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THEORY OF PLASTICITY WITHOUT SURFACE OF LOADING

V.S. Bondar

Moscow State University of Mechanical Engineering (MAMI),

Bolshaya Semenovskaya str. 38, Moscow, Russia Federation

e-mail: tm@mami.ru

Abstract. On the basis of hysteresis loop analysis (cyclic diagram) there are distinguished three parts, which characterize different behavior of stresses, that is distinguished three types of stresses. For each type of stresses there are formulated corresponding evolutionary equations for anisotropic hardening. For description isotropic hardening the evolutionary equation with parameter of saturation for second type stresses is introduced. The deviator of stresses is determined as sum of three types of stresses. For description of the non-linear process of accumulation damages the kinetic equation is introduced, basing on energetic principle, where in quality of energy, which expending on creating of damages in material there is energy equal work of second type stresses on full deformation field. The material functions, which completed the theory and subjected to experimental determining, are distinguished.

1. Introduction

Mathematical modeling of deformation processes with arbitrary complex cyclic loadings is constructed mainly on variants of theory of plastic flow with combined hardening for which the survey and analysis is reduced in [1-6]. The fundamental problem of constructing this variants are in formulating sufficiently adequate evolutionary equations for translating of loading surface center and also in non-onesignificance in experimental determining of the loading surface boundary. In the theory of plastic flow deformation is separated on elastic and plastic deformations. For determining the plastic deformation there is used associated with loading surface law of flow (gradiental law of plastic flow).

In present work on base of analysis of experimental results for cyclic loading, that is loop of cyclic hysteresis (cyclic diagram), there are three type stresses, for which is formulated appropriate evolutionary equations. Choosing three types of stresses answers to anisotropic hardening. For describing isotropic hardening there is introduced evolutionary equation for parameter of saturation for second type stresses. Deviator of full stresses is determined as the sum of three types stresses.

For describing non-linear processes of damage accumulation the kinetic equation of damage accumulation is introduced [1-4, 7], where as energy expending on setting up damages in material, the energy, equal work stresses second type on field of full deformations, is taken in.

The main feature of proposed variant of plastic flow theory is that deformation is common and not separates on elastic and plastic and also there is not used the conception of loading surface and correspondingly associative law of flow.

The material functions, which complete the variant of theory and which should be determined with experiments is distinguished.

2. Fundamental states and equations

For constructing the correlation of theory of plasticity the stabilize loop of hysteresis (Fig. 1) is considered in deviator components $3s_{11}/2$ and e_{11} for uniaxial extension – compression. Then on half cycle from point of change of deformation direction there is introduced the coordinate system: stress σ, deformation ε. The curve in coordinate σ, ε is included in himself elastic and plastic deformation without separating this deformations. Then the derivative $d\sigma/d\varepsilon$ is calculated and curve in coordinate $d\sigma/d\varepsilon$ and σ is constructed (Fig. 2).

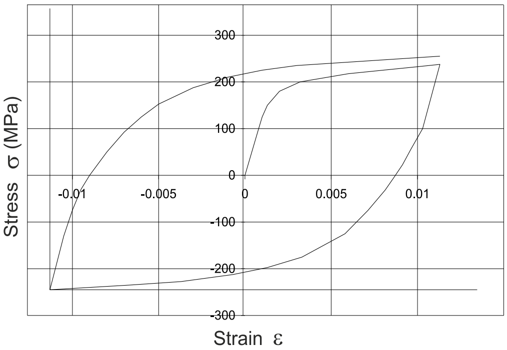

Fig. 1. Stabilize hysteresis loop.

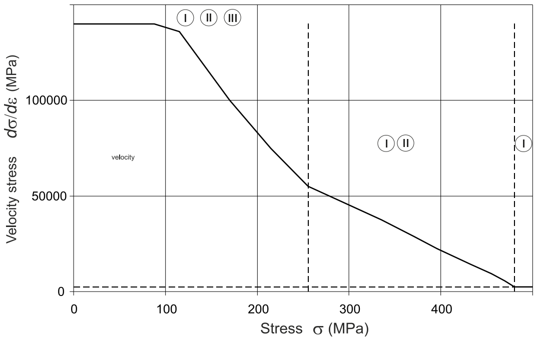

Fig. 2. The curve in coordinate $d\sigma/d\varepsilon$ and σ.

On the received curve may be to choose three parts, which are characterizing different behavior of stresses. On the first part the derivative has practically constant meaning and here for first type stress the evolutionary equation, similar equation Ishlinskii-Prager [8, 9] for backstresses of the first type [3, 7] is proposed:

$$\dot{s}_{ij}^{(1)} = \frac{2}{3} g^{(1)} \dot{e}_{ij}. \tag{1}$$

On the second part the derivative is changing for linear law and here for second type stresses evolutionary equation, similar equation Armstrong – Frederick – Kadachevich [10, 11] for backstresses of the second type [3, 7] is proposed:

$$\dot{s}_{ij}^{(2)} = \frac{2}{3} g^{(2)} \dot{e}_{ij} + g_s^{(2)} s_{ij}^{(2)} \dot{\varepsilon}_{u*}, \quad \left(\dot{\varepsilon}_{u*} = \left(\frac{2}{3} \dot{e}_{ij}\, \dot{e}_{ij} \right)^{1/2} \right). \tag{2}$$

Then on the third part the derivative is changed for non-linear law, which may be described with series evolutionary equations type equations Ohno – Wang [3, 7, 12] for backstresses of the third type [3, 7]

$$\dot{s}_{ij}^{(m)} = \frac{2}{3} g^{(m)} \dot{e}_{ij} \qquad (m = 3, \ldots, M). \tag{3}$$

The defining functions $g^{(1)}, g^{(2)}, g_s^{(2)}, g^{(m)}$, which is a member equations (1) – (3) is expressed through material functions in the following way [1-3, 7]:

$$g^{(1)} = E_s, g^{(2)} = \beta \sigma_s, \ g_s^{(2)} = -\beta, \tag{4}$$

$$g^{(m)} = \begin{cases} \beta^{(m)}\, \sigma_s^{(m)} \\ 0, \quad \text{если} \quad \sigma_и^{(\mathrm{m})} \geq \sigma_s^{(m)} \cap s_{ij}^{(m)} \dot{e}_{ij} > 0, \end{cases} \quad \left(\sigma_u^{(m)} = \left(\frac{3}{2} s_{ij}^{(m)} s_{ij}^{(m)} \right)^{1/2}, \ m = 3, \ldots M \right). \tag{5}$$

For description isotropic hardening and unhardening the dependence of saturation parameter σ for stresses second type from accumulated deformation ε_{u*} is introduced:

$$\sigma_s = \sigma_s\left(\varepsilon_{u*}\right) \quad \left(\varepsilon_{u*} = \int \dot{\varepsilon}_{u*} dt\right). \tag{6}$$

Here E_s, $\sigma_s\left(\varepsilon_{u*}\right)$, β, $\sigma_s^{(m)}, \beta^{(m)}$ - material functions, which is defined from experiment.

The deviator of stresses analogously model Novozhilov – Chaboche [13, 14] for backstresses is determined as sum of stresses of three types, for which there is own evolutionary equation:

$$s_{ij} = \sum_{m=1}^{M} s_{ij}^{(m)} . \tag{7}$$

Finally the equation for deviator of stresses with account (1-3) and (7) will be follow

$$\dot{s}_{ij} = \frac{2}{3} g \dot{e}_{ij} + g_s^{(2)} s_{ij}^{(2)} \dot{\varepsilon}_{u*} , \quad g = \sum_{m=1}^{M} g^{(m)} . \tag{8}$$

To equation (8) should be added the equation, which connecting sphere compositions of stress tensors $\sigma_0 = \sigma_{ii} / 3$ and deformations $\varepsilon_0 = \varepsilon_{ii} / 3$:

$$\sigma_0 = 3K\varepsilon_0 . \tag{9}$$

For description non-linear processes of damage accumulation the kinetic equation of damage accumulation, which based on energy principle, where in the capacity of energy, expended on making damages in material, the energy, equal the work of stresses second type on deformation field is introduced. The kinetic equation is introduced on the analogy of kinetic equation [7], basing on work of backstresses second type on field of plastic deformations. This kinetic equation of damage accumulation will have following view

$$\dot{\omega} = \alpha\, \omega^{\frac{\alpha-1}{\alpha}} \frac{s_{ij}^{(2)} \dot{e}_{ij}}{W_s} , \ \alpha = \left(\sigma_s / \sigma_u^{(2)}\right)^{n_\alpha} . \tag{10}$$

Here ω – measure of damage; W_s – failure energy; α and n_α - function and parameter of non-linearity for process of damage accumulation; $\sigma_u^{(2)}$ – intensity of second type stresses. Material function W_s and n_α must be determined with help of experiments.

Received variant theory of plasticity in distinct from other theory variants of plastic flow, has following peculiarities:

- the deformation is not separated on elastic and plastic;
- there is not necessity in introduction the surface of loading (yield points);
- there is not used associate law of plastic flow;
- there is not necessity in introduction the conditions for loading or unloading (the conditions elastic or elasto-plastic states);
- there is calculated the second type stresses, coinciding with second type backstresses, which is answered for process of damage accumulation.

For description of phenomen of ratcheting with unsymmetrical soft cyclic loadings the parameter E_s, entering in first evolutionary equation for first type stresses there is setting as depending on accumulate deformation ε_{u*} [3, 7].

In case of additional isotopic hardening with disproportional cyclic loading the parameter of saturation for second type stresses is assumed depending from parameter of load disproportionality [1–3].

Calculate – experimental method of determining material function is stated in work [7].

3. Conclusions

There are formulated basic states and equations for variant of plasticity theory without separating of deformation on elastic and plastic, without introduction of loading surface and associated law of plastic flow.

On basis of that, what the work of second type backstresses on field full deformations, as and work second type backstresses on field of plastic deformations, is universal characteristic of material failure for cyclic loading, there is formulated kinetic equation for non-linear accumulation damage processes.

There are chosen the material functions, which conclude the theory and which is founded with sufficiently simple base experiment and with method identification of material functions.

The variant of theory may be developed for description of phenomenon of ratcheting and for effect of additional isotropic hardening with non-proportional cyclic loadings.

The first calculated investigations of processes proportional and disproportional cyclic loadings have shown their reliable corresponding with experimental results.

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