THE NUMERICAL INVESTIGATIONS OF DOUBLE-SPAN
CONCRETE BEAMS STRENGTHENED WITH FIBER REINFORCED
PLASTICS ACROSS THE OBLIQUE SECTION

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Abstract. We suggest a method for finite-element modelling of beams strengthened with
fiber reinforced plastics (carbon fiber-reinforced plastics), which allows analyzing the
mechanism of cracks formation and forecasting the failure mode when planning physical
experiments. We have represented the comparison of schemes of crack formation obtained as
a result of numerical modelling.

1. Introduction
Technology of strengthening of concrete structures with fiber reinforced plastics (FRP) is
relatively new for Russia. The first large-scale project, strengthening of the scaffold bridge of
the third ring road in Moscow, dates back to 2001. Since then the technology has been
successfully applied in hundreds sites in the territory of Russia. Strengthening of oblique
sections is implemented by means of gluing the external FRP confinement reinforcement to
the zone of a dangerous oblique section of U-shaped stirrups. Russian methods of assessment
of stress-strain state of multi-span structures FRP strengthened across the oblique section are
currently being developed, while the foreign methods established by the regulations, ensure
(according to their authors) significant strength reserve and have been tested only on single-
span specimens.

2. Setting of the problem
For modeling of operation of a multi-span reinforced concrete structure strengthened with
polymer material across the oblique section, we have conducted a numerical experiment in the
programming and computing suite of the Ansys final element analysis. 9 double-span
reinforced concrete beams with 2460 mm in length and 120x220 (h) mm in section were
modeled in the Ansys Mechanical programming module (Fig. 2). The beams have the same
section spans (300 mm), but different confinement reinforcement rod spacing in the zone of
center support (without confinement reinforcement, rod spacing 85 mm, rod spacing
110 mm). Each beam was modeled in three variants – with strengthening at angle 90° to the
centerline, with strengthening at angle α (in the direction of the principal tensile stress in
unreinforced specimen) and without strengthening. The strengthening was specified in the
section span by U-shaped stirrups made from unidirectional carbon of cold (on-site)
solidification. The stirrups were set with the width of 50 mm and the pitch of 100 mm. The
scheme of specimens strengthening is shown in Fig. 1.

For modeling within the numerical experiment of the stress-strain state, the reinforced
concrete beam and composite material of strengthening were divided into volumetric (for
concrete modeling), flat (composite material) and axial elements (for modeling of
The size of the end element is adopted in accordance with the recommendations of the developer of the programming unit for such models and is equal to 10x10x10 mm.

The concrete was modeled with the use of the solid65 solid finite elements. As a criterion of strength, we chose the model of Willam-Warnke [1], which is the development of the tree invariant strength criterion by Geniev [2].

At the first stage of modeling, we used the diagrams of nonlinear strain of materials in accordance with the regulations: trilinear diagram for concrete and Prandtl diagram for steel in compliance with the design and construction specifications SP 63.13330.2012 [3]. Then we tested the reference specimens: concrete prisms for compression according to GOST 10180-2012 [4] and the samples of carbon fiber-reinforced plastic for tensile properties according to GOST 25.601-80 [5]. Real strain diagrams, obtained in the process of physical testing, were used for conducting the second stage of the numerical experiments. Then we tested the reference (unreinforced) specimens of beams and elaborated the concrete operating coefficients, particularly the coefficients of load transition through cracks and blind drains, level of stress and cracks formation.

Due to the complexity of defining the actual width of the crack opening using the methods of the finite element modeling for provision of contiguity of the finite element grid we used the method of “blurring” the crack zone to the finite elements group [6]. That’s why in the applied diagram of the material state after achieving the ultimate stress or strain by the material it is necessary to add the conventional recessive branch (Fig. 2). Thus, crack in the concrete is modeled through the decline in the stress-strain modulus value at a certain level of loads. Exactly this section of the diagram models the distribution of the cracks in a certain group of the finite elements thus providing the opportunity to assess the cracks formation peculiarities of the specimen.

Specifications of FRP were introduced for both the entire reinforcement system (carbon fiber-reinforced plastic filament together with the binding filament), accepted at the first stage according to the Manufacturer's Data Report and then according to the specimens of physical testing. The behavior of reinforcement material is provided only at the elastic stage.

For the calculation we used implied solver of differential equations. Search for solution of the nonlinear problem was implemented by iterative method of tangents (Newton–Raphson method) with automatic optimization of the approximation interval in the process of solution.
For integration of the solver into the Ansys we implement the method of conventional conjugate gradients. The convergence control was implemented by the load to the accuracy of 10 %.

3. Research Results
As a result of the numerical experiment we obtained the principal stress isofields and the corresponding strains. In Fig. 3 there are the depictions of oblique cracks in physical specimens and the strain isofields, which correspond to the principal tensile stress in the model fragment, created using Ansys.

![Image](image1.jpg)

![Image](image2.jpg)

![Image](image3.jpg)

**Fig. 3.** Shape of the critical oblique crack in the physical specimen and in the finite element model (unreinforced beam): a) the beam without confinement reinforcement, b) the beam with rod spacing equal to 110 mm, c) the beam with rod spacing equal to 85 mm.

As it is seen in Fig. 3, intensification of the confinement reinforcement increases the number of oblique cracks and the width of their opening decreases, which is generally agreed. Comparison of the strain isofields in the Ansys model fragment with depictions of the physical specimens shows that shape and number of oblique cracks satisfactorily coincide.

In the reinforced specimens the oblique crack first opens between the elements of reinforcement, which is confirmed by both the physical experiments and the finite elements models. The crack trajectory and angle are satisfactorily proven by conducting the physical experiments. Alongside with the maximum strains in the zone of the oblique crack formation, the peaks have also been discovered at the upper side of beam in the zone of maximum normal tensile stresses (Fig. 4). During the physical experiments, the destruction of the beams was accompanied by the indent of the concrete cones at the upper side of the beam.

4. Conclusion
The conducted experiments showed high efficiency of the Ansys programming and computing suite for analyzing the stress-strain state of structures considering nonlinear character of the materials strain. The results of the testing of the unreinforced beams and the material specimens have contributed to elaboration of the finite elements models and forecasting
expensive testing procedures of the reinforced specimens. The preliminary modeling of the reinforced specimens allowed avoiding some mistakes during the physical testing.

Fig. 4. Shape of the critical oblique crack in the physical specimen and in the finite element model (reinforced beam): a) the beam without confinement reinforcement strengthened by the carbon fiber-reinforced plastic stirrups with 50 mm in width, rod spacing equal to 110 mm at angle 90°, b) the beam with the confinement reinforcement rod spacing equal to 110 mm and carbon fiber-reinforced plastic stirrups reinforcement with 50 mm in width and rod spacing equal to 100 mm at angle 90°, c) the beam with the confinement reinforcement rod spacing equal to 110 mm and carbon fiber-reinforced plastic stirrups reinforcement with 50 mm in width and rod spacing equal to 100 mm at angle α.

References