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THE CONDITION OF TRANSITION TO UNSTABLE STATE (NECKING) OF A COMPRESSIBLE ELASTIC-PLASTIC MEDIUM

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Abstract. On the stress-strain curves obtained in simple tensile experiments for the metallic specimens a region of instability due to the formation of a neck is observed. In the theory of plasticity the conditions of transition to the unstable state and the appearance of the maximum point on the of stress-strain curve are defined. When this condition is derivate, the assumption of incompressibility of the material is accepted. However, this assumption cannot be justified, because in the neck region the numerous damages appear, i.e. the material becomes compressible. In the article, the condition of transition to unstable state for a compressible plastic medium is formulated.

1. Introduction

In solid mechanics, the basic mechanical characteristics of materials are determined, in particular, from experiments on simple tension. According to the results of measurements of force, current length and diameter of a specimen the values of the true stress $\sigma = P/F = \sigma_0 F_0 / F$ and logarithmic strain $\varepsilon = ln\, l / l_0$ are calculated (P is force, $\sigma_0 = P/F_0$ is engineering stress, l_0, F_0 are initial and l, F are current length and cross-section area of the specimen). Typical stress-strain curves for metallic specimens are shown on Fig. 1.

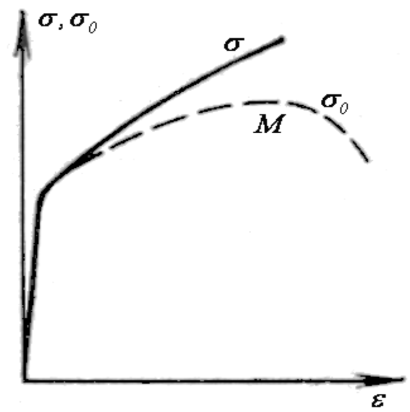

Fig. 1. Stress-strain curves. Solid line is tensile diagram in the true stresses; the dotted line is tensile diagram in engineering stresses.

It is usually assumed that the changes of cross-section area of the specimen during the deformation can be neglected, then $\sigma \approx \sigma_0$ and stress-strain curves are plotted in coordinates $\sigma_0 - \varepsilon$ (ε is engineering strain). At the point M on Fig. 1 the engineering stress reaches maximum, the neck is occurs on a specimen and the deformation process becomes unstable.

The descending region on the curve $\sigma_0 - \varepsilon$ is a result of a sharp decrease of a cross-section area of a specimen due to the formation of the neck. The character of a neck is determined by the properties of the material and is not the same for different materials (metals, polymers).

2. The definition of the condition of maximum achievement on the stress-strain curve for incompressible and compressible plastic medium

In the scientific literature [1, 2] to determine the condition of maximum achievement at the point M (Fig. 1) the material is considered as incompressible. Then $l_0F_0 = lF$ and $P = \sigma F = \sigma F_0\, e^{-\varepsilon}$, differentiating which for ε we will have

$$\frac{dP}{d\varepsilon} = F_0\, e^{-\varepsilon}\left(\frac{d\sigma}{d\varepsilon} - \sigma\right). \qquad (1)$$

When $dP / d\varepsilon = 0$, from (1) follows the ratio

$$\frac{d\sigma}{d\varepsilon} = \sigma\,, \qquad (2)$$

which is a maximum achievement condition at the point M (Fig. 1).

It is can be noted that when the expression (2) is derivate the assumption of incompressibility of the material is used, resulting a fixed maximum point on the stress-strain curve. At the same time in real metallic materials during plastic deformation, particularly in the area of instability, the maximum point is shifted, numerous damages are occur (pores, cracks), so the assumption of incompressibility of the material in the common case cannot be justified.

Let us define the compressibility condition using the current ratio of the transverse deformation ν: $\nu = -\varepsilon_y / \varepsilon_x = -\varepsilon_z / \varepsilon_x$ (ε_x is the longitudinal, ε_y, ε_z are transverse deformations of a cylindrical specimen). Then, taking into account the geometrical relationship $F_0 / F = \left(l / l_0\right)^{2\nu}$ [3], we receive

$$P = \sigma F = \sigma F_0\, e^{-2\nu\varepsilon}\,. \qquad (3)$$

Taking approximately $\nu = \nu(\sigma_0) = const$ and differentiating (3) for ε, we get the following expression

$$\frac{dP}{d\varepsilon} = F_0\, e^{-2\nu\varepsilon}\left(\frac{d\sigma}{d\varepsilon} - 2\nu\sigma\right), \qquad (4)$$

from which follows the condition of maximum achievement

$$\frac{d\sigma}{d\varepsilon} = 2\nu\sigma\,. \qquad (5)$$

For an incompressible material $\nu = 1/2$ and the relations (5) and (2) will coincide. According to the relation (5) the position of the maximum point on the stress-strain curve will shifted depending on the state of the material.

In the case of elastic-plastic medium $d\varepsilon = d\varepsilon^e + d\varepsilon^p$ (ε^e, ε^p are components of elastic and plastic deformation, $\varepsilon^e = \sigma / E$, E is elasticity modulus). The ratio between stress and strain can be determined by the following equation

$$\frac{dl}{l} = \frac{d\sigma}{E} + \varphi(\sigma)d\sigma\,. \qquad (6)$$

Integrating the equation (6), we obtain

$$ln\,\varepsilon = \frac{\sigma}{E} + \int_0^{\sigma} \varphi(\sigma)d\sigma\,. \qquad (7)$$

In the common case, inserting to (6) the expression $\sigma = \sigma_0\, e^{2\nu\varepsilon}$, we receive the equation written using the current ratio of the transverse deformation ν. For different values of ν it is possible to plot non-monotonic diagrams $\sigma_0 - \varepsilon$ and, so, to describe the experimental curves for metallic materials in coordinates engineering stress-strain. Further, this approach is applied for the case of rigid-plastic Ludwig medium with nonlinear hardening

$$\sigma = \sigma_T + b\varepsilon^m, \tag{8}$$

where σ_T is yield stress, b, m are constants.

Let us write the relation (8) through σ_0

$$\sigma_0 = \left(\sigma_T + b\varepsilon^m\right)e^{-2\nu\varepsilon}. \tag{9}$$

The theoretical curves (9) for $\sigma_T = 200\ MPa$, $b = 5\cdot 10^2\ MPa$, $m = 0{,}5$ and various values of ν are shown on Fig. 2.

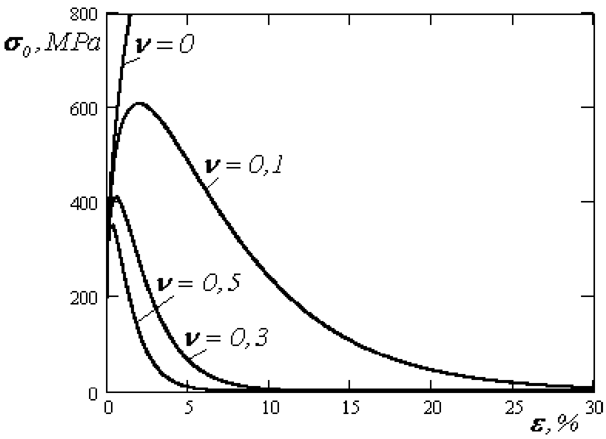

Fig. 2. The theoretical curves according relation (9).

3. Formulation of equations for a compressible elastic and elastic-plastic medium

Let us formulate the nonlinear equations for compressible viscous-elastic medium on the basis of the linear relations of Hooke's law and viscous Newton medium. It is known that using the linear relations the mechanical behavior of materials, in particular, polymers can be described only in limited temperature and force effects. In the common case should be operate the non-linear rheological equations. Using the proposed approach, we consider the behavior of a compressible elastic-viscous medium, which is generalized linear Hooke's model.

In the case of Hooke's law, the nonlinear version of elastic medium equations write down using the current ratio of the transverse deformation has the form

$$\sigma_0 = E\varepsilon e^{-2\nu\varepsilon} \tag{10}$$

Using equation (10) we can describe the nonlinear effects observed in experiments on simple tension. When $\nu = 1/2$ the equation (10) describes the behavior of an incompressible nonlinear media. When $\nu = 0$ the medium is linear elastic. Intermediate states correspond to elastic materials with different mechanical properties. On Fig. 3 the $\sigma_0 - \varepsilon$ diagrams for different values of the coefficient of transverse deformation are shown.

4. Conclusions

In the article, the transition condition is formulated for a compressible metal specimen in a state of instability in the region of neck formation. For an incompressible material this condition was first considered by Hill. The effect of compressibility is determined using the current ratio of the transverse deformation. Taking into account this coefficient are obtained the rheological equations for compressible media, generalizing the well-known relations for an incompressible plastic, elastic and elastic-plastic media. Analytical solutions are obtained and the corresponding theoretical stress-strain curves, depending on the current ratio of the transverse deformation are plotted. In particular, it is shown that for a compressible material in the region of instability the maximum point is shifted. In the case of a Hill's solution, there is a fixed maximum point on the stress-strain diagram.

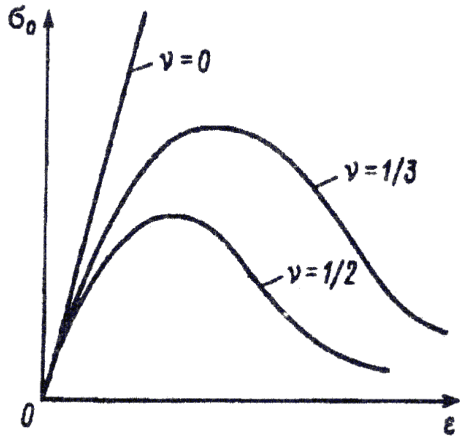

Fig. 3. The $\sigma_0 - \varepsilon$ diagrams for different values of the coefficient of transverse deformation: incompressible nonlinear medium ($\nu = 1/2$), linear elastic medium ($\nu = 0$).

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