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MECHANOCHEMICAL GROWTH OF AN ELLIPTICAL HOLE UNDER NORMAL PRESSURE

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Abstract. An approximate analytical benchmark for the problem of nonuniform mechanochemical wear of an infinite plane with an elliptical hole under internal normal pressure is presented. The solution is developed for the cases when the minor axis of the ellipse is greater than the half of the major one. The rate of material dissolution is supposed to be linearly dependent on the maximum principal stress at a corresponding point on the surface of the hole.

1. Introduction

The annual loss due to corrosion is estimated at about 3–6 per cent of the Gross Domestic Product of developed nations. Corrosion is defined as the degradation of a material or its properties due to an interaction with its environment. Corrosion can cause expensive and extremely dangerous damage of constructions from deep-water facilities to aircraft fuselages. The combined action of mechanical loads and chemically active media may initiate the process of stress corrosion, which is more severe than the simple superposition of damages induced by stresses and electrochemical corrosion acting separately [1-5]. With regard to general corrosion facilitated by stress, E.M. Gutman introduced the term “mechanochemical corrosion” [3].

There exists a number of different approaches to the problems of stress corrosion, based on physical and chemical mechanics of materials, thermodynamics, continuum mechanics, and fracture mechanics. E.M. Gutman proposed an exponential dependence of the rate of the anodic dissolution of deformed metal on the stress value [3]. On the basis of his theory several elegant mathematical models of the corrosion of pipe elements were developed [6-7]. The theory of the mechanochemical effect of dissolution in terms of the chemical affinity was developed by A.I. Rusanov. According to his work [4], dissolution rate is a quadratic function of strain-components. The case of dissolution (evaporation) of a bent plate was examined in details theoretically and experimentally; the effect of the strain sign related to the existence of surface tension was described [4]. Based on the concept of chemical affinity tensor, the authors of [8-10] studied the kinetics of the stress-assisted chemical reaction sustained by the diffusion of gas through an elastic solid. Some problems were solved there analytically and numerically. R.A. Arutyunan and A.R. Arutyunan studied the long-term durability of brittle materials defined by the development of corrosion cracks. The corrosion crack growth rate was supposed to be a power function of the stress intensity factor [11].

A lot of experimental data demonstrate a linear dependence of the metal corrosion rate on the effective stress [2]. Beginning with the pioneering work [12], this dependence is often used for engineering calculations. When corrosion rates depend on stresses, and stresses, in turn, depend on changing (due to corrosion) geometry of an element, one has to solve an initial boundary value problem with unknown boundaries. Such problems are mostly studied by numerical methods. However, several analytical solutions have been found for the uniform mechanochemical dissolution of structural elements, e.g. by the authors of [4, 6, 12-14].

Solutions presented in [15-21] for the stress corrosion of elastic or elastic–plastic thick-walled cylinders and spheres are applicable to the problems of corrosion of a large enough solid with a small cylindrical or spherical pore under internal pressure. However, those solutions do not allow to simulate the change in the shape of the pore. In the framework of the theory involved, the pore remains circular during the corrosion process. Nevertheless, the results presented below demonstrate that a nearly circular hole can grow non-uniformly under uniform normal pressure.

2. Problem formulation

Consider the first fundamental problem for a linearly elastic, isotropic infinite plane S with an elliptical hole bounded by the contour L with the semi-axes A and B $(A \geq B)$. The edge of the hole is supposed to be subjected to uniform normal pressure p and exposed to mechanochemical corrosion defined as a material dissolution. Stresses in S vanish at infinity. In this case the hole, described by the changing contour L, grows with time t.

Let A_0 and B_0 be the semi-axes of the ellipse L at the initial moment t_0. According to [2, 12], the rate v of corrosion is a linear function of the maximum principal stress $\sigma(s)$ at corresponding points along the surface:

$$v(s) = \frac{\mathrm{d}\delta(s)}{\mathrm{d}t} = a + m\sigma(s), \quad s \in L(t), \tag{1}$$

where a and m are empirically determined constants of corrosion kinetics; $\mathrm{d}\delta(s)$ is an increment (due to material dissolution) of the hole size in the direction of the normal to its edge at the point s. Moreover, $a = v_0 - m\sigma^{th}$, where v_0 is the rate of dissolution of unstressed metal and σ^{th} is the stress corrosion threshold. Note that in general, the constants a and m (i.e. σ^{th} and m) are different for positive and negative normal stresses.

It is required to study the possibility of constructing a closed-form expression for the change in the configuration of the hole with time.

3. Problem solution

Stress distribution on the elliptical contour L in the plane S under uniform normal pressure p applied to the hole surface have been found in [22] in terms of complex potentials by the use of the transformation of the region S on to the infinite plane with a circular hole, $|\varsigma| > 1$. The relevant transformation is

$$z = R\left(\varsigma + \tfrac{M}{\varsigma}\right), \; R > 0, \; 0 \leq M < 1, \tag{2}$$

where $z = x+iy$ and $\varsigma = \rho e^{i\theta}$. The ellipse L (with the centre at the origin of the coordinate system $0xy$) is then mapped on to the circle $|\varsigma| = 1$, so that $A = R\,(1+M)$ and $B = R\,(1–M)$.

Corresponding stress-components on the contour $|\varsigma| = \rho = 1$ can be found as

$$\sigma_{\theta\theta} = 4Mp\,\frac{\cos 2\theta - M}{1 - 2M\cos 2\theta + M^2} + p. \tag{3}$$

It can be seen from (3) that when $A>2B$, the stress $\sigma_{\theta\theta}$ is negative at the certain part of the hole edge where the curvature is small enough. In such situations, the corrosion behaviour of the plane with the hole can be quite different, depending on the values of the corrosion kinetics constants σ^{th} and m for tension and compression. It should be noted that in the case of plane stress, local instability may occur where the stresses are negative [23].

To avoid misunderstandings, we shall develop an analytical solution for the cases when

A<2B and corrosion dissolution is accelerated only by positive normal stresses (i.e. when the stress corrosion threshold for compression is greater than *p* in absolute value, while the threshold stress for tension is zero). In this case the effective stress is $\sigma(s)=\sigma_{\theta\theta}$.

Let us assume that the hole remains almost elliptical during the corrosion process. Then, equations (2) and (3) should hold true at any *t* for *A* and *B* (and consequently, *R* and *M*) varying with time. Therefore, we have to solve simultaneous equations (1)–(3), where the values of $\sigma(s)=\sigma_{\theta\theta}$, *A*, and *B* change synergetically.

Solution of this problem can be expressed in an implicit form through a new variable $\eta = A/B$:

$$t = t_0 - \frac{B_0}{a-3mp}\left(\frac{(\eta_0-1)^{a+mp}}{\eta_0^{2mp}}\right)^{\frac{1}{a-3mp}} \int_{\eta_0}^{\eta}\left(\frac{\eta^{2mp}}{(\eta-1)^{2a-2mp}}\right)^{\frac{1}{a-3mp}} d\eta, \tag{4}$$

where $\eta_0 = A_0/B_0$. Recall that this formula holds true while $\eta \le 2$.

It can be proved that the ratio η increases at *a<3mp* and decreases otherwise. Therefore, formula (4) can be used without additional checks if $A_0/B_0 \le 2$ and *a>3mp*. In this case, the ratio η runs from A_0/B_0 to 1 as time *t* tends to infinity.

Equation (4) gives a point-to-point correspondence between *t* and η. For every $\eta \le 2$ we can then find

$$B = B_0\left(\frac{\eta^{2mp}(\eta_0-1)^{a+mp}}{\eta_0^{2mp}(\eta-1)^{a+mp}}\right)^{\frac{1}{a-3mp}}; \qquad A = \eta B. \tag{5}$$

Thus, we obtain a one-to-one relationship, defined by (4)–(5), between *t*, *A*, and *B* within the specified restrictions.

If $A_0 = B_0 = R_0$, then the shape of the hole remains circular during the corrosion process and its radius *R* grows with the constant rate

$$\frac{\mathrm{d}R}{\mathrm{d}t} = a + mp$$

for any values R_0, a, m, and *p*. Therefore, $R = R_0 + (a+mp)t$.

As one can see, during the process of mechanochemical wear, the stress concentration factor near the elliptical hole increases at *a<3mp* and decreases otherwise; and it remains constant for *A*=*B* anyway. Add that in the case of 3-D problem [24-26] or for nonlinear material [27] corrosion behaviour in the vicinity of an elliptic hole can be quite different. In the presence of numerous holes or any other phase boundaries, we should take into account their possible interaction [28, 29].

4. Conclusion

An approximate analytical benchmark is obtained for the problem of nonuniform mechanochemical dissolution of an elastical plane with an elliptic hole under internal pressure applied to the edge of the hole. The solution is applicable for the cases when the minor axis of the ellipse is greater than the half of the major one.

The calculation results show that over time nearly circular holes can become more elongated under mechanochemical corrosion conditions. At the same time, when the mechanochemical effect is weak enough, elliptical holes can turn almost circular.

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