Svanadze (2014) [30].

In the present investigation the reflection of plane wave in heat conducting transversely isotropic micropolar viscoelastic half space has been discussed. The amplitude ratios of various waves corresponding to reflected waves are obtained numerically when different waves are incident. The effect of viscosity on the amplitude ratios are shown graphically for specific model. Some particular results are also deduced.

2. Basic equations
The basic equations in a homogeneous anisotropic micropolar viscothermoelastic solid in the absence of body forces, body couples, and heat sources are given by
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(a) The constitutive relations.

\[ t_{kl} = \overline{A}_{klmn} E_{mn} + \overline{G}_{klmn} \Psi_{mn} - \beta_{kl} (1 + \tau_1 \frac{\partial}{\partial t}) T, \]

\[ m_{kl} = \overline{B}_{klmn} E_{mn} + \overline{G}_{mnkl} \Psi_{mn}, \]

\[ q_k = K_{kl} T_k, \]

for Lord and Shulman (L-S) theory

\[ \rho \eta T_k = \rho C^* T + \beta_{kl} T_k \mu_{i,j}, \quad (q_{k,k} + \tau_0 q_k) = K_{kl} T_k, \]

for Green and Lindsay (G-L) theory

\[ \rho \eta T_k = \rho C^* (T + \tau_0 \dot{T}) + \beta_{kl} T_k \mu_{i,j}, \quad q_{k,k} = K_{kl} T_k. \]

The deformation and wryness tensor are defined by the following:

\[ E_{kl} = u_{kl} + \varepsilon_{mn} \varphi_{mn}, \quad \Psi_{kl} = \varphi_{kl}. \]

(b) Balance laws.

\[ t_{kl,k} = \rho \ddot{u}_k, \]

\[ m_{kl,k} + \varepsilon_{mn} t_{mn} = \rho f \ddot{\phi}_k, \quad i, j, k = 1, 2, 3, \]

\[ q_{k,k} = \rho T_0 \dot{\eta}, \]

Assuming the viscoelastic nature of the material is described by the Voigt (1887) [1] model of linear elasticity, we replace the micropolar elastic constants \( \overline{A}_{ijkl}, \overline{B}_{ijkl}, \overline{G}_{ijkl} \) by their complex moduli as

\[ \overline{A}_{ijkl} = A_{ijkl} + A_{ijkl}^* \frac{\partial}{\partial t}, \quad \overline{B}_{ijkl} = B_{ijkl} + B_{ijkl}^* \frac{\partial}{\partial t}, \quad \overline{G}_{ijkl} = G_{ijkl} + G_{ijkl}^* \frac{\partial}{\partial t}, \]

where \( t_{kl}, m_{kl}, K_{kl} \) are the stress tensor, couple stress tensor and thermal conductivity tensor, respectively; \( q_k \) is the heat flux vector; \( \eta \) is the entropy; \( T \) is the absolute temperature; \( C^* \) is the specific heat at constant strain; \( \tau_0, \tau_1 \) are thermal relaxation times; \( \rho \) is the bulk mass density; \( j \) is the microinertia; \( \mu_{i,j}, \phi_k \) are components of displacement vector and microrotation vector; \( \beta_{kl} \) is thermal elastic coupling tensor; \( A_{klmn}, B_{klmn}, G_{klmn} \) are characteristic constants of material, where \( \overline{A}_{klmn}, \overline{B}_{klmn}, \overline{G}_{klmn} \) satisfies the symmetric properties

\[ \overline{A}_{klmn} = \overline{A}_{mnkl}, \quad \overline{B}_{mnkl} = \overline{B}_{klmn}, \quad \overline{G}_{klmn} \]

and \( \beta_{kl} = \beta_{lk} \), and \( K_{kl} = K_{lk} \). Tensor \( \overline{G}_{klmn} \) does not possess this property; it forms a pseudotensor, inversion of the coordinate system changing its sign. In a centrosymmetric body, all components of \( \overline{G}_{klmn} \) vanish.

For Lord-Shulman (1967) [31] theory \( \tau_1 = 0, \eta_0 = 1 \); for Green-Lindsay (1972) [32] theory, \( \tau_1 > 0, \eta_0 = 0 \) and for coupled thermoelasticity theory, \( \tau_0 = \tau_1 = 0, \eta_0 = 0 \). The thermal relaxation times \( \tau_0 \) and \( \tau_1 \) satisfy the inequality \( \tau_0 \geq \tau_1 \geq 0 \) for G-L theory only. However, it
has been proved by Strunin (2001) [33] that the inequalities are not mandatory for $\tau_0$ and $\tau_1$ to follow.

We introduce the dimensionless quantities as

\[
x_i' = \frac{\omega^*}{c_1} x_i, \quad u_i' = \frac{\omega^*}{c_1} u_i, \quad \phi_2' = \frac{\varphi_2}{K_1}, \quad m_{ij}' = \frac{c_1}{B_{35}} m_{ij}, \quad t_{ij}' = \left( \frac{t_i}{A_{11}} \right), \quad \tau_0' = \omega^* \tau_0,
\]

\[
t' = \omega^* t, \quad T' = \frac{T}{T_0}, \quad \omega^* = \frac{X}{\rho j}, \quad c_1' = \frac{A_{11}}{\rho}, \quad \tau_0' = \omega^* \tau_0.
\]

(12)

where \( \omega^* \) is the characteristic frequency of the material.

3. Formulation and solution of the problem

Following Slaughter (2002) [34], we use the appropriate transformation on the system of equations (1)-(3) and (4)-(6) to deduce the equations for transversely isotropic micropolar viscothermoelastic medium initially in an undeformed state and at uniform temperature $T_0$.

We take the origin of the coordinate system on the top plane surface and $x_3$-axis is pointing normally into the half space, which is thus represented by $x_3 \geq 0$. We consider plane waves in a plane such that all particles on a line parallel to $x_2$-axis are equally displaced, so all partial derivatives with respect to the variable $x_2$ would be zero.

For two dimensional problem, we take $\vec{u} = (u_1, 0, u_3), \quad \vec{\phi} = (0, \phi_2, 0)$ so that

\[
\begin{align*}
\bar{A}_{11} \frac{\partial^2 u_1}{\partial x_1^2} + \bar{A}_{13} \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \bar{A}_{35} \frac{\partial^2 u_1}{\partial x_3^2} + K_1 \frac{\partial \phi_2}{\partial x_3} = \beta_1 \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \\
\bar{A}_{66} \frac{\partial^2 u_3}{\partial x_1^2} + \bar{A}_{65} \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \bar{A}_{35} \frac{\partial^2 u_3}{\partial x_3^2} + K_2 \frac{\partial \phi_2}{\partial x_1} = \beta_1 \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x_3} = \rho \frac{\partial^2 u_3}{\partial t^2}, \\
\beta_{77} \frac{\partial^2 \phi_2}{\partial x_1^2} + \beta_{66} \frac{\partial^2 \phi_2}{\partial x_3^2} = \chi \phi_2 + K_1 \frac{\partial u_1}{\partial x_3} + K_2 \frac{\partial u_3}{\partial x_1} = \rho \frac{\partial^2 \phi_2}{\partial t^2}, \\
K_1' \frac{\partial^3 T}{\partial x_1^2} + K_3' \frac{\partial^3 T}{\partial x_3^2} = \rho c^* \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + T_0 \left( \frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \beta_1 \frac{\partial u_1}{\partial x_1} + \beta_1 \frac{\partial u_3}{\partial x_2} \right),
\end{align*}
\]

(13)-(16)

\[
t_{33} = \bar{A}_{13} \frac{\partial u_1}{\partial x_1} + \bar{A}_{33} \frac{\partial u_3}{\partial x_3} - \beta_1 \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) T,
\]

(17)

\[
t_{31} = \bar{A}_{65} \frac{\partial u_1}{\partial x_1} + K_1 \phi_2 + \bar{A}_{35} \frac{\partial u_3}{\partial x_3} + m_{32} = \bar{A}_{66} \frac{\partial \phi_2}{\partial x_3},
\]

(18, 19)

where $\beta_i = \bar{A}_{11} \alpha_i + \bar{A}_{33} \alpha_3, \quad \beta_3 = \bar{A}_{11} \alpha_1 + \bar{A}_{33} \alpha_3, \quad K_1 = \bar{A}_{56} - \bar{A}_{35}, \quad K_2 = \bar{A}_{66} - \bar{A}_{56}, \quad X = K_3 - K_1, \quad \alpha_i, \alpha_3$ are the coefficients of linear thermal expansion, we have used the notations $11 \rightarrow 1, \quad 33 \rightarrow 3, \quad 12 \rightarrow 7, \quad 13 \rightarrow 6, \quad 23 \rightarrow 5$ for the material constants.

We take plane-wave solution as

\[
(u_1, u_3, \phi_2, T) = (\vec{u}_1, \vec{u}_3, \vec{\phi}_2, \vec{T}) e^{i(\vec{x} \cdot \vec{n} + \rho \vec{s}_0 - \epsilon t)}.
\]

(20)
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Substituting the values of $u_1$, $u_3$, $\varphi_2$ and $T$ from equation (20) in the equations (13)-(16) and with the aid of (12) and after some simplification we obtain

$$c^8 + A_1 c^6 + A_2 c^4 + A_3 c^2 + A_4 = 0,$$

where

$$A_i = \omega \frac{c_i f_i + f_2}{a_i \omega^2 - a_i}, \quad A_2 = \frac{f_2 \omega^2 + (f_1 + f_2) \omega^2 + f_2}{a_i \omega^2 - a_i}, \quad A_3 = \frac{f_2 \omega^2 + f_2}{a_i \omega^2 - a_i}, \quad A_4 = \frac{f_2 \omega^2}{a_i \omega^2 - a_i},$$

$$f_1 = a_1 - a_1^2 - a_3 b_1, \quad f_2 = a_8 - a_3 - a_4 b_1 + a_5 b_4 + a_6 b_5 + b_5 a_8, \quad f_3 = a_9 - a_1 a_3 + a_1 b_1 + a_2 b_4 b_5 + a_3 b_5 b_6,$$

$$f_4 = a_6 + b_4 a_1^2 + b_5 + b_5 b_6 + b_5 a_7 + b_5 a_8 b_5 a_8,$$

$$f_6 = a_1 b_1 - a_1 a_3 + b_1 a_4 + a_2 b_4 b_5 - a_1 b_5 + b_5 a_8,$$

$$f_8 = a_1 b_1 - a_1 a_3 + b_1 a_4 + a_2 b_4 b_5 - a_1 b_5 + b_5 a_8,$$

$$f_9 = a_1 b_1 - a_1 a_3 + b_1 a_4 + a_2 b_4 b_5 - a_1 b_5 + b_5 a_8,$$

and define the wave propagation velocity $A_i$ as

$$p_{(0,0)}$$

denotes the unit propagation vector.

The characteristic equation (21) having the complex coefficients yields the four complex roots. The complex phase velocities $c_i$, $(i=1,2,3,4)$ of the quasi-waves will be varying with the direction of phase propagation. We take the complex velocity $c_i = c_{Ki} + ic_{Li}$ and define the wave propagation velocity $V_i = \frac{c_{Ki}^2 + c_{Li}^2}{c_{Ki}}$ and the attenuation quality factor $Q^{-1} = -2c_i / c_{Ki}$. These waves become homogeneous wave if they have same direction of wave propagation and attenuation vector of these waves. These waves are called quasi-waves because polarization may not be along the dynamic axes. The wave velocity $V_i$, $(i=1,2,3,4)$ are
named as quasi-longitudinal displacement (qLD) wave, quasi transverse microrotational (qTM), quasi-transverse displacement (qTD) wave and quasi-thermal wave (qT) that are propagating with the descending phase velocities respectively.

4. Reflection at the free surface

Let us consider free surface \( x_3 = 0 \) for transversely isotropic micropolar viscothermoelastic half-space occupying the region \( x_3 > 0 \) (Fig. 1).

We assume qLD or qTD or qTM or qT wave is incident at the interface and each Incident wave will generate reflected qLD, qTD, qTM and qT waves in free surface. The total displacement, microrotations and temperature distributions are

\[
\begin{align*}
u_i &= \sum_{j=1}^{8} A_j e^{i \omega \left( x_1 \sin \theta_0 - x_3 \cos \theta_0 \right)/\gamma_j}, \quad \nu_3 = \sum_{j=1}^{8} B_j e^{i \omega \left( x_1 \sin \theta_0 - x_3 \cos \theta_0 \right)/\gamma_j}, \\
\phi_j &= \sum_{j=1}^{8} C_j e^{i \omega \left( x_1 \sin \theta_0 - x_3 \cos \theta_0 \right)/\gamma_j}, \quad T = \sum_{j=1}^{8} D_j e^{i \omega \left( x_1 \sin \theta_0 - x_3 \cos \theta_0 \right)/\gamma_j}, \quad (j = 1, \ldots, 8),
\end{align*}
\]

where \( \omega \) is the angular frequency. The subscripts 1, \ldots, 4 denote the quantities corresponding to incident qLD, qTD, qTM and qT wave, respectively, whereas the subscripts 5, \ldots, 8, respectively, denote the corresponding reflected wave and

\[
B_j = \frac{A_{j+4}}{A_j}, \quad C_j = \frac{A_{j+4} - A_{j+3}}{A_j}, \quad D_j = \frac{A_{j+3}}{A_j}, \quad (j = 1, \ldots, 8),
\]

where \( A_1, A_2, A_3 \) are obtained as

\[
\begin{align*}
A_1 &= - \xi^3 b_{11} b_{12} b_{13} \left( d_1 \omega - b_6 \xi \right) + ( b_3 + b_8 \xi^2 ) \left( b_3 b_{11} + b_8 \xi b_{12} \right) \left( \xi^2 b_3 b_{12} \right) - \xi^4 b_3 b_{12} b_{13}, \\
A_2 &= \xi^3 \left( d_1 \omega - b_6 \xi \right) + ( b_3 + b_8 \xi^2 ) \left( b_3 b_{11} + b_8 \xi b_{12} \right) \left( \xi^2 b_3 b_{12} \right), \\
A_3 &= \xi^3 \left( d_1 \omega - b_6 \xi \right) + ( b_3 + b_8 \xi^2 ) \left( b_3 b_{11} + b_8 \xi b_{12} \right) \left( \xi^2 b_3 b_{12} \right).
\end{align*}
\]

The expressions for \( A_i, i = 1, 2, \ldots, 15 \) are obtained from the expressions for \( a_i, i = 1, 2, \ldots, 15 \) given in equation (22) on substituting the values for \( p_1 \) and \( p_2 \).

For incident qLD wave \( p_1 = \sin \theta_1, \quad p_3 = -\cos \theta_1 \); for incident qTD wave \( p_1 = \sin \theta_2, \quad p_3 = -\cos \theta_2 \); for incident qTM wave \( p_1 = \sin \theta_3, \quad p_3 = -\cos \theta_3 \); for incident qT wave
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$p_i = \sin e_i$, $p_3 = -\cos e_i$; for reflected qLD wave $p_i = \sin e_i$, $p_3 = \cos e_i$; for reflected qTD wave $p_i = \sin e_i$, $p_3 = \cos e_i$; for reflected qTM wave $p_i = \sin e_i$, $p_3 = \cos e_i$; for reflected qT wave $p_i = \sin e_i$, $p_3 = \cos e_i$.

5. Boundary conditions

Mechanical condition:

Mechanical conditions are the vanishing of the normal stress, tangential stress, tangential couple stress components and these can be written mathematically as

\[ t_{33} = t_{31} = m_{32} = 0 \quad \text{at} \quad x_3 = 0. \]  

(25)

Thermal condition:

\[ \frac{\partial T}{\partial x_3} + hT = 0, \]  

(26)

where $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ refers to isothermal boundaries.

The boundary conditions given by equations (25) to (26) can be satisfied for all values of $x_i$ and $t$, therefore we have

\[ E_i(x_1, 0, t) = E_2(x_1, 0, t) = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots = E_8(x_1, 0, t) \]  

(27)

From the equations (23) and (27), we have

\[ \frac{\sin e_1}{c_1} = \frac{\sin e_2}{c_2} = \ldots \ldots \ldots \ldots \ldots = \frac{\sin e_8}{c_8} = \frac{1}{c}, \]  

(28)

which corresponds to the Snell’s law and

\[ \xi_1 c_1 = \xi_2 c_2 = \ldots \ldots \ldots \ldots \ldots = \xi_8 c_8 = \omega, \]  

(29)

where $e_1 = e_5$, $e_2 = e_6$, $e_3 = e_7$, and $e_4 = e_8$, and $c_1 = c_5$, $c_2 = c_6$, $c_3 = c_7$, and $c_4 = c_8$.

Substituting the values of $u_i, u_3, \phi_2, T$ from (23) in boundary conditions (25)-(26) and with the help of equations (17) - (19), (28), and (29) yield

\[ \sum_{j=1}^{8} A_{ij} A_j = 0 \quad , \quad (i = 1, \ldots, 4), \]  

(30)

where

\[ A_{11} = -d_{18} F_1 + r d_5 F_2 - t_1 F_3 \quad (j = 1, \ldots, 4); \]  
\[ A_{12} = d_{15} F_2 + r d_5 F_2 - t_1 F_3 \quad (j = 1, \ldots, 4); \]  
\[ A_{13} = d_{15} F_2 + s_5 d_{17} \quad (j = 1, \ldots, 4); \]  
\[ A_{14} = -d_{18} F_1 + r d_5 F_2 - t_1 F_3 \quad (j = 5, \ldots, 8); \]  
\[ A_{21} = d_{15} F_2 + r d_5 F_2 - t_1 F_3 \quad (j = 1, \ldots, 4); \]  
\[ A_{22} = -d_{18} F_1 + r d_5 F_2 - t_1 F_3 \quad (j = 5, \ldots, 8); \]  
\[ A_{23} = -d_{15} s_5 F_2 \quad (j = 1, \ldots, 4); \]  
\[ A_{24} = -d_{18} F_1 + r d_5 F_2 - t_1 F_3 \quad (j = 5, \ldots, 8); \]  
\[ A_{31} = d_{15} F_2 + r d_5 F_2 - t_1 F_3 \quad (j = 1, \ldots, 4); \]  
\[ A_{32} = -d_{15} s_5 F_2 \quad (j = 1, \ldots, 4); \]  
\[ A_{33} = d_{15} F_2 + s_5 d_{17} \quad (j = 1, \ldots, 4); \]  
\[ A_{34} = -d_{18} F_1 + r d_5 F_2 - t_1 F_3 \quad (j = 5, \ldots, 8); \]  
\[ A_{41} = d_{15} F_2 + r d_5 F_2 - t_1 F_3 \quad (j = 1, \ldots, 4); \]  
\[ A_{42} = -d_{15} s_5 F_2 \quad (j = 1, \ldots, 4); \]  
\[ A_{43} = -d_{18} F_1 + r d_5 F_2 - t_1 F_3 \quad (j = 5, \ldots, 8); \]  
\[ A_{44} = -d_{18} F_1 + r d_5 F_2 - t_1 F_3 \quad (j = 5, \ldots, 8); \]

where $F_i = \frac{\sin e_j}{c_j}$, $F_2 = \frac{\cos e_j}{c_j}$, $F_3 = d_{13}(1 + \tau \omega)$.

For incidence of qLD wave $A_2 = A_3 = A_4 = 0$. We obtain a system of four non-homogeneous equations in four unknowns which on simplification yield

\[ Z_i = \frac{A_{i+4}}{A_i} = \frac{\Delta_i}{\Delta} \quad (i = 1, \ldots, 4), \]  

(31)
For incidence of qTD wave $A_1 = A_3 = A_4 = 0$ and we have

$$Z_i = \frac{A_i + A_4}{A_1} = \frac{\Delta_i^2}{\Delta} \quad (i = 1, \ldots, 4),$$

(32)

For incidence of qTM wave $A_1 = A_2 = A_4 = 0$ and we obtain

$$Z_i = \frac{A_i + A_4}{A_3} = \frac{\Delta_i^3}{\Delta} \quad (i = 1, \ldots, 4).$$

(33)

For incidence of qT wave $A_1 = A_2 = A_3 = 0$ and which yield

$$Z_i = \frac{A_i + A_4}{A_4} = \frac{\Delta_i^4}{\Delta} \quad (i = 1, \ldots, 4),$$

(34)

where $\Delta = |A_{i + 1}|_{4 \times 4}$ and $\Delta_i^p (i = 1, 2, \ldots, 4)$ ($p = 1, \ldots, 4$) are obtained by replacing, respectively, the 1st, 2nd, ..., 4th columns of $\Delta$ by $[-A_{1p}, -A_{2p}, -A_{3p}, -A_{4p}]^T$.

6. Numerical results and discussion

To illustrate the numerical results graphically, the value for relevant parameters for transversely isotropic micropolar viscothermoelastic medium are taken as

- $A_{11} = 1.78 \times 10^{10} \text{Nm}^{-2}$, $A_{33} = 1.843 \times 10^{10} \text{Nm}^{-2}$, $A_{55} = 4.357 \times 10^{10} \text{Nm}^{-2}$, $A_{66} = 4.42 \times 10^{10} \text{Nm}^{-2}$,
- $A_{13} = 7.59 \times 10^{10} \text{Nm}^{-2}$, $A_{66} = 4.357 \times 10^{10} \text{Nm}^{-2}$, $B_{77} = 2.63 \times 10^9 \text{N}$, $B_{66} = 5.648 \times 10^9 \text{N}$,
- $\rho = 1.74 \text{kg/m}^3$, $T_0 = 2.98 \text{K}$, $C^* = 1.04 \text{Cal/K}$, $K_1^* = 1.7 \text{Cal/K}$, $f = 0.02 \text{m}^2$, $\tau_0 = 0.4 \text{s}$, $\tau_1 = 0.8 \text{s}$.

For a particular model of heat conducting transversely isotropic micropolar viscothermoelastic solid half space we take $R_i = \chi(1-i\tau_i)$, where $\chi = A_{11}$, $A_{33}$, $A_{55}$, $A_{66}$, $B_{66}$, $B_{77}$ and $R_1 = 10.05$, $R_2 = 2.6$, $R_3 = 3.2$, $R_4 = 7.9$, $R_5 = 7.03$, $R_6 = 2.01$, $R_7 = 1.7$, $R_8 = 5.8$.

![Fig. 2. Amplitude Ratio $|Z_2|$ when qLD wave is incident.](image1)

![Fig. 3. Amplitude Ratio $|Z_3|$ when qLD wave is incident.](image2)
Figures 2-5 show the variation of amplitude ratio $|Z_i|$, $(i = 1, 2, 3, 4)$ with respect to frequency when qLD wave is incident. Figures 6-9 show the variation of amplitude ratio $|Z_i|$, $(i = 1, 2, 3, 4)$ with respect to frequency when qTM wave is incident. Figures 10-13 show the variation of amplitude ratio $|Z_i|$, $(i = 1, 2, 3, 4)$ with respect to frequency when qTD wave is incident.

Figures 13-17 show the variation of amplitude ratio $|Z_i|$, $(i = 1, 2, 3, 4)$ with respect to frequency when qT wave is incident. In all figures, effect of viscosity along with relaxation times has been shown for transversely isotropic micropolar thermoelastic half space and transversely isotropic micropolar viscothermoelastic half space. In case of L-S theory, the solid lines corresponds to transversely isotropic micropolar thermoelastic half space is MTIT(L-S) and dotted line corresponds to transversely isotropic micropolar thermoelastic half space is MTIT(L-S).
viscothermoelastic half space is MTIV(L-S). In case of G-L theory the solid lines with node corresponds to transversely isotropic micropolar thermoelastic half space is MTIT(G-L) and dotted line with node corresponds to transversely isotropic micropolar viscothermoelastic half space is MTIV(G-L).

When \( q_{LD} \) wave is incident. Figure 2 shows the variation of amplitude ratio \( |Z_i| \) with respect to frequency when \( q_{LD} \) wave is incident. The values of amplitude ratio \( |Z_i| \) is more for MTIT(L-S) as compared to MTIV(L-S) for \( 0 \leq \omega \leq 80 \) due to viscous effect and finally becomes dispersionless. The values of amplitude ratio \( |Z_i| \) increase monotonically for MTIT(G-L) in \( 0 \leq \omega \leq 10 \) and decrease gradually for \( 11 \leq \omega \leq 100 \), for MTIV(G-L), the values oscillate for the region \( 0 \leq \omega \leq 10 \) and as \( \omega \geq 10 \), the values increase.

In Figure 3, the value of amplitude ratio \( |Z_2| \) for MTIT(L-S) oscillates initially and then increase linearly, and for MTIV(L-S) the value increase slowly as \( \omega \) increases. For
MTIV(G-L), the value of $|Z_2|$ initially oscillates and become stationary as $\omega$ increases, whereas for MTIT(G-L) the value initially oscillates, increases instantaneously remain stationary as $\omega$ goes on increasing.

In Figure 4 the value of amplitude ratio $|Z_3|$ oscillates for MTIV(L-S) with initial values of $\omega$ and then remain stationary. The value of $|Z_3|$ for MTIT(L-S), MTIT(G-L), MTIV(G-L) increases monotonically as $\omega$ increases except for MTIT(L-S) case in which it has some stationary values at the particular value of $\omega$.

**Fig. 12.** Amplitude Ratio $|Z_3|$ when qTD wave is incident.

**Fig. 13.** Amplitude Ratio $|Z_4|$ when qTD wave is incident.

**Fig. 14.** Amplitude Ratio $|Z_1|$ when qT wave is incident.

**Fig. 15.** Amplitude Ratio $|Z_2|$ when qT wave is incident.

In Figure 5 the trend and behavior of variation MTIT(L-S), MTIV(L-S), MTIT(G-L) and MTIV(G-L) are same for all the values of $\omega$. The value of $|Z_4|$ for MTIV(L-S) are smaller in comparison to other cases.

**When qTM wave is incident.** In Figure 6 the amplitude ratio $|Z_1|$ oscillates for $0 \leq \omega \leq 10$ and attains peak values for MTIT(G-L) and instantly vanishes for $\omega \geq 25$. For
MTIV (L-S) and MTIV (G-L) the value of $|Z_1|$ oscillates for $0 \leq \omega \leq 10$ and vanishes for $\omega \geq 10$. Similar behavior is observed for MTIT(L-S) and MTIT(G-L) theories within $20 \leq \omega \leq 35$ range of frequency.

In Figure 7 the values of amplitude ratio $|Z_2|$ initially oscillates for $0 \leq \omega \leq 10$ and decreases as $\omega$ increases for MTIV (L-S) and MTIV (G-L). For MTIT(L-S) the value of amplitude ratio is maximum for $\omega \leq 20$ and in case of MTIT (G-L), its value slowly decreases and becomes constant near the boundary surface.

In Figure 8 the values of $|Z_3|$ oscillates in the interval $0 \leq \omega \leq 15$ for all the cases attaining the maximum value for MTIV(L-S) and as $\omega$ increase further the behavior of $|Z_3|$ is similar and all values of $|Z_3|$ decrease due to viscosity effect for both the theories.

In Figure 9 the values of amplitude ratio $|Z_4|$ near the boundary surface oscillates for both theories and attaining maxima for MTIV(G-L) theory and then decreases as frequency increases. The trend and behavior of $|Z_4|$ as $\omega \geq 30$ is similar as $|Z_3|$ for $\omega \geq 22$.

When qTD wave is incident. In Figure 10 the amplitude ratio $|Z_1|$ oscillates for all the cases when $0 \leq \omega \leq 20$ and then become stationary for $\omega \geq 20$. The value of $|Z_1|$ decreases due to viscosity effect.

Figure 11 shows the behavior and variation of $|Z_2|$ when qTD wave is incident as that of $|Z_1|$ when qTD wave is incident for $0 \leq \omega \leq 20$ and as $\omega \geq 20$, the values are stationary for all the cases, but the value of $|Z_2|$ for MTIT(L-S) is more than MTIV(L-S) and the value of $|Z_2|$ for MTIV(G-L) is less than that of MTIT(G-L) showing the viscosity effect.

In Figure 12 the value of amplitude ratio $|Z_3|$ is minimum for MTIV(G-L) as compared to MTIT(G-L) theory and the values of $|Z_3|$ attains maximum for MTIT(L-S) as compared to MTIV(L-S) theory, and then flatten out near the boundaries for both the cases.

In Figure 13 the trend and behavior is similar but the value of $|Z_4|$ is higher for MTIT(L-S) theory due to viscous effect.
When qT wave is incident. In Figure 14 the value of $|Z_1|$ for MTIV(L-S) and MTIV(G-L) are less in comparison to MTIT(L-S) and MTIT(G-L) due to viscosity effect for $0 \leq \omega \leq 10$ and as $\omega \geq 20$ the values are stationary for all the cases.

In Figure 15 the value of $|Z_2|$ is more for MTIV(L-S) and MTIV(G-L) as compared to MTIT(L-S) and MTIT(G-L) initially. Its value instantaneously increase and decrease as $\omega$ increases respectively. The values of MTIT(L-S) and MTIT(G-L) oscillates and become stationary as $\omega$ increases.

In Figure 16 the value of $|Z_3|$ for MTIV(L-S) and MTIV(G-L) is opposite to MTIT(L-S) and MTIT(G-L) initially. Also the value of $|Z_3|$ for MTIV(L-S) and MTIV(G-L) decreases as $\omega$ increases.

In Figure 17 the value of $|Z_4|$ for MTIV(L-S) and MTIV(G-L) are more than MTIT(L-S) and MTIT(G-L) initially, in the intermediate range of $\omega$ the value of $|Z_4|$ are small. As $\omega \geq 25$ the values of $|Z_4|$ for MTIV(L-S) and MTIV(G-L) lie between MTIT(L-S) and MTIT(G-L).

7. Conclusion
In the present investigation, reflections of plane waves in transversely isotropic micropolar thermoelastic solid have been discussed. The trend of variation and behavior of the amplitude ratio $|Z_1|$ is similar for incidence of qLD, qTD, qTM and qT with change in their magnitude values. The value of amplitude ratio $|Z_1|$ for MTIT(L-S) remains more in comparison with MTIT(G-L) for incidence of qTD, qTM and qT, except for qLD, where reverse behavior occurs for the value of $|Z_1|$.

The behavior of $|Z_4|$ is different for all the incident waves, however, the value of amplitude ratio $|Z_4|$ is more in case of G-L theory due to viscosity effect, for the incident of qLD and qTM waves, but for the remaining incident waves the value of amplitude ratio $|Z_4|$ is more for L-S theory.

From the present investigation, it is concluded, that the values of amplitude ratios shows sharp oscillations at initial frequency for incident qT, qTD, qTM as compared to qLD waves. An appreciable effect of viscosity and relaxation time are noticed on amplitude ratios of various reflected waves.

References