

## DYNAMIC BEHAVIOUR OF CONCRETE AND MORTAR AT HIGH STRAIN RATES

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**Abstract.** In this paper a uniform interpretation of the rate effects of fracture of the concrete and mortar is given on the basis of a structural-temporal approach based on the notion of incubation time. It is shown that temporal dependences of both materials are well calculated using the incubation time criterion. Different relation of ultimate stresses for concrete and mortar in quasi static and high rate loading conditions is observed.

### Nomenclature

$F(t)$  - intensity of the local force field;

$F_c$  - static limit of the local force field;

$\tau$  - incubation time;

$\alpha$  - a parameter sensitive to the level (or amplitude) of the intensity of the local force field;

$H(t)$  - Heaviside function;

$\sigma(t)$  - stress;

$\sigma_c^{compr}$  - static compressive strength defect-free material;

$E$  - Young's modulus;

$\dot{\sigma}$  - stress rate;

$\dot{\epsilon}$  - strain rate;

$\sigma_d$  - limit stress;

$\rho$  - mass density of material;

$\nu$  - Poisson coefficient;

$V_0$  - projectile velocity in impact experiments.

### 1. Introduction

Nowadays, bridge demolition often occurs as a result of strong winds, plane crashes on runways, failures of building roofs due to loud noise, accidents in factories, or explosive damage to buildings. One of the major problems to catastrophic failure under intense sudden overloading isn't considered impermanence of fracture process. The other reason can be widely applied (because it is cheap), and is usually faulty material in buildings – concrete (the basis of any foundation) and mortar (in relation to composition it has various uses), which connects a variety of construction materials. An analysis of dynamically loaded structures requires an understanding of the behaviour of the mechanical properties of mortar and aggregated concrete at high strain rates. Therefore a study of the behaviour of concrete and mortar at high rates of deformation is necessary.

Modelling the behaviour of concrete and mortar in conditions of high-speed loading conducted on different experimental facilities: standard testing machines (low  $10^{-1} c^{-1}$ ), split Hopkinson bar [1] ( $10^1 - 10^2 c^{-1}$ ), classic Taylor impact [2] in quasi-static (low  $10^{-1} c^{-1}$ ) and dynamic conditions (for brittle materials this is not used in practice [3]) ( $10^4 - 10^5 c^{-1}$ ), and plate impact (above  $10^4 - 10^5 c^{-1}$ ). It has been observed [4] for the high-speed curve of concrete (for the cases of compression and tension) that the dynamic strength of the material begins to grow in the range  $10^0 - 10^3 c^{-1}$  of strain rates. On the basis of test data (dynamic strength, strain rate), it can be constructed a high-speed curve that characterizes the dependence of the dynamic strength of strain rate material and combines the material behaviour at various levels of quasi-static and dynamic conditions. It should be noted that in fact the appearance of the curve depends on the history of the load; however, as will be shown below, it can be obtained based on the incubation time criterion and its material parameters, which are invariant to any impact history.

At the equilibrium compression material is characterized by static strength [5]; however, in the case of dynamic loading the limiting stress is different depending on the strain rate. In this paper, it is proposed to use a notion of the incubation time. It is a characteristic time of the fracture process invariably for any strain rate. On the basis of the incubation time criterion, it possible to predict the behaviour of fracture stress over a wide range of rates. Grote et al. [6] observed that concrete strength becomes greater than that of mortar, although in the case of static processes the opposite is true. In [6], this phenomenon is explained by the acute effect on the plate impact constraints of compressive loads that inhibit crack growth, although the quasi-static uniaxial compression increases the probability of cracking. It is interesting to note that in the static tests constraint loads also occur, but their influence on the experiment plate impact is so great there, as described in [6]. It is shown that the improvement of the properties of concrete at high strain rates relative to mortar can be explained without the influence of hydrostatic stress.

In the present research it is considered the experimental data of mortar on a split Hopkinson bar ( $10^2 - 10^3 c^{-1}$ ) and tests of mortar and concrete by plate impact ( $10^4 - 10^5 c^{-1}$ ), obtained in [6]. It is shown that the dynamic strength of mortar is in the range of 60–180 MPa when tested on a Hopkinson bar, and in plate impact experiments the ultimate dynamic strength of mortar and concrete achieved 1.2 GPa and 1.5 GPa, respectively. On the basis of these results in this paper we evaluate incubation time for mortar and plot high-speed curve for mortar which describes the dependence of the limiting of compressive stress on the strain rate. The behaviour of concrete and mortar is compared under quasi-static and dynamic deformation conditions.

## 2. The incubation time criterion

Consider, as proposed in [7-14], description methods of the process of dynamic brittle fracture of materials, where incubation time is introduced related to the properties of the material to describe the dynamic tests in the process of breaking. The character of the incubation time establishes relaxation process propagation of microdefects in the structure of material and its irreversible deformation.

The fracture criterion based on the concept of incubation time, proposed in [7-14], makes it possible to predict the unstable behaviour of the dynamic-strength characteristics observed in experiments on the dynamic fracture of solids. The fracture criterion can be written in the following general form:

$$\frac{1}{\tau} \cdot \int_{t-\tau}^t \left( \frac{F(t')}{F_c} \right)^\alpha dt' \leq 1. \quad (1)$$

Here,  $F(t)$  is the intensity of the local force field causing the fracture (or structural transformation) of the medium,  $F_c$  is the static limit of the local force field, and  $\tau$  is the incubation time associated with the dynamics of the relaxation processes preceding the fracture. It actually characterizes the strain (stress) rate sensitivity of the material. The fracture time  $t^*$  is defined as the time at which the equality sign is reached in Eq. (1). The parameter  $\alpha$  characterizes the sensitivity of the material to the intensity (amplitude) of the force field causing the fracture (or structural transformation). Often,  $\alpha = 1$  gives a good agreement with test data.

One of the possible means of interpreting and determining the parameter  $\tau$  is proposed here in the example of the mechanical rupture of a material. Let us assume that a standard test specimen made of the material in question is subjected to tension and is broken into two parts under a stress  $P$  arising at a certain time  $t = 0$ :  $F(t) = PH(t)$ , where  $H(t)$  is the Heaviside step function. In the case of quasi-brittle fracture, the material would unload, and the local stress at the break point would decrease rapidly (but not instantaneously) from  $P$  to 0. In this case, a corresponding unloading wave is generated that propagates over the sample and can be detected by standard (e.g., interferometry) methods. The stress variation at the break point can be conditionally represented by the relation  $\sigma(t) = P - Pf(t)$ , where  $f(t)$  varies from 0 to 1 (Fig. 1) within a certain time interval  $T$ .

The case  $f(t) = H(t)$  corresponds to the classical strength theory. In other words, according to the classical approach, rupture occurs instantaneously ( $T = 0$ ). In practice, the rupture of a material (sample) is a process in time, and the function  $f(t)$  describes the *micro-scale level* kinetics of the transition from a conditionally defect-free state ( $f(0) = 0$ ) to a completely broken state at the given point  $f(t^*) = 1$  that can be associated with the macro-fracture event. On the other hand, application of the fracture criterion (1) to the *macro-level* situation ( $F(t) = PH(t)$ ), gives the time to fracture  $t^* = T = \tau$  at  $P = F_c$ . The physical meaning of incubation time is related to the relaxation process of growth of microdefects in the structure of material, which provides its non-reversible deformation. In this case characteristic time of relaxation can be considered as the incubation time [14].

This duration can be measured experimentally by statically fracturing the samples and controlling the rupture process by different possible methods, e.g., by measuring the time of the increase in pressure at the unloading wave front based on the recorded velocity profile of points (by interferometry) on the sample boundary.

### 3. Determining the dependence of critical compressive stress on strain rate based on the incubation time criterion

Let us consider the application of the incubation time approach for calculating material strength under compression at different strain rates. As a rule, the concept of strength defines the growth limit value of the local stresses (maximum stress), at which the material does not fall into decay. In the incubation time approach, dynamic strength is characterized by the incubation time, but not maximum stress.

Grote et al. [6] conducted experiment a split Hopkinson bar tests on the defect-free samples mortar uniaxial stress state. The concept “defect-free” sample determined the material that doesn’t contain specially created defects and concentrators such as cracks and oblique cut-outs. Grote et al [6] found in the stress-strain rate diagram of mortar that fracture stress of material increases with strain rate. This effect will be showed for example plotting high-speed mortar curve on the basis of the incubation time criterion in a simplified form with test data [6].

The dynamic compression stress is predicted on the criterion, which in case the defect-free sample takes the following form:

$$\int_{t-\tau}^t \sigma(s) ds \leq \sigma_c^{compr} \cdot \tau, \quad (2)$$

where  $\sigma(s)$  is the time dependence of the average compressive stress in the specimen,  $\sigma_c^{compr}$  is the static compressive strength of the material, and  $\tau$  is the incubation time of fracture of the material under compression.

According to [6], an increase in the compressive stress with strain rate can be assumed to be linear until it reaches the maximum value, so that

$$\sigma(t) = \dot{\sigma} \cdot t \cdot H(t) = E \cdot \dot{\epsilon} \cdot t \cdot H(t), \quad (3)$$

where  $\dot{\sigma}$  is the stress rate,  $H(t)$  is the Heaviside function,  $E$  is Young's modulus,  $\dot{\epsilon}$  is the strain rate, and we propose that  $\dot{\sigma}$  and  $\dot{\epsilon}$  are constant.

The left-hand side of (2) with (3) is written as follows:

$$\int_{t-\tau}^t E \cdot \dot{\epsilon} \cdot s \cdot H(s) ds = \begin{cases} \frac{E \cdot \dot{\epsilon}}{2} \cdot t^2, & 0 < t < \tau; \\ \frac{E \cdot \dot{\epsilon}}{2} \cdot (2 \cdot \tau \cdot t - \tau^2), & t > \tau. \end{cases}$$

Material fracture occurs if equality is attained in criterion (2). It is constructed the time dependence of the fracture of material on the strain rate:

$$t_*(\dot{\epsilon}) = \begin{cases} \sqrt{\frac{2 \cdot \sigma_c^{compr} \cdot \tau}{E \cdot \dot{\epsilon}}}, & 0 < t < \tau; \\ \frac{\sigma_c^{compr}}{E \cdot \dot{\epsilon}} + \frac{\tau}{2}, & t > \tau. \end{cases} \quad (4)$$

Defining ultimate stress  $\sigma_d$  as a stress at the moment of fracture  $\sigma_d = \sigma(t_*)$ , it is obtained the following for the limiting stress from (3) and (4):

$$\sigma_d = \begin{cases} \sqrt{2 \cdot E \cdot \dot{\epsilon} \cdot \tau \cdot \sigma_c^{compr}}, & \frac{2 \cdot \sigma_c^{compr}}{E \cdot \tau} < \dot{\epsilon} < \infty; \\ \frac{E \cdot \dot{\epsilon} \cdot \tau}{2} + \sigma_c^{compr}, & \dot{\epsilon} < \frac{2 \cdot \sigma_c^{compr}}{E \cdot \tau}. \end{cases} \quad (5)$$

In general, function (5) describes the behaviour of the fracture stress of the material in two cases: in the slow (equilibrium) process and in the fast dynamic deformation (monotonically increasing function). It should be noted that the ultimate fracture stress (5) at different strain rates is determined by the influence of Young's modulus, the static strength of defect-free specimen and the incubation time of material.

Grote et al. [6] have carried out quasi-static compression tests on specimens of concrete and mortar uniaxial stress state at strain rate  $10^{-3} c^{-1}$ , plate impact experiments at medium strain rates of the order  $10^4 - 10^5 c^{-1}$  of concrete and mortar, and experiments on a split Hopkinson bar at strain rates of the order  $10^2 - 10^4 c^{-1}$  of samples of mortar. It was determined during the quasi-static test the static strength of mortar specimens in compression 46 MPa and Young's modulus 20 GPa. Cylindrical samples of mortar with a circular diameter of 19.05 mm (or 12.7 mm) were used. Strain rates in these tests were achieved in the range of 250 to 1700  $c^{-1}$ .

Using the function (5) and the test data [6] about the dynamic strength and strain rate, it is calculated the incubation time for the mortar specimens  $\tau = 6.5 \mu\text{s}$  as an arithmetical middling from all samples. On the basis of computed values dependence critical compressive stress on the strain rate (5) is plotted in Fig. 1.

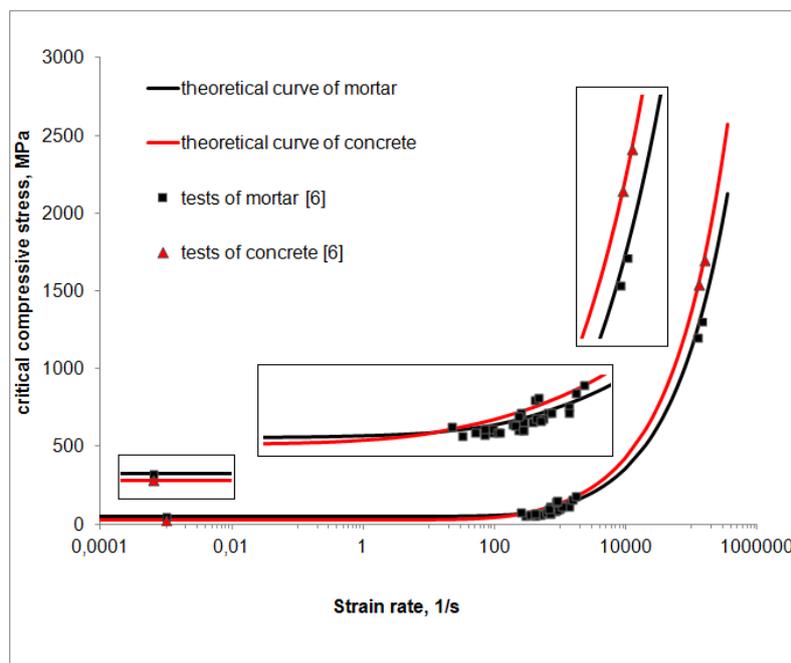


Fig. 1. Dependence critical compressive stress on strain rate.

#### 4. Discussion

Critical compressive stress increases with the strain rate of the material. It is regarded the right point of mortar specimens in Fig. 1 at which the strain rate is  $1700 \text{ c}^{-1}$  and fracture stress is four times above static compressive strength. It should be noted that the dynamic strength of material begins to increase drastically with a strain rate of the order  $10^2 - 10^3 \text{ c}^{-1}$ , as observed in [4].

In general, the continuous curve in Fig. 1 corresponds experimental points [6]. On the basis of this curve, it can get the average value of the dynamic strength of the material at different strain rates. The calculation shows that the incubation time criterion allows a gradient junction to be made from static to dynamic.

Another test configuration, providing high strain rates, has also been conducted in [6]. According to this scheme, specimens in the form of circular discs of concrete and mortar at a diameter of 76.2 mm and thickness of 10 mm are subjected to loading. The projectile tube contains a specimen in the front part and there is a gap between the specimen and the projectile. Under pressure from helium gas, the projectile starts to move at the velocity of a steel anvil plate whose thickness is 13.5 mm and whose hardness measures approximately 65 on the Rockwell C scale. The velocities of particles on the rear surface of the steel anvil plate were measured using a laser interferometer. On the basis velocities of particles on the rear surface the average stress was calculated. In the case of plate impact experiments [6], authors as a dynamic strength took average stress until the arrival of the reloading wave. The material used was assumed to be homogeneous and its behaviour linear elastic.

It is shown in Table 1 the data for specimens of concrete and mortar quasi-static testing and experimentation by plate impact [6].

The impact experiment showed that concrete was stronger than mortar by 30 %.

Table 1. Comparison of the behaviour of concrete and mortar in quasi-static tests and plate impact experiments [6].

Material	$\rho$ , kg/m <sup>3</sup>	$\nu$	$V_0$ , m/s	$\sigma_d$ , GPa	$E$ , GPa	$\sigma_c$ , MPa	$\tau$ , $\mu$ s
Concrete	2600	0.29	290	1.55	45	30	6.5
			330	1.7			
Mortar	2100	0.2	290	1.2	20	46	6.5
			330	1.3			

Grote et al. [6] explain the effect of the existence of high confining pressure in the specimens. These stresses are growth drivers of micro-cracks in static compression that critical compression stress reduces; also, these stresses are retard growth factors of microdefects in tests at high strain rates whereby material strength increases. Concrete is more inhomogeneous than In the case of high strain rates this influence of modulus becomes determinant mortar, so the fracture stress of mortar is less than that of concrete in plate impact experiments.

The results in Table I demonstrate that the aggregated concrete has smaller static strength and larger elastic modulus compared to mortar. It is seen that the static compressive strength does not influence much on the critical stress at high strain rates of the order  $10^4 - 10^5 c^{-1}$ . Here it is considered that the local stress history follows the linear law (3) where Young's modulus defines the growth rate of compressive stress. Thus, it turns out that the competitive "strength inversion" between concrete and mortar is defined by the existence of incubation period and by the difference in elastic modulus. Taking in mind similar values of  $\tau$  for mortar and aggregated concrete we get that the greater is Young's modulus the bigger is ultimate critical stress, which is attained during the incubation period. It has little effect at low impact velocities, but in the case of high strain rates this influence of modulus becomes determinant.

## 5. Conclusions

The structural time approach allows the critical compressive stress of material at different strain rates to be predicted using the incubation time of the fracture as a material parameter.

On the basis of experimental data from split Hopkinson bar tests [6] it was determined the incubation time of mortar. The stain rate dependency of mortar and aggregated concrete in a wide range of strain rates was calculated by means of the incubation time criterion. It should be emphasized that the sharp increase in the critical compressive stress is observed at strain rates of the order  $10^0 - 10^3 c^{-1}$ .

Behaviour of both materials concrete and mortar over the course of quasi-static tests  $10^{-3} c^{-1}$  and plate impact experiments  $10^4 - 10^5 c^{-1}$  was investigated and compared. It is obtained that limiting compressive stress of mortar at a low strain rate was smaller than that of concrete, but at a high strain rate the opposite is true. Finally, it should be noted that the reason for this "strength inversion" effect can be the combined influence of incubation time and material elastic modulus along with growth of strain rate.

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