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THE INFLUENCE OF STRESS DISTRIBUTION ON STABILITY OF THE DISPLACEMENT OF CONICAL INDENTER INTO THE SOIL MEDIUM

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Abstract. Plane-parallel displacement of conical bodies in an elastoplastic medium on the basis of the hypothesis of locality is simulated. The parameters of the quadratic velocity model of local interaction are determined by solving the one-dimensional problem of the expansion of a spherical cavity. It is shown that the radial and angular velocity dependent on the stress distribution along the cone at oblique impact. We analyze the behavior of the angular velocity of rotation in time at different positions the center of gravity of a sharp cone.

1. Introduction

Solution of problems of impact and penetration at an angle to the free surface is carried out mainly numerical and analytical methods [1-3], among which a large enough class is based on the hypothesis of local interaction [4]. The results of calculations of strength and kinematic characteristics of the interaction of conical strikers with the soil at oblique impact within the model of local interaction (LIM) and three-dimensional formulation are analyzed [5-9]. Quite good qualitative and quantitative agreement between the results match the axial force components are showed [10, 11]. However, the error in determining the angular rotation velocity obtained in this model is small only in the initial stage of penetration [11]. The soil error increases significantly with the development of penetration after full immersion head of the cone into soil. In this paper the influence of the stress distribution along the lateral surface of the conical striker on the stability of displacement in a resistant medium is analyzed.

2. Formulation of the problem of motion of the body

The equations of motion and rotation of a plane figure around the center of mass C projections on the axes of the coordinate system associated with the body have the form [10, 11]

$$M\left(\dot{v}_{Cr'}-\omega v_{Cz'}\right)=F_{r'},\ M\left(\dot{v}_{Cz'}+\omega v_{Cr'}\right)=F_{z'},\ J_C\dot{\omega}=K, \tag{1}$$

where ($v_{Cr'}$, $v_{Cz'}$) and ($F_{r'}$, $F_{z'}$) - the projection of vectors $\boldsymbol{v}_C$ and F on the axis of moving coordinate system r' и z', ω - the projection of the angular velocity ω on the axis perpendicular to the plane of motion ($\omega_{r'}=\omega_{z'}=0$) and passing through the center of mass; dot denotes differentiation with respect to time.

In accordance with one implementation LIM assumed that the normal stress acting on the lateral surface of a cone, can be represented as a quadratic function of the speed, shear is defined by Coulomb's law:

$$\sigma_n / \rho_0 = -H\left(v_n\right)\left(Av_n^2 + Bv_n + C\right), \quad \sigma_\tau = -\operatorname{sgn} v_\tau k_f \left|\sigma_n\right|, \tag{2}$$

where H - the Heaviside function, A, B and C - constant coefficients depending on the physical and mechanical properties of the medium, the velocity of the striker and the other components, ρ_0 - the initial density of the medium, k_f - the coefficient of friction. If $v_n < 0$, than $\sigma_n = 0$, which corresponds to the separation of the medium from the body. Coefficients of LIM A, B and C (2) in accordance with the known approach (spherical cavity expansion approximation), are determined by solving the problem of the expansion of a spherical cavity in a given range of speeds [12, 13].

For the model (2) after the moment of full immersion of the body and if it is flow without separation of the motion for the components of the vector of resistance force and moment of forces are recorded [10] as follows:

$$F_{r'}/S_0 = v_{Cr'} f_{r'}^0 + \omega f_{r'}^\omega, \qquad F_{z'}/S_0 = f_{z'}^0 + \omega f_{z'}^\omega, \quad K/S_0 = v_{Cr'} k^0 + \omega k^\omega,$$

$$f_{r'}^0 = \cos^2\beta\gamma_1, \; f_{r'}^\omega = \left(\tfrac{1}{3}H - H\sin^2\beta + z_C\cos^2\beta\right)\gamma_1,$$

$$f_{z'}^0 = \left(C - Bv_{Cz'}\sin\beta + Av_{Cz'}^2\sin^2\beta + \tfrac{1}{2}Av_{Cr'}^2\cos^2\beta\right), \tag{3}$$

$$f_{z'}^\omega = A\left(\tfrac{1}{2}\gamma_2 H^2\omega + \left(\tfrac{1}{3} - \sin^2\beta\right)\left(\omega z_C + v_{r'}\right)H + \cos^2\beta\left(\tfrac{1}{2}\omega z_C + v_{Cr'}\right)z_C\right),$$

$$k^0 = \left(-\tfrac{2}{3}H + (z_C + H)\cos^2\beta\right)\gamma_1,$$

$$k^\omega = \left(\gamma_2 H^2 - \tfrac{4}{3}z_C H + 2z_C H\cos^2\beta + z_C^2\cos^2\beta\right)\gamma_1,$$

where indicated: $\gamma_1 = Av_{Cz'} - \tfrac{1}{2}B/\sin\beta$, $\gamma_2 = \tfrac{1}{2}\sin^2\beta\, tg^2\beta - \tfrac{1}{3}\sin^2\beta + \tfrac{1}{6}\cos^2\beta$, $S_0 = \pi R^2$ - base area of the body.

The system of equations of plane of motion rigid body inertia (1) - (3) is performed [10, 11] by methods of Runge-Kutta fourth order for given initial conditions.

3. The equations of motion in the fixed axial velocity

The system of equations (1) - (3) at the developed stage of penetration and fixed axial velocity is reduced to a system of two linear ordinary differential equations respectively to ω and $v_{Cr'}$. When $\beta = \pi/6$, we will have:

$$\dot{\omega} = a_{11}\omega + a_{12}v_{Cr'}, \qquad \dot{v}_{Cr'} = a_{21}\omega + a_{22}v_{Cr'}, \quad a_{11} = \frac{3}{2}\left(\frac{3}{2}\mu^2 - \frac{1}{3}\mu + \frac{1}{6}\right)\frac{\gamma_1}{J_C},$$

$$a_{12} = \frac{\sqrt{3}}{4}\left(\frac{1}{3} - 3\mu\right)\frac{\gamma_1}{J_C}, \qquad a_{21} = \frac{\sqrt{3}}{4}\left(\frac{1}{3} - 3\mu\right)\frac{\gamma_1}{M} + v_{Cz'}, \; a_{22} = \frac{3}{4}\frac{\gamma_1}{M}, \tag{4}$$

where adopted: $z_C = -\mu H$, $\gamma_1 = Av_{Cz'} - B$.

The Cauchy problem for equations (4) with zero initial conditions has an analytical solution that allows you to determine the dependence of the angular velocity of the parameters of the problem. The characteristic equation of the system of equations (4) when $\mu < 1/9$ has a complex conjugate roots, the solution is oscillatory, but the amplitude of oscillation decays in some time, as the movement of the cone is stable [7]. When $\mu = 1/9$ the coefficient turns to zero $a_{12} = 0$, the roots of the characteristic equation are real and distinct, the solution decreases in absolute value. For large values μ of the criterion of stability of motion when fixed axial velocity [7] is violated. At a variation the coefficients A and B of LIM (2) deteriorating give the corresponding axial force resistance in the problem of normal impact. Thus, the angular velocity of rotation is mainly determined by the position of the center of mass of the body, fixed in the above problem of oblique penetration (it is assumed that a isotropic cone and $\mu = 1/4$).

For a more reliable determination of the angular speed of rotation of the conical striker model (2) is modified. Since the assumed dependence of the normal stress on the speed along the side surface of a body of revolution, the resistance force of the circular cone with a constant velocity when $\omega = 0$ is equal

$$F_z = 2\pi\, tg^2\beta \int_{-H}^{0} \sigma_n \left(H + z\right) dz = \sigma_n S_0 ,$$

when σ_n – normal stress, constant along the lateral surface of the cone and is determined only by the speed of its movement and coefficients *A*, *B* и *C*.

However, the same value of the axial force resistance is obtained by different from the constant stress distribution, for example:

$$F_z = 2\pi\, tg^2\beta \int_{-H}^{0} \hat{\sigma}_n \left(H + z\right) dz = \sigma_n S_0 ,\ \hat{\sigma}_n = \left(\left(1+\delta\right) - 2\delta \left(1 + \frac{z}{H}\right)^2 \right) \sigma_n . \qquad (5)$$

Thus, δ is an additional parameter influencing angular velocity, which is the solution of equations (4), taking into account the modifications of LIM (5), with almost the same values of the axial force resistance. As shown in [13], the stress distribution along the lateral surface of the sharp cone, qualitatively and quantitatively close to (5).

4. Results of numerical calculations

We consider the problem of penetration of inertia with an initial velocity V_0 =150 m/s of the conical striker with a half-angle $\beta = \pi / 6$ and the mass *M*=40 g at an angle $\theta = \pi / 3$ to the surface of the medium. The radius of the base of the cone *R*=0.01 м, height $H = R/tg\beta$, the coordinates of the center of mass $r_C = 0$, $z_C = -H/4$ ($z_C = -\mu H$, $\mu = 1/4$), the moment of inertia $J_c = \frac{3}{80} M \left(4R^2 + H^2\right)$.

Only the axial component of the velocity vector $V_z = -V_0 = -150$ m/s is nonzero at the initial time. The values of the coefficients of the three-member LIM (7) equals: *A*=1.2, *B*=0.95V_0 and *C*= 0.034V_0^2 [10, 11], the coefficient of friction k_f =0.

Figure 1 summarizes the results of comparing LIM (2) and its modifications (5) for δ =0.5. We analyze the behavior of the angular velocity of rotation in time at different positions the center of mass of a cone with half-angle $\beta = \pi / 6$ penetrating at an angle $\theta = \pi / 3$ to the free surface of the soil.

Effect of δ >0 on the angular velocity is close to the effects of displacement of the center of mass in the region of instability of motion. Thus, the characteristic equation of the system (4) has different real roots with $\mu = 1/5$ in the modified model (5) with δ =0.5, while in the original model (2) has different real roots with $\mu = 1/9$. Similarly, the boundary violation criterion for the stability is shifting with frozen axial velocity [7] from the value $z_C = -0.064H$ in LIM (2) to $z_C = -0.15H$ from modified model (5).

5. Conclusion

Thus, the first obtained results indicate that the change of the distribution of stresses along the generatrix of the cone must be considered when analyzing the stability of displacement of extended bodies in soil medium. To answer the question about the nature of the distribution of normal stresses along the contact surface requires further study.

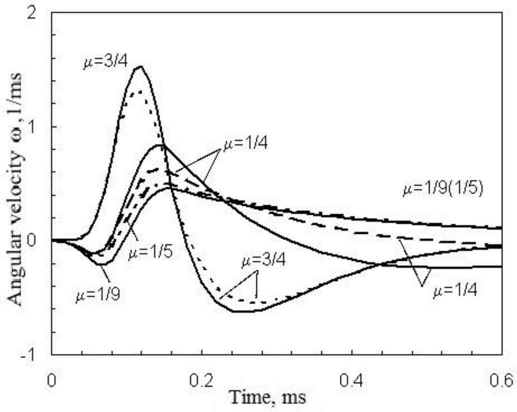

Fig. 1. Rotation angular velocity obtained at different positions of the center of mass ($\mu = -z_C / H$): solid curves correspond to LIM (2) at constant pressure, bar - its modifications (5), taking into account the linear contact pressure distribution.

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