EXAMINATION OF HARDENING CURVES DEFINITION METHODS
IN TORSION TEST

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Abstract. Torsion tests allow studying the rheological properties of materials over a wide range of strain and strain rate values as well as temperature. A key moment in construction of hardening curves is the interpretation of experimental data, which are usually the torque – angle of twist curves. However, there are a number of independent methods to obtain interpretation of experimental data. In addition, there is no single approach to determining the degree of equivalent strain in torsion test. The aim of this paper is to review existing hardening curves definition methods in torsion test and to examine them with the use of finite element modelling.

Keywords: material testing; torsion; rheological properties; strain resistance; equivalent strain.

1. Introduction
Strain resistance $\sigma_s$ is a key characteristic of the material that is necessary for the successful solution of problems associated with the improvement of existing and the development of new metal forming processes. In general, the material’s strain resistance depends on the ongoing processes of speed and strain hardening, as well as the relaxation processes observed simultaneously with this and determined by material’s temperature. According to [1] strain resistance at some instant of time can be represented by the following expression:

$$\sigma_s = \sigma_s(\varepsilon, \dot{\varepsilon}, \theta, \varepsilon(t)),$$

where $\varepsilon$ and $\dot{\varepsilon}$ are the degree of strain and strain rate respectively, $\theta$ is the temperature of material, and $\varepsilon(t)$ is the function that determines the loading history. In the processes of cold deformation, it is assumed that the strain resistance is determined only by the strain hardening process, i.e.

$$\sigma_s = \sigma_s(\varepsilon).$$

The material hardening curves are determined during the tensile, compression or torsion tests of the specimens. The method of torsion testing is characterized by the possibility of achieving large values of the degree of strain without neck formation, as well as the absence of the negative effect of friction forces on the test results that are typical for the tensile and compression tests respectively. The characteristic of the material properties in the torsion test is the shear strain resistance:

$$\tau_s = \sigma_s/\sqrt{3}. $$

According to [2, 3], the data on the strain resistance of materials obtained in tensile, compression and torsion tests differ from each other. In this case, the difference between the values of $\sigma_s$ reaches 30-40%, which contradicts the single curve hypothesis. This indicates a
lack of knowledge of the process of test specimens in torsion and indicates no reliable experimental data processing algorithm which allows to convert the data obtained in the form of a curve of torque - angle of twist of the sample to the form (1) or (2).

The purpose of the paper is to review and verify with the use of computer modeling of the existing methods for processing experimental data obtained during the torsion test and to determine the rheological properties of the material.

2. Methods for determining the hardening curves by the torsion

In one of the first works on determining shear stresses in the torsion the Nadai technique was proposed [4, 5]. The result of the experiment obtained relationship between the torque \( M \) and the angle of twist of the sample \( \varphi \). Experimental data processing performed by the formula:

\[
\tau_s = \frac{1}{2 \pi r^3} \left[ 3M + \gamma \frac{dM}{d\gamma} \right] = \frac{1}{2 \pi r^3} \left[ 3M + \varepsilon \frac{dM}{d\varepsilon} \right],
\]

where \( \gamma \) is the magnitude of shear strain, and \( r \) is the radius of the working part of the sample. Not contradicting the Nadai technique, in the formula, instead of the shear strain \( \gamma \), the value of the equivalent strain \( \varepsilon \) was used in this paper. According to [6], this technique is actively used at present to determine the rheological properties of materials that are insensitive to high-speed hardening. For materials whose strain resistance is determined by the strain rate and the temperature conditions of the process, there is a technique [7, 8]. According to this technique, the shear strain resistance of material is found by the formula:

\[
\tau_s = \frac{M}{2 \pi r^3} [3 + n + m],
\]

where \( n \) and \( m \) are the coefficients that determine the logarithmic dependence of the torque \( M \) on the twisting angle of the active gripper \( \varphi \) and the rate of change of the twisting angle \( d\varphi/dt \), respectively:

\[
n = \left( \frac{d \ln M}{d \ln \varphi} \right)_{d\varphi/dt = \text{const}},
\]

\[
m = \left( \frac{d \ln M}{d \ln (d\varphi/dt)} \right)_{\varphi = \text{const}}.
\]

In [9], a technique was proposed for processing experimental data obtained in the torsion test, according to which the shear strain resistance is determined by the expression:

\[
\tau_s = \frac{3M}{2 \pi r^3}.
\]

There are other methods for processing experimental data for formation hardening curves [10–12]. However, they are not widely used.

Important in deciphering the experimental data is the precise determination of the equivalent degree of strain accumulated in the sample at each instant of time. However, at present there is a situation that there is no single approach to calculating this quantity. The most widespread expression was [13], known as the equivalent shear strain of von Mises:

\[
\varepsilon = \frac{\gamma}{\sqrt{3}}.
\]

In [1] the equivalent degree of strain in the torsion test is given by:

\[
\varepsilon = \frac{n \gamma}{\sqrt{3}}.
\]
The work [14] presents an approach to determining the degree of strain based on the Hencky’s theory:

\[ \varepsilon = \frac{2}{\sqrt{3}} \ln \left[ \sqrt{1 + \frac{\gamma^2}{4} + \frac{\gamma}{2}} \right]. \]  

(11)

In formulas (9-11), the shear strain \( \gamma \) corresponds to the angle of rotation of the lines deposited on the surface of the specimen before testing along the axis:

\[ \gamma = \arctg \left( r \frac{\varphi}{l} \right), \]  

(12)

where \( \varphi \) – angle of twist of the sample, equal to the angle of rotation of the grippers of the testing machine, \( l \) – length of the working cylindrical part of the sample.

Known works in which a working part of the specimen is formed as a recess along the radius [15]. It is asserted that when using such samples, the drift phenomenon of the section with maximum deformation is absent and during deformation up to the moment of sample destruction the maximum deformation is localized in the cross section of the neck with the smallest diameter. The value of the angle of rotation of the lines deposited on the surface of the sample with a recess along the radius are determined by the formula:

\[ \gamma = \arctg \left( r \frac{dz}{dr} \right), \]  

(13)

where \( z \) is the direction of the coordinate axis, which coincides with the axis of the sample.

Determination of the strain rate during decipherment of the experimental data is carried out according to the known expression:

\[ \xi = \frac{d\varepsilon}{dt}. \]  

(14)

3. Research methods

To study the torsion test, the finite element method implemented in the software product Deform-3D was used. Samples with a cylindrical recess and a radial recess was used. The dimensions of the samples are shown in Fig. 1.

![Fig. 1. Models of samples for setting research objectives.](image)

The workpiece material DIN-C45 is selected from the Deform-3D materials database. The use of this material is due to the description of its rheological properties in a sufficient range of the degree of the strain from 0 to 1, the strain rate from 1.6 to 40 s\(^{-1}\), and also the temperature from 20 to 1370°C. The study implements the isothermal formulation of finite
element modeling problems, which corresponds to the basic idea of studying rheological properties at fixed temperatures. The torsion of samples in the Deform-3D program is defined by the motion of the grippers with equal and constant angular velocity $\pi \text{ rad/s}$ in opposite directions. The description of the conditions of contact of the workpiece with the tool is given without taking friction forces into account, because The slip of the metal particles of the sample along the surface of the captures is absent. In total, 4 problems were solved for modeling the process of testing samples with different geometry of the working part for torsion at temperatures of 20 and 900°C.

Based on the simulation results, the value of $\tau_s$ was calculated using formulas (4, 5 and 8). In this case, the value of the equivalent degree of deformation $\varepsilon$ was determined in accordance with (9-11). In this way it was not only the problem of evaluating the methods for determining the hardening curves of materials by the torsion method, but also the adequacy of various approaches to the calculation of the equivalent degree of deformation. The calculated values of $\tau_s$ were compared with the value of the shear deformation resistance, which was specified in the material properties during the simulation. For a more accurate comparison of the calculated and given values of $\tau_s$, the tabulated data on the deformation resistance were subjected to bilinear interpolation at the corresponding values of the strain and strain rate at a given temperature. The shear strain $\gamma$ for cylindrical samples was determined by the formula (12), and the samples with a recess along the radius were determined by the formula (13).

4. Results of the study and discussion

The results of the simulation are given in Table 1 as values of the coefficients of pair correlation of the values of $\tau_s$ calculated by known methods and specified in the properties of the material.

The data in Table 1 indicate the failure of the method for calculating the degree of deformation on the basis of the Hencky theory with the aim of establishing the hardening curves of the material. None of the formulas for calculating shear stresses in this case makes it possible to reproduce the hardening curve at the corresponding values of the strain and strain rate. The difference in the values of the shear strain resistance is observed up to 3 times as shown in Fig. 2.

![Fig. 2. Shear stresses for a cylindrical sample, 900°C.](image)
Table 1. Values of the coefficients of pair correlation.

<table>
<thead>
<tr>
<th>A sample form</th>
<th>Temperature</th>
<th>Model for calculating the degree of deformation</th>
<th>Technique for processing experimental data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tau_s = \frac{1}{2\pi r^3} \left[ \frac{3M + \varepsilon}{d\varepsilon} \right] $</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{\gamma}{\sqrt{3}}$</td>
<td>$\tau_s = \frac{M}{2\pi r^3} [3 + n + m]$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{t_g \gamma}{\sqrt{3}}$</td>
<td>$\tau_s = \frac{3M}{2\pi r^3}$</td>
</tr>
<tr>
<td>20°C cylindrical sample</td>
<td>20°C</td>
<td>$\varepsilon = \frac{\gamma}{\sqrt{3}}$</td>
<td>0,95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{t_g \gamma}{\sqrt{3}}$</td>
<td>0,96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{2}{\sqrt{3}} \ln \left[ \sqrt{1 + \frac{\gamma^2}{4} + \frac{\gamma}{2}} \right]$</td>
<td>0,48</td>
</tr>
<tr>
<td></td>
<td>900°C</td>
<td>$\varepsilon = \frac{\gamma}{\sqrt{3}}$</td>
<td>0,33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{t_g \gamma}{\sqrt{3}}$</td>
<td>0,98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{2}{\sqrt{3}} \ln \left[ \sqrt{1 + \frac{\gamma^2}{4} + \frac{\gamma}{2}} \right]$</td>
<td>0,58</td>
</tr>
<tr>
<td></td>
<td>900°C</td>
<td>$\varepsilon = \frac{\gamma}{\sqrt{3}}$</td>
<td>-0,04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{t_g \gamma}{\sqrt{3}}$</td>
<td>0,22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{2}{\sqrt{3}} \ln \left[ \sqrt{1 + \frac{\gamma^2}{4} + \frac{\gamma}{2}} \right]$</td>
<td>0,30</td>
</tr>
<tr>
<td>20°C sample with a radial recess</td>
<td>20°C</td>
<td>$\varepsilon = \frac{\gamma}{\sqrt{3}}$</td>
<td>-0,04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{t_g \gamma}{\sqrt{3}}$</td>
<td>0,22</td>
</tr>
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<td></td>
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<td>$\varepsilon = \frac{2}{\sqrt{3}} \ln \left[ \sqrt{1 + \frac{\gamma^2}{4} + \frac{\gamma}{2}} \right]$</td>
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<td>0,22</td>
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<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{t_g \gamma}{\sqrt{3}}$</td>
<td>0,17</td>
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<tr>
<td></td>
<td></td>
<td>$\varepsilon = \frac{2}{\sqrt{3}} \ln \left[ \sqrt{1 + \frac{\gamma^2}{4} + \frac{\gamma}{2}} \right]$</td>
<td>0,33</td>
</tr>
</tbody>
</table>
The best result of reproducing the hardening curve from the data on the dependence of the torque on the twisting angle is observed on cylindrical samples at a temperature of 20°C. In this case, the models for calculating the equivalent degree of strain (9) and (10) give an identical result. The relative difference between the calculated and given values of the shear strain resistance does not exceed 6% (see Fig. 3).

![Fig. 3. Shear stresses for a cylindrical sample, 20°C.](image)

When the test temperature of the cylindrical sample is increased to 900°C, despite the sufficiently high values of the pair correlation coefficients, the hardening curve can not be reproduced for any of the considered methods (see Fig. 4). The relative difference between the calculated and given values of the shear strain resistance reaches 85–427%, depending on the model for calculating the equivalent strain and the technique for processing the experimental data. The large deviation of the calculated values of the strain resistance is explained by the increase in the effect on it of the strain rate at elevated temperatures.

![Fig. 4. Shear stresses for a cylindrical sample, 900°C.](image)
In the case of using sample with a recess along the radius, it is possible to calculate the value of shear strain $\gamma$ with greater accuracy. However, as shown by the simulation results, the hardening curve of the material can be reproduced only with the use of the technique [10] regardless of the strain degree model (9) or (10) (see Fig. 5). The relative difference between the calculated and given values of the shear strain resistance does not exceed 6%. With an increase in the temperature of the sample to 900°C, none of the methods for processing experimental data makes it possible to determine the properties of the material from experimental data.

**Fig. 5.** Shear stresses for a sample with a recess along the radius, 20°C.

5. **Conclusions**
1. Calculation of the equivalent strain degree based on Hencky’s theory for the purpose of establishing hardening curves is untenable.
2. In the case of cold deformation, the experimental data processing technique (8) provides the possibility of calculating the strain resistance for both cylindrical specimens and samples with a recess along the radius.
3. The techniques (4) and (5) do not ensure the convergence of the results of calculating the strain resistance in the cold state with the specified properties of the material when using specimens with a recess along the radius.
4. None of the considered methods for processing experimental data makes it possible to study the properties of materials in the hot state.
5. In order to study the rheological properties of materials in the hot state, it is necessary to develop a reliable algorithm for processing experimental data obtained from the results of torsion test.

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**References**


