Materials Physics and Mechanics 21 (2014) 147-167 Received: July 2, 2014

# REFLECTION AND TRANSMISSION OF PLANE WAVES AT THE LOOSELY BONDED INTERFACE OF AN ELASTIC SOLID HALF-SPACE AND A MICROSTRETCH THERMOELASTIC DIFFUSION SOLID HALF-SPACE

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**Abstract.** The problem of reflection and refraction phenomenon due to plane waves incident obliquely at the loosely bonded interface between an elastic solid half-space and the microstretch thermoelastic diffusion solid half-space is discussed. It is assumed that the interface behaves like a dislocation, which preserves the continuity of traction while allowing a finite amount of slip. The amplitude ratios of various reflected and transmitted waves to that of incident wave are obtained. These amplitude ratios are further used to find the expressions of energy ratios of various reflected and refracted waves to that of incident wave. Numerical results have been obtained for a particular model to study the variation of energy ratios with respect to angle of incidence. The effect of relaxation times and loosely bonding parameters are shown with energy ratios for a specific model. The law of conservation of energy at the loosely boundary interface is verified. Some special cases of interest have been deduced from the present investigation.

## 1. Introduction

Due to the unidentified nature of the layers beneath the earth's surface and for the purpose of theoretical investigations, various appropriate mathematical models are to be considered. In the problem of reflection and refraction of plane waves at the boundaries between two half-spaces, it is generally considered that the half-spaces are in welded contact at the interface. Therefore, it is reasonable approximation to consider the presence of a thin layer of viscous liquid at the interface and two half-spaces are caused to be loosely bounded. Murty [1, 2] studied reflection and refraction of elastic waves through a loosely bonded interface, i.e., an interface at which the two media are not perfectly bonded, by assuming the continuity of the traction and that a finite amount of slip can take place at the interface.

Eringen [3] developed the theory of a micropolar elastic solid with stretch in which he has taken the effect of axial stretch during the rotation of molecules. Eringen [4, 5] also developed the theory of a thermo-microstretch elastic solid and fluids, in which he included microstructural expansions and contractions. Microstretch continuum is a model for Bravais lattice with a basis on the atomic level, and a two-phase dipolar solid with a core on the

macroscopic level. For example, composite materials reinforced with chopped elastic fibres, porous media whose pores filled with gas or inviscid liquid, asphalt or other elastic inclusions and "solid-liquid" crystals etc. should be characterizable by microstretch solids. A comprehensive review on the micropolar continuum theory has been given in his book by Eringen [6].

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include the thermal effect by Eringen [7] and Nowacki [8]. Now it is known as the micropolar coupled thermoelasticity. Dost and Tabarrok [9] have presented the generalized micropolar thermal elasticity by using Green-Lindsay theory. Chanderashekhariah [10] developed a heat-flux dependent generalized theory of micropolar thermoelasticity.

The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Nowacki [11-14] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [15] and Olesiak and Pyryev [16], respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer. Gawinecki and Szymaniec [17] proved a theorem about global existence of the solution for a nonlinear parabolic thermoelastic diffusion problem. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief et al. [18] on the basis of the variational principle equations, under restrictive assumptions on the elastic coefficients. Recently, Kumar and Kansal [19] developed the basic equation of anisotropic thermoelastic diffusion based upon Green-Lindsay model.

Many researches have investigated the problems of reflection and transmission of plane waves at the interface of two dissimilar media. Notable among them are Borejko et al. [20], Wu and Lundberg [21], Sinha and Elsibai [22], Singh [23], Kumar and Singh [24] discussed a problem on reflection and transmission of elastic waves at the loosely bonded interface between an elastic solid half-space and a micropolar elastic solid half-space, Kumar and Panchal [25] studied the problem of reflection and transmission of plane waves incident on the interface between the loosely bonded elastic solid and micropolar cubic crystal half-spaces. In recent times, Sharma and Bhargava [26] investigated the propagation of thermoelastic plane waves at an imperfect boundary of thermal conducting viscous liquid/generalized thermoelastic solid.

In the present paper, the reflection and refraction phenomenon of plane waves at a loosely boundary interface between an elastic solid half-space and a microstretch thermoelastic diffusion solid half-space has been analyzed. In microstretch thermoelastic diffusion solid medium, potential functions are introduced to represent four coupled longitudinal waves and two coupled transverse waves. The amplitude ratios of various reflected and refracted waves to that of incident wave are derived. The expressions for the energy ratios of reflected and transmitted waves are obtained. The graphical representation is given for these energy ratios for different values of the bonding parameter.

## 2. Basic equations

Following Sherief et al. [18], Eringen [6], Kumar and Kansal [19] the equations of motion and the constitutive relations in a homogeneous isotropic microstretch thermoelastic diffusion solid in the absence of body forces, body couples, stretch force, and heat sources are given by

$$\left(\lambda+2\mu+K\right)\nabla\left(\nabla.\vec{u}\right)-\left(\mu+K\right)\nabla\times\nabla\times\vec{u}+K\nabla\times\vec{\varphi}+\lambda_O\nabla\varphi^*$$
$$-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{2.1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\varphi})-\gamma\,\nabla\times(\nabla\times\vec{\varphi})+K\nabla\times\vec{u}-2K\vec{\varphi}=\rho\, j\,\frac{\partial^2\vec{\varphi}}{\partial t^2}, \tag{2.2}$$

$$\alpha_0\nabla^2\varphi^*+\nu_1\left(T+\tau_1\dot{T}\right)+\nu_2\left(C+\tau^1\dot{C}\right)-\lambda_1\varphi^*-\lambda_0\nabla.\vec{u}=\frac{\rho\, j_0}{2}\frac{\partial^2\varphi^*}{\partial t^2} \tag{2.3}$$

$$K^*\nabla^2T=\beta_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\nabla.\dot{\vec{u}}+\nu_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\dot{\varphi}^*+\rho C^*\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+aT_0\left(\dot{C}+\gamma_1\ddot{C}\right), \tag{2.4}$$

$$D\beta_2\varepsilon_{kk,ii}+D\nu_2\varphi^*_{,ii}+Da\left(T+\tau_1\dot{T}\right)_{,ii}+\left(\dot{C}+\varepsilon\tau^0\ddot{C}\right)-Db\left(C+\tau^1\dot{C}\right)_{,ii}=0, \tag{2.5}$$

and constitutive relations are

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\varepsilon_{ijr}\varphi_r\right)+\lambda_o\delta_{ij}\varphi^*-\beta_1(1+\tau_1\frac{\partial}{\partial t})T\delta_{ij}-\beta_2(1+\tau^1\frac{\partial}{\partial t})C\delta_{ij}, \tag{2.6}$$

$$m_{ij}=\alpha\varphi_{r,r}\delta_{ij}+\beta\varphi_{i,j}+\gamma\varphi_{j,i}+b_0\varepsilon_{mji}\varphi^*_{,m}, \tag{2.7}$$

$$\lambda_i^*=\alpha_0\varphi^*_{,i}+b_0\varepsilon_{ijm}\varphi_{j,m}, \tag{2.8}$$

where $\lambda,\mu,\alpha,\beta,\gamma,K,\lambda_o,\lambda_1,\alpha_o,b_o$ are material constants; $\rho$ is the mass density; $\vec{u}=(u_1,u_2,u_3)$ is the displacement vector and $\vec{\varphi}=(\varphi_1,\varphi_2,\varphi_3)$ is the microrotation vector; $\varphi^*$ is the scalar microstretch function; $T$ and $T_0$ are the small temperature increment and the reference temperature of the body chosen such that $|T/T_0|\ll 1$; $C$ is the concentration of the diffusion material in the elastic body; $K^*$ is the coefficient of the thermal conductivity; $C^*$ is the specific heat at constant strain; $D$ is the thermoelastic diffusion constant; $a$, $b$ are, respectively, coefficients describing the measure of thermo- diffusion and of mass diffusion effects; $\beta_1=(3\lambda+2\mu+K)\alpha_{t1}$; $\beta_2=(3\lambda+2\mu+K)\alpha_{c1}$; $\nu_1=(3\lambda+2\mu+K)\alpha_{t2}$; $\nu_2=(3\lambda+2\mu+K)\alpha_{c2}$; $\alpha_{t1},\alpha_{t2}$ are coefficients of linear thermal expansion; and $\alpha_{c1},\alpha_{c2}$ are the coefficients of linear diffusion expansion; $j$ is the microintertia; $j_o$ is the microinertia of the microelements; $t_{ij}$ and $m_{ij}$ are components of stress and couple stress tensors respectively; $\lambda_i^*$ is the microstress tensor; $e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right)$ are components of infinitesimal strain; $e_{kk}$ is the dilatation; $\delta_{ij}$ is the Kronecker delta; $\tau^0,\tau^1$ are diffusion relaxation times with $\tau^1\geq\tau^0\geq 0$ and $\tau_0,\tau_1$ are thermal relaxation times with $\tau_1\geq\tau_0\geq 0$. Here $\tau_0=\tau^0=\tau_1=\tau^1=\gamma_1=0$ for Coupled Thermoelasitc (CT) model, $\tau_1=\tau^1=0,\ \varepsilon=1,\ \gamma_1=\tau_0$ for Lord-Shulman (L-S) model and $\varepsilon=0,\ \gamma_1=\tau^0$, where $\tau^0>0$ for Green-Lindsay (G-L) model. In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time respectively.

The basic equation of motion and stress-strain relation in a homogeneous isotropic elastic solid are written as

$$\left(\lambda^e+\mu^e\right)\nabla.\nabla\vec{u}^e+\mu^e\nabla^2\vec{u}^e=\rho^e\frac{\partial^2\vec{u}^e}{\partial t^2}, \tag{2.9}$$

$$t_{ij}{}^e=2\mu^e\,e_{ij}{}^e+\lambda^e\,e_{kk}{}^e\,\delta_{ij}, \tag{2.10}$$

where $\lambda^e$, $\mu^e$ are Lame's constants, $\vec{u}^e$ is the displacement vector, $\rho^e$ is density, $e^e_{kk}$ is the dilatation, $t_{ij}{}^e$ and $e^e_{ij} = \frac{1}{2}\left(u^e_{i,j} + u^e_{j,i}\right)$ are components of stress and strain tensors respectively.

## 3. Formulation of the problem

We consider an isotropic elastic solid half-space ($M_1$) lying over a homogeneous isotropic, microstretch generalized thermoelastic diffusion solid half-space ($M_2$). The origin of the cartesian coordinate system $(x_1, x_2, x_3)$ is taken at any point on the plane surface (interface) and $x_3$-axis point vertically downwards into the microstretch thermoelastic diffusion solid half-space. The elastic solid half-space ($M_1$) occupies the region $x_3 \le 0$ and the region $x_3 \ge 0$ is occupied by the microstretch themoelastic diffusion solid half-space ($M_2$) as shown in Fig. 1. We consider plane waves in the $x_1x_3$-plane with wave front parallel to the $x_2$-axis. For two- dimensional problem, we take

$$\vec{u} = (u_1, 0, u_3),\ \vec{\varphi} = (0, \varphi_2, 0),\ \vec{u}^e = (u^e_1, 0, u^e_3). \tag{3.1}$$

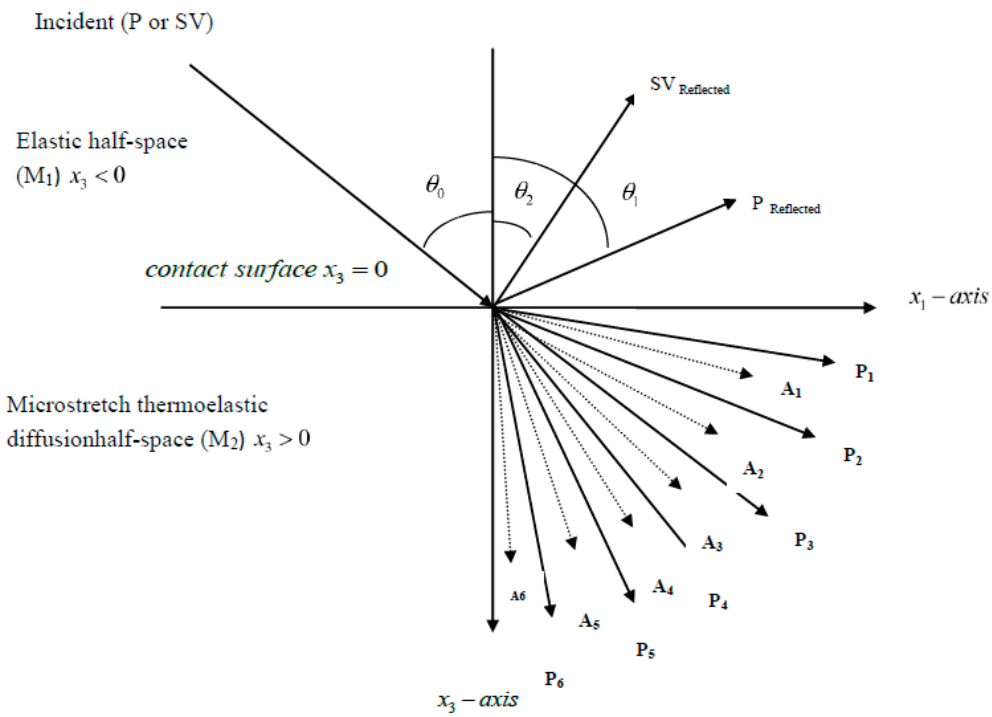


**Fig. 1.** Geometry of the problem.

We define the following dimensionless quantities

$$\left(x_1', x_3'\right) = \frac{\omega^*}{c_1}\left(x_1, x_3\right),\ \left(u_1', u_3'\right) = \frac{\rho c_1 \omega^*}{\beta_1 T_o}\left(u_1, u_3\right),\ t_{ij}' = \frac{t_{ij}}{\beta_1 T_o},\ t_{ij}^{e'} = \frac{t^e_{ij}}{\beta_1 T_o},\ T' = \frac{T}{T_o},\ t' = \omega^* t,$$

$$\tau_o' = \omega^* \tau_o,\ \tau^{0'} = \omega^* \tau^0,\ \left(u_1^{e'}, u_3^{e'}\right) = \frac{\rho c_1 \omega^*}{\beta_1 T_o}\left(u^e_1, u^e_3\right),\ \tau_1' = \omega^* \tau_1,\ \tau^{1'} = \omega^* \tau^1,\ \varphi^{*'} = \frac{\rho c_1^2}{\beta_1 T_o}\varphi^*,$$

$$\lambda_i^{*'} = \frac{\omega^* \lambda_i^*}{c_1 \beta_1 T_o},\ \varphi_2' = \frac{\rho c_1^2}{\beta_1 T_o}\varphi_2,\ C' = \frac{\beta_2 C}{\rho c_1^2},\ m_{ij}' = \frac{\omega^*}{c_1 \beta_1 T_o} m_{ij}, \tag{3.2}$$

where $\omega^* = \dfrac{\rho C^* c_1^2}{K^*}$, $c_1^2 = \dfrac{\lambda + 2\mu + K}{\rho}$, $\omega^*$ is the characteristic frequency of the medium,

Making use of (3.1) in equations (2.1)-(2.5), with the aid of dimensionless quantities defined by (3.2) and after suppressing the primes, we obtain

$$\delta^2 \frac{\partial e}{\partial x_1} + \left(1-\delta^2\right)\nabla^2 u_1 - \zeta_1^* \frac{\partial \varphi_2}{\partial x_3} + \zeta_3^* \frac{\partial \varphi^*}{\partial x_1} - \tau_t^1 \frac{\partial T}{\partial x_1} - \zeta_2^* \tau_c^1 \frac{\partial C}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{3.3}$$

$$\delta^2 \frac{\partial e}{\partial x_3} + \left(1-\delta^2\right)\nabla^2 u_3 + \zeta_1^* \frac{\partial \varphi_2}{\partial x_1} + \zeta_3^* \frac{\partial \varphi^*}{\partial x_3} - \tau_t^1 \frac{\partial T}{\partial x_3} - \zeta_2^* \tau_c^1 \frac{\partial C}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{3.4}$$

$$\zeta_1 \nabla^2 \varphi_2 + \zeta_2 \left( \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) - \zeta_3 \varphi_2 = \frac{\partial^2 \varphi_2}{\partial t^2}, \tag{3.5}$$

$$\left(\delta_1^2 \nabla^2 - \chi_1^*\right)\varphi^* - \chi_2^* e + \chi_3^* \tau_t^1 T + \chi_4^* \tau_c^1 C = \frac{\partial^2 \varphi^*}{\partial t^2}, \tag{3.6}$$

$$\nabla^2 T = \ell_1^* \tau_e^0 \frac{\partial e}{\partial t} + \ell_2^* \tau_e^0 \frac{\partial \varphi^*}{\partial t} + \tau_t^0 \frac{\partial T}{\partial t} + \ell_3^* \tau_c^0 \frac{\partial C}{\partial t}, \tag{3.7}$$

$$q_1^* \nabla^2 e + q_4^* \nabla^2 \varphi^* + q_2^* \tau_t^1 \nabla^2 T + \tau_f^0 \frac{\partial C}{\partial t} - q_3^* \tau_c^1 \nabla^2 C = 0, \tag{3.8}$$

where

$$\zeta_1 = \frac{\gamma}{j\rho c_1^2}, \quad \zeta_2 = \frac{K}{j\rho\omega^{*2}}, \quad \zeta_3 = \frac{2K}{j\rho\omega^{*2}}, \quad \zeta_1^* = \frac{K}{\rho c_1^2}, \quad \zeta_2^* = \frac{\rho c_1^2}{\beta_1 T_0}, \quad \zeta_3^* = \frac{\lambda_0}{\rho c_1^2}, \quad \delta^2 = \frac{\lambda+\mu}{\rho c_1^2},$$

$$\ell_1^* = \frac{T_0 \beta_1^2}{\rho K^* \omega^*}, \quad \ell_2^* = \frac{\beta_1 T_0 \nu_1}{\rho K^* \omega^*}, \quad \ell_3^* = \frac{\rho c_1^4 a}{\beta_2 K^* \omega^*}, \quad q_1^* = \frac{D\omega^* \beta_1^2}{\rho c_1^4}, \quad q_2^* = \frac{D\omega^* \beta_2 a}{\beta_1 c_1^2}, \quad q_3^* = \frac{Db\omega^*}{c_1^2},$$

$$q_4^* = \frac{D\nu_2 \beta_2 \omega^*}{\rho c_1^4}, \quad \chi_1^* = \frac{2\lambda}{\rho j_0 \omega^{*2}}, \quad \chi_2^* = \frac{2\lambda_0}{\rho j_0 \omega^{*2}}, \quad \chi_3^* = \frac{2\nu_1 c_1^2}{j_0 \beta_1 \omega^{*2}}, \quad \chi_4^* = \frac{2\nu_2 \rho c_1^4}{j_0 \beta_1 \beta_2 T_0 \omega^{*2}}, \quad \delta_1^2 = \frac{c_2^2}{c_1^2},$$

$$c_2^2 = \frac{2\alpha_0}{\rho j_0}, \quad \tau_t^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \quad \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \quad \tau_f^0 = 1 + \varepsilon \tau^0 \frac{\partial}{\partial t}, \quad \tau_t^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \quad \tau_e^0 = 1 + \varepsilon \tau_0 \frac{\partial}{\partial t},$$

$$\tau_c^0 = 1 + \gamma_1 \frac{\partial}{\partial t}, \; e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}, \; \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}.$$

We introduce the potential functions $\phi$ and $\psi$ through the relations

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}, \tag{3.9}$$

in the equations (3.3)-(3.8), we obtain

$$\nabla^2 \phi + \zeta_3^* \varphi^* - \tau_t^1 T - \zeta_2^* \tau_c^1 C = \ddot{\phi}, \tag{3.10}$$

$$\left(1-\delta^2\right)\nabla^2 \psi + \zeta_1^* \varphi_2 = \ddot{\psi}, \tag{3.11}$$

$$\left(\zeta_1 \nabla^2 - \zeta_3\right)\varphi_2 - \zeta_2 \nabla^2 \psi = \ddot{\varphi}_2, \tag{3.12}$$

$$\left(\delta_1^2 \nabla^2 - \chi_1^*\right)\varphi^* - \chi_2^* \nabla^2 \phi + \chi_3^* \tau_t^1 T + \chi_4^* \tau_c^1 C = \ddot{\varphi}^*, \tag{3.13}$$

$$\nabla^2 T = \tau_e^0\left(\ell_1^*\nabla^2\dot{\phi} + \ell_2^*\dot{\varphi}^*\right) + \tau_t^0\dot{T} + \ell_3^*\tau_c^0\dot{C}, \tag{3.14}$$

$$q_1^*\nabla^4\phi + q_4^*\nabla^2\varphi^* + q_2^*\tau_t^1\nabla^2 T - q_3^*\tau_c^1\nabla^2 C + \tau_f^0\dot{C} = 0, \tag{3.15}$$

For the propagation of plane waves in $x_1x_3-$ plane, we assume

$$\left\{\phi, T, C, \varphi^*, \psi, \varphi_2\right\}(x_1, x_3, t) = \left\{\bar{\phi}, \bar{T}, \bar{C}, \bar{\varphi}^*, \bar{\psi}, \bar{\varphi}_2\right\} e^{-i\omega t}, \tag{3.16}$$

where $\omega$ is the angular frequency.
Substituting the values of $\phi, \psi, T, C, \varphi^*, \varphi_2$ from equation (3.16) in the equations (3.10)-(3.15), we obtain

$$\left(\nabla^2 + \omega^2\right)\bar{\varphi} + \zeta_3^*\bar{\varphi}^* - \tau_t''\bar{T} - \zeta_2^*\tau_c''\bar{C} = 0, \tag{3.17}$$

$$\left(\left(1-\delta^2\right)\nabla^2 + \omega^2\right)\bar{\psi} + \zeta_1^*\bar{\varphi}_2 = 0, \tag{3.18}$$

$$\zeta_2\nabla^2\bar{\psi} + \left(-\omega^2 - \zeta_1\nabla^2 + \zeta_3\right)\bar{\varphi}_2 = 0, \tag{3.19}$$

$$-\chi_2^*\nabla^2\bar{\varphi} + \chi_3^*\tau_t''\bar{T} + r_1\bar{\varphi}^* + r_2\bar{C} = 0, \tag{3.20}$$

$$\ell_1^*\tau_e^{10}\nabla^2\bar{\varphi} + \left(\tau_t^{10} - \nabla^2\right)\bar{T} + \ell_2^*\tau_e^{10}\bar{\varphi}^* + \ell_3^*\tau_c^{10}\bar{C} = 0, \tag{3.21}$$

$$q_1^*\nabla^4\bar{\varphi} + q_4^*\nabla^2\varphi^* + q_2^*\tau_t''\nabla^2\bar{T} + \left(\tau_f^{10} - q_3^*\tau_c''\nabla^2\right)\bar{C} = 0, \tag{3.22}$$

where $r_1 = \delta_1^2\nabla^2 - \chi_1^* + \omega^2$, $r_2 = \chi_4^*(1-i\omega)$, $\tau_t^{11} = (1-i\omega\tau_1)$, $\tau_c^{11} = (1-i\omega\tau^1)$, $\tau_e^{10} = -i\omega(1-i\omega\varepsilon\tau_0)$, $\tau_t^{10} = -i\omega(1-i\omega\tau_0)$, $\tau_c^{10} = -i\omega(1-i\omega\gamma)$, $\tau_f^{10} = -i\omega(1-i\omega\varepsilon\tau^0)$.
The system of equations (3.17), (3.20)-(3.22) has a non-trivial solution if the determinant of the coefficients $\left[\bar{\varphi}, \bar{T}, \bar{C}, \bar{\varphi}^*\right]^T$ vanishes, which yields to the following polynomial characteristic equation

$$\nabla^8 + B_1\nabla^6 + B_2\nabla^4 + B_3\nabla^2 + B_4 = 0, \tag{3.23}$$

where $B_i = A_i/A$ for $(i = 1,2,3,4)$, $A = g_1^* - a_{14}g_{14}^*$, $A_1 = g_2^* + g_1^*\omega^2 - a_{12}g_6^* + a_{13}g_9^* - a_{14}g_{12}^*$, $A_2 = g_3^* + g_2^*\omega^2 - a_{12}g_7^* + a_{13}g_{10}^* - a_{14}g_{13}^*$, $A_3 = g_4^* + g_3^*\omega^2 - a_{12}g_8^* + a_{13}g_{11}^*$, $A_4 = g_4^*\omega^2$,
and

$g_1^* = -\delta_1^2 a_{46}, g_2^* = a_{23}a_{46} - a_{24}a_{43} + \delta_1^2(a_{32}a_{46} + a_{45} + a_{34}a_{42})$, $g_4^* = a_{45}(a_{22}a_{33} + a_{23}a_{32})$,
$g_3^* = -a_{33}(a_{22}a_{46} + a_{24}a_{42}) - a_{23}(a_{32}a_{46} + a_{45} + a_{34}a_{42}) + a_{43}(a_{24}a_{32} - a_{22}a_{34}) - \delta_1^2 a_{32}a_{45}$,
$g_6^* = \delta_1^2(a_{31}a_{46} + a_{41}a_{34})$, $g_7^* = a_{33}(-a_{21}a_{46} - a_{24}a_{41}) - a_{23}(a_{31}a_{46} + a_{34}a_{41}) + \left(a_{24}a_{31} - a_{21}a_{34}\right)a_{43} - \delta_1^2 a_{31}a_{45}$,
$g_8^* = a_{45}(a_{23}a_{31} + a_{21}a_{33})$, $g_9^* = a_{24}a_{41} + a_{21}a_{46}$,
$g_{10}^* = -a_{21}(a_{32}a_{46} + a_{45} + a_{34}a_{42}) + a_{22}(a_{31}a_{46} + a_{34}a_{41}) + a_{24}(a_{31}a_{42} - a_{32}a_{41})$,
$g_{11}^* = a_{45}(a_{21}a_{32} - a_{22}a_{31})$, $g_{12}^* = -\left(a_{23}a_{41} + a_{21}a_{43}\right) + \delta_1^2(a_{31}a_{42} - a_{41}a_{32})$, $a_{45} = \tau_f^{10}$, $a_{46} = q_3^*\tau_c^{11}$,
$g_{13}^* = a_{33}(a_{22}a_{41} - a_{21}a_{42}) + a_{23}(a_{32}a_{41} - a_{31}a_{42}) + a_{43}(a_{21}a_{32} - a_{22}a_{31})$, $g_{14}^* = \delta_1^2 a_{41}$, $a_{11} = \nabla^2 + \omega^2$,

$a_{21}=-\chi_2^*,\quad a_{31}=\ell_1^*\tau_e^{10},\quad a_{41}=q_1^*,\quad a_{12}=-\tau_t^{11},\quad a_{22}=\chi_3^*\tau_t^{11},\quad a_{32}=\tau_t^{10},\quad a_{42}=q_2^*\tau_t^{11},\quad a_{13}=\zeta_3^*,$
$a_{23}=\chi_1^*-\omega^2,\ a_{33}=\ell_2^*\tau_e^{10},\ a_{14}=-\zeta_2^*\tau_c^{11},\ a_{24}=r_2,\ a_{34}=\ell_3^*\tau_c^{10},\ a_{43}=q_4^*\nabla^2,\ a_{44}=(a_{45}-a_{46}\nabla^2)$.

The general solution of equation (3.23) can be written as

$$\bar{\varphi}=\sum_{i=1}^{4}\bar{\varphi}_i\,, \tag{3.24}$$

where the potentials $\bar{\varphi}_i,\ i=1,2,3,4$ are solutions of wave equations, given by

$$\left[\nabla^2+\frac{\omega^2}{V_i^2}\right]\bar{\varphi}_i=0,\ i=1,2,3,4 \tag{3.25}$$

Here $\left(V_i^2, i=1,2,3,4\right)$ are the velocities of four longitudinal waves in the descending order and derived from the roots of the biquadratic equation in $V^2$, named as, longitudinal displacement wave (LD), thermal wave (T), mass diffusion wave (MD) and longitudinal microstretch wave (LM), given by

$$\left(B_4V^8-B_3\omega^2V^6+B_2\omega^4V^4-B_1\omega^6V^2+\omega^8\right)=0\,. \tag{3.26}$$

Using the equation (3.24) in the equations (3.17), (3.20)-(3.22), with the aid of equations (3.16) and (3.25), we obtain the general solutions for $\varphi, T, C$ , and $\varphi^*$ as

$$(\varphi,T,C,\varphi^*)=\sum_{1}^{4}(1,k_{1i},k_{2i},k_{3i})\bar{\varphi}_i\,, \tag{3.27}$$

where $k_{1i}=(g_6^*\omega^6-g_7^*\omega^4V_i^2+g_8^*\omega^2V_i^4)/k^d$, $k_{2i}=(-g_{14}^*\omega^8+g_{12}^*\omega^6V_i^2-g_{13}^*\omega^4V_i^4)/(V_i^2k^d)$,
$k_{3i}=-(g_9^*\omega^6+g_{10}^*\omega^4V_i^2+g_{11}^*\omega^2V_i^4)/k^d$, $k^d=(g_1^*\omega^6+g_2^*\omega^4V_i^2+g_3^*\omega^2V_i^4+g_4^*V_i^6),\ i=1,2,3,4$.

The system of equations (3.18)-(3.19) has a non-trivial solution if the determinant of the coefficients $\left[\bar{\psi},\bar{\varphi}_2\right]^T$ vanishes, which yields the following polynomial characteristic equation

$$\nabla^4+A^*\nabla^2+B^*=0\,, \tag{3.28}$$

where $A^*=\left(\omega^2\zeta_1+\zeta_1^*\zeta_2-\left(1-\delta^2\right)\left(\zeta_3+\omega^2\right)\right)/\left(1-\delta^2\right)\zeta_1$, $B^*=\omega^2\left(\omega^2-\zeta_3\right)/\left(1-\delta^2\right)\zeta_1$.

The general solution of equation (3.28) can be written as

$$\bar{\psi}=\sum_{i=5}^{6}\bar{\psi}_i\,, \tag{3.29}$$

where the potentials $\bar{\psi}_i,\ i=1,2$ are solutions of wave equations, given by

$$\left[\nabla^2+\frac{\omega^2}{V_i^2}\right]\bar{\psi}_i=0,\ i=5,6\,. \tag{3.30}$$

Here $\left(V_i^2, i=5,6\right)$ are the velocities of two coupled transverse waves in the descending order and derived from the root of quadratic equation in $V^2$, named as, transverse displacement wave (CD I) and transverse microrotational wave (CD II), given by

$$\left(B^{*}V^{4}-A^{*}\omega^{2}V^{2}+\omega^{4}\right)=0. \tag{3.31}$$

Making use of equation (3.29) in the equations (3.18)-(3.19) with the aid of equations (3.16) and (3.30), the general solutions for $\psi$ and $\varphi_2$ are obtained as

$$\{\psi,\varphi_2\}=\sum_{i=5}^{6}\{1,n_{1i}\}\bar{\psi}_i \ , \tag{3.32}$$

where $n_{1i}=\dfrac{\zeta_2\omega^2}{\left(\zeta_3-\omega^2\right)V_i^2+\zeta_1\omega^2}$ for $i=5,6$.

Making the use of (3.1) in the equation (2.9), with the aid of dimensionless quantities (3.2) and after suppressing the primes, yield

$$\left(\frac{\alpha^{e^2}-\beta^{e^2}}{c_1^2}\right)\left(\frac{\partial e^e}{\partial x_1}\right)+\frac{\beta^{e^2}}{c_1^2}\nabla^2u_1^e=\ddot{u}_1^e, \tag{3.33}$$

$$\left(\frac{\alpha^{e^2}-\beta^{e^2}}{c_1^2}\right)\left(\frac{\partial e^e}{\partial x_3}\right)+\frac{\beta^{e^2}}{c_1^2}\nabla^2u_3^e=\ddot{u}_3^e, \tag{3.34}$$

where $e^e=\left(\dfrac{\partial u_1^e}{\partial x_1}+\dfrac{\partial u_3^e}{\partial x_3}\right)$, and $\alpha^e=\sqrt{\left(\lambda^e+2\mu^e\right)/\rho^e}$, $\beta^e=\sqrt{\mu^e/\rho^e}$ are velocities of longitudinal wave (P-wave) and transverse wave (SV-wave) corresponding to $M_1$, respectively.

The components of $u_1^e$ and $u_3^e$ are related by the potential functions as:

$$u_1^e=\frac{\partial\varphi^e}{\partial x_1}-\frac{\partial\psi^e}{\partial x_3},\ u_3^e=\frac{\partial\varphi^e}{\partial x_3}+\frac{\partial\psi^e}{\partial x_1}, \tag{3.35}$$

where $\varphi^e$ and $\psi^e$ satisfy the wave equations as

$$\nabla^2\phi^e=\frac{\ddot{\phi}^e}{\alpha^2},\ \nabla^2\psi^e=\frac{\ddot{\psi}^e}{\beta^2}, \tag{3.36}$$

and $\alpha=\alpha^e/c_1$, $\beta=\beta^e/c_1$.

## 4. Reflection and refraction

We consider a plane harmonic wave (P or SV) propagating through the isotropic elastic solid half-space and is incident at the interface $x_3=0$ as shown in Fig. 1. Corresponding to each incident wave, two homogeneous waves (P and SV) are reflected in an isotropic elastic solid and six inhomogeneous waves (LD, T, MD, LM, CD I and CD II) are transmitted in isotropic microstretch thermoelastic diffusion solid half-space.

In elastic solid half-space, the potential functions satisfying equation (3.36) can be written as

$$\phi^e=A_0^e e^{\{i\omega(x_1\sin\theta_0+x_3\cos\theta_0)/\alpha-t\}}+A_1^e e^{\{i\omega(x_1\sin\theta_1-x_3\cos\theta_1)/\alpha-t\}}, \tag{4.1}$$

$$\psi^e = B_0^e e^{\{i\omega(x_1 \sin\theta_0 + x_3 \cos\theta_0)/\beta - t\}} + B_1^e e^{\{i\omega(x_1 \sin\theta_2 - x_3 \cos\theta_2)/\beta - t\}} . \tag{4.2}$$

The coefficients $A_0^e$ ($B_0^e$), $A_1^e$ and $B_1^e$ are amplitudes of the incident P (or SV), reflected P and reflected SV waves respectively.

Following Borcherdt [27], in a homogeneous isotropic microstretch thermoelastic diffusion half-space, potential functions satisfying equations (3.25) and (3.30) can be written as

$$\left(\varphi, T, C, \varphi^*\right) = \sum_{i=1}^{4} \left\{1, k_{1i}, k_{2i}, k_{3i}\right\} B_i e^{(\vec{A}_i.\vec{r})} e^{\{i(\vec{P}_i.\vec{r} - \omega t)\}} , \tag{4.3}$$

$$\left(\psi, \phi_2\right) = \sum_{i=5}^{6} \{1, n_{ip}\} B_i \, e^{(\vec{A}_i.\vec{r})} e^{\{i(\vec{P}_i.\vec{r} - \omega t)\}} , \tag{4.4}$$

The coefficients $B_i$, $i = 1,2,3,4,5,6$ are the amplitudes of refracted waves. The propagation vector $\vec{P}_i$, $i = 1,2,3,4,5,6$ and attenuation $\vec{A}_i$ factor ($i = 1,2,3,4,5,6$) are given by

$$\vec{P}_i = \xi_R \hat{x}_1 + dV_{iR} \hat{x}_3 , \quad \vec{A}_i = -\xi_I \hat{x}_1 - dV_{iI} \hat{x}_3 , \; i = 1,2,3,4,5,6 \tag{4.5}$$

where

$$dV_i = dV_{iR} + i\, dV_{iI} = \text{p.v.} \left( \frac{\omega^2}{V_i^2} - \xi^2 \right)^{1/2} , \; i = 1,2,3,4,5,6 \tag{4.6}$$

and $\xi = \xi_R + i\xi_I$ is the complex wave number. The subscripts R and I denote the real and imaginary parts of the corresponding complex number and p.v. stands for the principal value of the complex quantity derived from square root. $\xi_R \geq 0$ ensures propagation in positive $x_1$-direction. The complex wave number $\xi$ in the microstretch thermoelastic diffusion medium is given by

$$\xi = \left|\vec{P}_i\right| \sin\theta_i' - i \left|\vec{A}_i\right| \sin(\theta_i' - \gamma_i) , \; i = 1,2,3,4,5,6 \tag{4.7}$$

where $\gamma_i$, $i = 1,2,3,4,5,6$ is the angle between the propagation and attenuation vector and $\theta_i'$, $i = 1,2,3,4,5,6$ is the angle of refraction in medium II.

## 5. Boundary conditions

Murti [2] proposed boundary conditions for a loosely bonded interface with the assumption that there is a thin interface layer of viscous liquid between two half-spaces.

Following Murty [2] and Kumar et al. [25], the appropriate boundary conditions at the loosely bonded interface $x_3 = 0$ are defined by

Continuity of the normal stress component

$$t_{33}^e = t_{33} . \tag{5.1$_1$}$$

Continuity of the tangential stress component

$$t_{31}^e = t_{31} . \tag{5.1$_2$}$$

Continuity of the normal displacement component

$$u_3^e = u_3 . \tag{5.1$_3$}$$

Vanishing of the tangential couple stress component

$$m_{32} = 0. \tag{5.1$_4$}$$

Vanishing of the microstress component

$$\lambda_3^* = 0 . \tag{5.1$_5$}$$

Vanishing the temperature gradient

$$\frac{\partial T}{\partial x_3} = 0 . \tag{5.1$_6$}$$

Vanishing the mass concentration

$$\frac{\partial C}{\partial x_3} = 0 . \tag{5.1$_7$}$$

Proportionality between the shear stress and the slip

$$t_{31} = \eta \frac{\partial \dot{u}_1}{\partial x_3} . \tag{5.1$_8$}$$

Following Kumar et al. [25], boundary condition $(5.1)_8$ may be rewritten as

$$t_{31} = ik \frac{V_0}{\sin\theta_0} \left( \frac{\psi^*}{1-\psi^*} \right) \left( u_1 - u_1^e \right), \tag{5.1$_9$}$$

where $\eta$ be the coefficient of viscosity, $\theta_0 = \sin^{-1}\left( \frac{kV_0}{\omega} \right)$ is the angle of the incidence, and $V_0$ is the velocity of the incident wave. Here, $(V_0, \theta_0)$ is replaced with $(V_j, \theta_j)$, where $j$ takes values from 1 to 2 depending upon the type of the incident wave. It is convenient to introduce a variable $\psi^*$ $(0 \le \psi^* \le 1)$ such that, $\psi^* = 0$ corresponds to a smooth, surface $\psi^* = 1$ corresponds to a welded interface, and any intermediate value corresponds to a loosely bonded interface between two half-spaces. This $\psi^*$ may be considered as a bonding constant. Making use of potentials given by equations (4.1)-(4.4), we find that the boundary conditions are satisfied if and only if

$$\xi_R = \frac{\omega \sin\theta_0}{V_0} = \frac{\omega \sin\theta_1}{\alpha} = \frac{\omega \sin\theta_2}{\beta} \tag{5.2}$$

and

$$\xi_I = 0. \tag{5.3}$$

where

$$V_0 = \begin{cases} \alpha, \text{ for incident } P-\text{wave} \\ \beta, \text{ for incident } SV-\text{wave} \end{cases}, \tag{5.4}$$

It means that waves are attenuating only in $x_3$-direction. From equation (4.7), it implies that if $\left|\vec{A}_i\right| \neq 0$, then $\gamma_i^{'} = \theta_i^{'}$, $i = 1,2,3,4,5,6$, that is, attenuated vectors for the six refracted waves are directed along the $x_3$-axis.

Using equations (4.1)-(4.4) in the boundary conditions (5.1) and with the aid of equations (3.9), (3.35), (5.2)-(5.4), we get a system of eight non-homogeneous equations which can be written as

$$\sum_{j=1}^{8} d_{ij} Z_j = g_i, \tag{5.5}$$

where $Z_j = \left|Z_j\right| e^{i\psi_j^*}$, $\left|Z_j\right|$, $\psi_j^*$, $j = 1,2,3,4,5,6,7,8$ represent amplitude ratios and phase shift of reflected P-, reflected SV-, refracted LD-, refracted T-, refracted MD-, refracted LM-, refracted CD I -, refracted CD II - waves to that of amplitude of incident wave, respectively.

$$d_{11} = \left[ 2\mu^e \left(\frac{\xi_R}{\omega}\right)^2 - \rho^e c_1^2 \right], \qquad d_{12} = \left[ 2\mu^e \left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_\beta}{\omega}\right) \right], \qquad d_{17} = \left[ (2\mu + K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_5}{\omega}\right) \right],$$

$$d_{1j} = \left[ \lambda \left(\frac{\xi_R}{\omega}\right)^2 + \rho c_1^2 \left(\frac{dV_j}{\omega}\right)^2 + \rho c_1^2 \left( \frac{k_{1j}\tau_t^{''} + k_{3j}\tau_c^{''}}{\omega^2} \right) - \frac{\lambda_0 k_{2j}}{\omega^2} \right] \quad \text{for} \quad (j = 3,4,5,6)$$

$$d_{18} = \left[ (2\mu + K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_6}{\omega}\right) \right], \qquad d_{21} = 2\mu^e \left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_\alpha}{\omega}\right), \qquad d_{22} = \mu^e \left[ \left(\frac{dV_\beta}{\omega}\right)^2 - \left(\frac{\xi_R}{\omega}\right)^2 \right],$$

$$d_{2j} = (2\mu + K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_j}{\omega}\right) \text{ for } (j = 3,4,5,6), \quad d_{27} = \left[ \mu \left(\frac{\xi_R}{\omega}\right)^2 - (\mu + K)\left(\frac{dV_5}{\omega}\right)^2 - \frac{Kn_{5p}}{\omega^2} \right],$$

$$d_{28} = \left[ \mu \left(\frac{\xi_R}{\omega}\right)^2 - (\mu + K)\left(\frac{dV_6}{\omega}\right)^2 - \frac{Kn_{6p}}{\omega^2} \right], \; d_{31} = -\left(\frac{dV_\alpha}{\omega}\right), \; d_{32} = \left(\frac{\xi_R}{\omega}\right), \; d_{3j} = -\left(\frac{dV_{j-2}}{\omega}\right) \text{ for}$$

$(j = 3,4,5,6)$, $d_{37} = -\left(\frac{\xi_R}{\omega}\right)$, $d_{38} = -\left(\frac{\xi_R}{\omega}\right)$, $d_{4j} = 0$ for $(j = 1,2)$, $d_{4j} = b_o k_{2(j-2)}\left(\frac{\xi_R}{\omega}\right)$ for

$(j=3,4,5,6)$, $d_{4j} = \gamma\left(\frac{dV_{j-2}}{\omega}\right) n_{(j-2)p}$ for $(j=7,8)$, $d_{5j} = 0$ for $(j = 1,2)$, $d_{5j} = \alpha_O k_{2(j-2)}\left(\frac{dV_{j-2}}{\omega}\right)$

for $(j=3,4,5,6)$, $d_{6j} = 0$ for $(j=1,2,7,8)$, $d_{5j} = -b_o n_{(j-2)p}\left(\frac{\xi_R}{\omega}\right)$ for $(j=7,8)$, $d_{6j} = k_{1(j-2)}\left(\frac{dV_{j-2}}{\omega}\right)$

for $(j = 3,4,5,6)$, $d_{7j} = 0$ for $(j = 1,2,7,8)$, $d_{7j} = k_{3(j-2)}\left(\frac{dV_{j-2}}{\omega}\right)$ for $(j = 3,4,5,6)$,

$$d_{81} = \left(\frac{\xi_R}{\omega}\right), \qquad d_{82} = \left(-\frac{dV_\beta}{\omega}\right), \qquad d_{8j} = -\left(\frac{\xi_R}{\omega}\right)\left[ 1 + c_1 (2\mu + K)\left(\frac{1-\psi^*}{\psi^*}\right)\left(\frac{dV_j}{\omega}\right)\left(\frac{\xi_R}{\omega}\right) \right] \quad \text{for}$$

$$(j = 3,4,5,6), \; d_{87} = \left[ \left(\frac{dV_5}{\omega}\right) + c_1 \left(\frac{1-\psi^*}{\psi^*}\right)\left(\frac{\xi_R}{\omega}\right)\left( (\mu + K)\left(\frac{dV_5}{\omega}\right)^2 - \mu\left(\frac{\xi_R}{\omega}\right)^2 \right) \right],$$

$$d_{88} = \left[ \left(\frac{dV_6}{\omega}\right) + c_1 \left(\frac{1-\psi^*}{\psi^*}\right)\left(\frac{\xi_R}{\omega}\right)\left( (\mu + K)\left(\frac{dV_6}{\omega}\right)^2 - \mu\left(\frac{\xi_R}{\omega}\right)^2 \right) \right],$$

$$\left(\frac{dV_\alpha}{\omega}\right)=\left[\frac{1}{\alpha^2}-\left(\frac{\xi_R}{\omega}\right)^2\right]^{1/2}=\left[\frac{1}{\alpha^2}-\left(\frac{\sin\theta_0}{V_0}\right)^2\right]^{1/2},$$

$$\left(\frac{dV_\beta}{\omega}\right)=\left[\frac{1}{\alpha^2}-\left(\frac{\xi_R}{\omega}\right)^2\right]^{1/2}=\left[\frac{1}{\beta^2}-\left(\frac{\sin\theta_0}{V_0}\right)^2\right]^{1/2}, \qquad \left(\frac{dV_j}{\omega}\right)=p.v.\left[\frac{1}{V_j^{\,2}}-\left(\frac{\sin\theta_0}{V_0}\right)^2\right]^{1/2} \quad \text{for}$$

$(j=1,2,3,4,5,6)$.

Here p.v. is evaluated with restriction $dV_{jI} \geq 0$ to satisfy decay condition in the microstretch thermoelastic diffusion medium. The coefficients $g_i$, $for\ (i=1,2,3,4,5,6,7,8)$ on the right side of the equation (5.5) are given by

(i) For incident P-wave

$$g_i=(-1)^i d_{i1} \text{ for } (i=1,2),\ g_i=(-1)^{i+1}d_{i1} \text{ for } (i=3,8),\ g_i=0 \text{ for } (i=4,5,6,7), \tag{5.6}$$

(ii) For incident SV-wave

$$g_i=(-1)^{i+1}d_{i2} \text{ for } (i=1,2),\ g_i=-d_{i2} \text{ for } (i=3,8),\ g_i=0 \text{ for } (i=4,5,6,7). \tag{5.7}$$

Now we consider a surface element of unit area at the interface between two media. The reason for this consideration is to calculate the partition of energy of the incident wave among the reflected and refracted waves on the both sides of surface. Following Achenbach [28], the energy flux across the surface element, that is, rate at which the energy is communicated per unit area of the surface is represented as

$$P^* = t_{lm}\, l_m\, \dot{u}_l, \tag{5.8}$$

where $t_{lm}$ is the stress tensor, $l_m$ are the direction cosines of the unit normal $\hat{l}$ outward to the surface element and $\dot{u}_l$ are the components of the particle velocity. The time average of $P^*$ over a period, denoted by $< P^* >$, represents the average energy transmission per unit surface area per unit time. Thus, on the surface with normal along $x_3$-direction, the average energy intensities of the waves in the elastic solid are given by

$$< P^{*e} > = \mathrm{Re} < t >_{13}^{e} .\mathrm{Re}\,(\dot{u}_1^e) + \mathrm{Re} < t >_{33}^{e} .\mathrm{Re}\,(\dot{u}_3^e). \tag{5.9}$$

Following Achenbach [28], for any two complex functions f and g, we have

$$< \mathrm{Re}\,(f).\mathrm{Re}\,(g) > = ½\ \mathrm{Re}\,(f\,\bar{g}\,). \tag{5.10}$$

The expressions for energy ratios $E_i$, $i$ = 1, 2 for the reflected P and reflected SV are given by

$$E_i=-\frac{< P_i^{*e} >}{< P_0^{*e} >} \quad for\ (i=1,2), \tag{5.11}$$

where $< P_1^{*e} > = \dfrac{\omega^4\rho^e c_1^2}{2\alpha}|Z_1|^2\,\mathrm{Re}(\cos\theta_1)$, $< P_2^{*e} > = \dfrac{\omega^4\rho^e c_1^2}{2\beta}|Z_2|^2\,\mathrm{Re}(\cos\theta_2)$ and

(i) For incident P- wave

$$< P_0^{*e} > = -\frac{\omega^4\rho^e c_1^2\cos\theta_0}{2\alpha}, \tag{5.12}$$

(ii) For incident SV- wave

$$< P_0^{*e} > = -\frac{\omega^4 \rho^e c_1^2 \cos\theta_0}{2\beta}, \tag{5.13}$$

are the average energy intensities of the reflected P-, reflected SV-, incident P- and incident SV-waves respectively. In equation (5.11) negative sign is taken because the direction of reflected waves is opposite to that of incident wave.

For microstretch thermoelastic diffusion medium, the average energy intensities of the waves on the surface with normal along $x_3$-direction, are given by

$$< P_{ij}^* > = \mathrm{Re} < t >_{13}^{(i)} \mathrm{Re}(\dot{u}_1^{(j)}) + \mathrm{Re} < t >_{33}^{(i)} \mathrm{Re}(\dot{u}_3^{(j)}) + \mathrm{Re} < m >_{32}^{(i)} \mathrm{Re}(\dot{\phi}_2^{(j)}) + \mathrm{Re} < \lambda_3^* >^{(i)} \mathrm{Re}(\dot{\varphi}^{*(j)}). \tag{5.14}$$

The expressions for the energy ratios $E_{ij}$ *for* $(i, j = 1,2,3,4,5,6)$ for the refracted waves are given by

$$E_{ij} = \frac{< P_{ij}^* >}{< P_0^{*e} >} \text{ for } (i, j = 1,2,3,4,5,6), \tag{5.15}$$

where

$$<P_{ij}^*> = -\frac{\omega^4}{2}\mathrm{Re}\left[(2\mu+K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_i}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) + \left\{\begin{array}{l}\lambda\left(\frac{\xi_R}{\omega}\right)^2 + \rho c_1^2\left(\frac{dV_i}{\omega}\right)^2 + \\ \rho c_1^2\left(\frac{k_{1i}\tau_t'' + k_{3i}\tau_c''}{\omega^2}\right) - \frac{\lambda_0 k_{2i}}{\omega^2}\end{array}\right\}\left(\frac{d\bar{V}_j}{\omega}\right) + \frac{\alpha_0 k_{2i}\bar{k}_{2j}\omega^{*2}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right)\right] Z_{i+2}\bar{Z}_{j+2}$$

for $(i, j = 1,2,3,4)$,

$$< P_{ij}^* > = -\frac{\omega^4}{2}\mathrm{Re}\left[\begin{array}{l}-\left(\mu\left(\frac{\xi_R}{\omega}\right)^2 - (\mu+K)\left(\frac{dV_i}{\omega}\right)^2 + \frac{Kn_{ip}}{\omega^2}\right)\left(\frac{d\bar{V}_j}{\omega}\right) + \\ (2\mu+K)\left(\frac{dV_i}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) + \frac{\omega^{*2}\gamma n_{ip}\bar{n}_{jp}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right)\end{array}\right] Z_{i+2}\bar{Z}_{j+2}$$

for $(i, j = 5,6)$,

$$<P_{i5}^*> = \frac{-\omega^4}{2}\mathrm{Re}\left[-(2\mu+K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_i}{\omega}\right)\left(\frac{d\bar{V}_5}{\omega}\right) + \left\{\begin{array}{l}\lambda\left(\frac{\xi_R}{\omega}\right)^2 + \rho c_1^2\left(\frac{dV_i}{\omega}\right)^2 \\ +\rho c_1^2\left(\frac{k_{1i}\tau_t'' + k_{3i}\tau_c''}{\omega^2}\right) - \frac{\lambda_0 k_{2i}}{\omega^2}\end{array}\right\}\left(\frac{\bar{\xi}_R}{\omega}\right) - \frac{b_0\omega^{*2}\bar{n}_{5p}k_{2i}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\right] Z_{i+2}\bar{Z}_7$$

for $(i = 1,2,3,4)$,

$$< P_{i5}^* > = -\frac{\omega^4}{2}\mathrm{Re}\left[\begin{array}{l}-\left(\mu\left(\frac{\xi_R}{\omega}\right)^2 - (\mu+K)\left(\frac{dV_i}{\omega}\right)^2 + \frac{Kn_{ip}}{\omega^2}\right)\left(\frac{d\bar{V}_5}{\omega}\right) \\ +(2\mu+K)\left(\frac{dV_i}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) - \frac{\omega^{*2}\gamma n_{ip}\bar{n}_{5p}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right)\end{array}\right] Z_{i+2}\bar{Z}_7 \quad \text{for } (i = 5,6),$$

$$<P_{i6}^{*}> = -\frac{\omega^4}{2}\,\mathrm{Re}\left[-(2\mu+K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_i}{\omega}\right)\left(\frac{d\bar{V}_6}{\omega}\right)+\left\{\begin{array}{l}\lambda\left(\frac{\xi_R}{\omega}\right)^2+\rho c_1^2\left(\frac{dV_i}{\omega}\right)^2\\ +\rho c_1^2\left(\frac{k_{1i}\tau_t''+k_{3i}\tau_c''}{\omega^2}\right)-\frac{\lambda_0 k_{2i}}{\omega^2}\end{array}\right\}\left(\frac{\bar{\xi}_R}{\omega}\right)-\frac{b_o\omega^{*2}\bar{n}_{6p}k_{2i}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\right]Z_{i+2}\bar{Z}_8$$

for $(i = 1, 2, 3, 4)$,

$$< P_{i6}^{*} > = -\frac{\omega^4}{2}\,\mathrm{Re}\left[\begin{array}{l}-\left(\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_i}{\omega}\right)^2+\frac{Kn_{ip}}{\omega^2}\right)\left(\frac{d\bar{V}_6}{\omega}\right)\\ +(2\mu+K)\left(\frac{dV_i}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)-\frac{\omega^{*2}\gamma n_{ip}\bar{n}_{6p}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right)\end{array}\right]Z_{i+2}\bar{Z}_8 \qquad \text{for } (i = 5, 6),$$

$$<P_{5j}^{*}> = \frac{-\omega^4}{2}\,\mathrm{Re}\left[\begin{array}{l}\left(\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_5}{\omega}\right)^2+\frac{K\,n_{5p}}{\omega^2}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)\\ +\left\{(2\mu+K)\left(\frac{dV_5}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\right\}\left(\frac{d\bar{V}_j}{\omega}\right)-\frac{\omega^{*2}b_o n_{5p}\bar{k}_{2j}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\end{array}\right]Z_7\bar{Z}_{j+2} \qquad \text{for } (j = 1, 2, 3, 4),$$

$$<P_{5j}^{*}> = -\frac{\omega^4}{2}\,\mathrm{Re}\left[\begin{array}{l}-\left(\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_j}{\omega}\right)^2+\frac{Kn_{5p}}{\omega^2}\right)\left(\frac{d\bar{V}_j}{\omega}\right)+\\ (2\mu+K)\left(\frac{dV_5}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)+\frac{\gamma n_{5p}\bar{n}_{jp}\omega^{*2}}{\rho c_1^4\omega^2}\left(\frac{dV_5}{\omega}\right)\end{array}\right]Z_7\bar{Z}_{j+2} \qquad \text{for } (j = 5, 6),$$

$$<P_{6j}^{*}> = -\frac{\omega^4}{2}\,\mathrm{Re}\left[\begin{array}{l}\left(\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_6}{\omega}\right)^2+\frac{Kn_{6p}}{\omega^2}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)\\ +\left\{(2\mu+K)\left(\frac{dV_6}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\right\}\left(\frac{d\bar{V}_j}{\omega}\right)-\frac{\omega^{*2}b_o n_{6p}\bar{k}_{2j}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\end{array}\right]Z_8\bar{Z}_{j+2} \qquad \text{for } (j = 1, 2, 3, 4),$$

$$<P_{6j}^{*}> = -\frac{\omega^4}{2}\,\mathrm{Re}\left[\begin{array}{l}-\left(\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_j}{\omega}\right)^2+Kn_{jp}\right)\left(\frac{d\bar{V}_j}{\omega}\right)+\\ (2\mu+K)\left(\frac{dV_6}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)+\frac{\gamma n_{6p}\bar{n}_{jp}\omega^{*2}}{\rho c_1^4\omega^2}\left(\frac{dV_6}{\omega}\right)\end{array}\right]Z_8\bar{Z}_{j+2} \qquad \text{for } (j = 5, 6). \tag{5.16}$$

The diagonal entries of energy matrix $E_{ij}$ in equation (5.15) represent the energy ratios of the waves, whereas sum of the non-diagonal entries of $E_{ij}$ give the share of interaction energy among all the refracted waves in the medium and is given by

$$\mathrm{E_{RR}} = \sum_{i=1}^{6}\left(\sum_{j=1}^{6}E_{ij} - E_{ii}\right). \tag{5.17}$$

The energy ratios $E_i$, $i$ = 1, 2, diagonal entries and sum of non-diagonal entries of energy matrix $E_{ij}$, that is, $E_{11}, E_{22}, E_{33}, E_{44}, E_{55}, E_{66}$, and $E_{RR}$ yield the conservation of incident energy across the interface, through the relation

$$E_1 + E_2 + E_{11} + E_{22} + E_{33} + E_{44} + E_{55} + E_{66} + E_{RR} = 1. \tag{5.18}$$

## 6. Particular cases

**(i)** Take $\tau^0 > 0, \varepsilon = 0$ and $\gamma_1 = \tau^0$ in equation (5.17), along with (5.11) and (5.15) yield the expressions of energy ratios for elastic/ microstretch thermoelastic diffusion solid half-spaces with two relaxation times.

**(ii)** Using $\tau_1 = \tau^1 = 0,\ \gamma_1 = \tau_0$ and $\varepsilon = 1$ in equations (5.17), together with (5.11) and (5.15) gives the corresponding results for elastic/ microstretch thermoelastic diffusion solid half-spaces with one relaxation time.

**(iii)** On taking $\tau_0 = \tau^0 = \tau_1 = \tau^1 = \gamma_1 = 0$ in equations (5.17), (5.11) and (5.15) provide the corresponding expression at elastic/ microstretch thermoelastic diffusion solid half-spaces with Coupled Thermoelastic (CT) theory.

**(iv)** If we neglect the stretch effect, the boundary conditions will be reduced to the $(5.1)_1$-$(5.1)_4$, $(5.1)_6$-$(5.1)_8$ and we obtain the energy ratios for elastic/ micropolar thermoelastic diffusion solid half-spaces and these results will be identical, if we directly solve the problem independently.

**(v)** If mass concentration effect is neglected, the boundary conditions in present problem will be reduced to $(5.1)_1$-$(5.1)_6$, $(5.1)_8$ and we obtain the energy ratios for elastic/microstretch generalized thermoelastic solid half-spaces. The resulting expressions obtained due to these changes will be similar, if we investigate the problem directly.

## 7. Numerical results and discussion

The analysis is conducted for a magnesium crystal-like material. Following Eringen [29], the values of micropolar parameters are $\lambda = 9.4\times10^{10}\ \mathrm{Nm^{-2}}$, $\mu = 4.0\times10^{10}\ \mathrm{Nm^{-2}}$, $K = 1.0\times10^{10}\ \mathrm{Nm^{-2}}$, $\rho = 1.74\times10^{3}\ \mathrm{Kgm^{-3}}$, $j = 0.2\times10^{-19}\mathrm{m^2}$, $\gamma = 0.779\times10^{-9}\mathrm{N}$. Thermal and diffusion parameters are given by $C^* = 1.04\times10^{3}\mathrm{JKg^{-1}K^{-1}}$, $\mathrm{K}^* = 1.7\times10^{6}\mathrm{Jm^{-1}s^{-1}K^{-1}}$, $\alpha_{t1} = 2.33\times10^{-5}\mathrm{K^{-1}}$, $\alpha_{t2} = 2.48\times10^{-5}\mathrm{K^{-1}}$, $T_0 = .298\times10^{3}\mathrm{K}$, $\tau_1 = 0.01$, $\tau_0 = 0.02$, $\alpha_{c1} = 2.65\times10^{-4}\mathrm{m^3Kg^{-1}}$, $\alpha_{c2} = 2.83\times10^{-4}\mathrm{m^3Kg^{-1}}$, $a = 2.9\times10^{4}\mathrm{m^2s^{-2}K^{-1}}$, $b = 32\times10^{5}\mathrm{Kg^{-1}m^5s^{-2}}$, $\tau^1 = 0.04$, $\tau^0 = 0.03$, $D = 0.85\times10^{-8}\mathrm{Kgm^{-3}s}$ and, the microstretch parameters are taken as $j_o = 0.19\times10^{-19}\mathrm{m^2}$, $\alpha_o = 0.779\times10^{-9}\mathrm{N}$, $b_o = 0.5\times10^{-9}\mathrm{N}$, $\lambda_o = 0.5\times10^{10}\mathrm{Nm^{-2}}$, $\lambda_1 = 0.5\times10^{10}\mathrm{Nm^{-2}}$. Following Bullen [30], the numerical data of granite for elastic medium is given by $\rho^e = 2.65\times10^{3}\mathrm{Kgm^{-3}}$, $\alpha^e = 5.27\times10^{3}\mathrm{ms^{-1}}$, $\beta^e = 3.17\times10^{3}\mathrm{ms^{-1}}$.

The MATLAB software 7.04 has been used to determine the values of energy ratios $E_i,\ i = 1,2$ and energy matrix $E_{ij},\ i,j = 1,2,3,4,5,6$ for some particular values of bonding parameter $\psi^*(0, 0.75\ \&\ 1)$. The energy ratios of reflected and transmitted waves are calculated in the previous section for different values of incident angle ($\theta_o$) ranging from $0^0$ to $90^0$ for fixed frequency $\omega = 2\times\pi\times100\ \mathrm{Hz}$. Corresponding to incident P wave, the variation of energy ratios with respect to angle of incident have been plotted in Figures (2)-(10). Similarly, corresponding to SV waves, the variation of energy ratios with respect to angle of incident have been plotted in Figures (11)-(19). In all figures, the words LS and GL symbolize the graphs of L-S and G-L theories in microstretch thermoelastic diffusion medium for the smooth surface $(\psi^* = 0)$, loosely bonded interface $(\psi^* = 0.75)$ and welded interface $(\psi^* = 1)$ and are represented by  ,  ,  and  ,  ,  respectively.

**Incident P-wave.**

Figures 2-10 depict the variation of energy ratios with the angle of incidence $(\theta_0)$ for P waves.

Figure 2 exhibits the variation of energy ratio $E_1$ with the angle of incidence ($\theta_o$). For each bonding parameter, the values of $E_1$ for both cases LS and GL decrease monotonically for the range $0^0 \le \theta_0 \le 40^0$ and consistently for $40^0 \le \theta_0 < 80^0$ and then increase sharply at $\theta_o = 80^0$, with difference in their magnitude values. Figure 3 depicts the variation of energy ratio $E_2$ with $\theta_o$ and it shows nearly opposite behavior to that of $E_1$, the values of $E_2$ increase with the increase in $\theta_o$ from $0^0$ to $50^0$ and then decrease monotonically for the range $80^0 \le \theta_0 \le 90^0$ for all the cases. Figure 4 depicts the variation of energy ratio $E_{11}$ with $\theta_o$ and it shows that the behavior and trend of variation are similar for LS and GL cases, but the values $E_{11}$ for the GL case are higher than LS case for smooth surface, loosely bonded interface and welded interface. Figure 5 exhibits the variation of energy ratio $E_{22}$ with $\theta_o$ and it shows that the values of $E_{22}$ are higher corresponding to $\psi^* = 0$ for both LS and GL cases which is also true for $\psi^* = 0.75 \,\&\, 1$, but, trends of the graph in case of loosely bonded and welded interface are contrary to that of smooth surface.

Figure 6 depicts the variation of energy ratio $E_{33}$ with $\theta_o$ and it indicates the values of $E_{33}$ in case of LS are very large as compared to the GL for the bonding parameter $\psi^* = 0.75$, while, the trend of the graphs for all other cases are similar which increase consistently within the range $0^0 \le \theta_0 < 80^0$ and decrease sharply near the end values of $\theta_o$ considered. Figure 7 depicts the variation of energy ratio $E_{44}$ with $\theta_o$ and it shows that the values of $E_{44}$ oscillates for the whole range of $\theta_o$ for all particular values of the bonding parameter and a significant difference in the magnitude values for both cases LS and GL can be perceived. Figure 8 exhibits the variation of energy ratio $E_{55}$ with $\theta_o$ and it indicates the behavior of the graphs in case of loose bonding is closely same to that of $\psi^* = 1$ and nearly opposites to that of $\psi^* = 0$ for both LS and GL cases. In case of smooth surface the value of $E_{55}$ first decrease suddenly within the range $0^0 \le \theta_0 < 15^0$, shows a consistent variation in the values and then decrease further for $80^0 \le \theta_0 \le 90^0$. Figure 9 shows the variation of $E_{66}$ with $\theta_o$ and it indicates that the behavior of the curves for value of $E_{66}$ is nearly same to that of the $E_{33}$ as shown in Fig. 6, however, the corresponding values are different in magnitude. For smooth surface, the values of $E_{66}$ in case of GL are higher as compared to LS case, but the trend of variation is opposite from loose bonded and welded interface. Figure 10 shows the variation of interaction energy ratio $E_{RR}$ with $\theta_o$ and it indicates the values of $E_{RR}$ for all the cases show a similar trend for different values of bonding parameter which decrease for $0^0 \le \theta_0 < 80^0$ and suddenly shows a sharp increase as $\theta_o$ increases further. It is further noticed that the interaction energy in both LS and GL cases attain the stationary values near the end of range for $\psi^* = 0.75 \,\&\, 1$.

**Incident SV-wave**

Figures 11-19 depict the variation of energy ratios with the angle of incidence $(\theta_0)$ for SV waves.

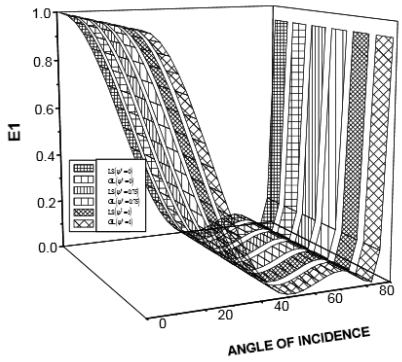


**Fig. 2.** Variation of energy ratio $E_1$ w.r.t. angle of incidence P-wave.

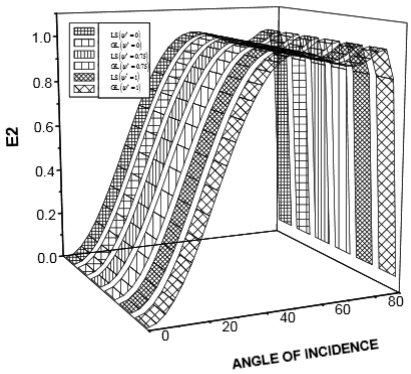


**Fig. 3.** Variation of energy ratio $E_2$ w.r.t. angle of incidence P-wave.

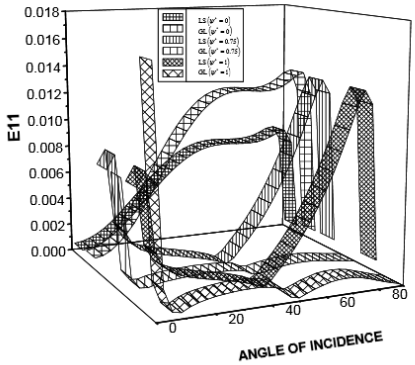


**Fig. 4.** Variation of energy ratio $E_{11}$ w.r.t. angle of incidence P-wave.

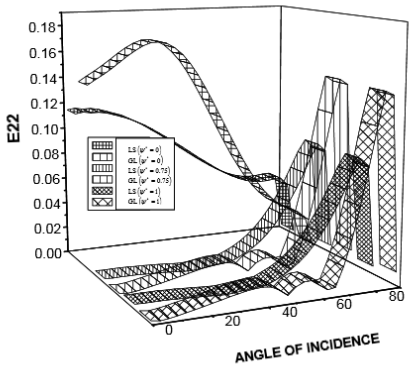


**Fig. 5.** Variation of energy ratio $E_{22}$ w.r.t. angle of incidence P-wave.

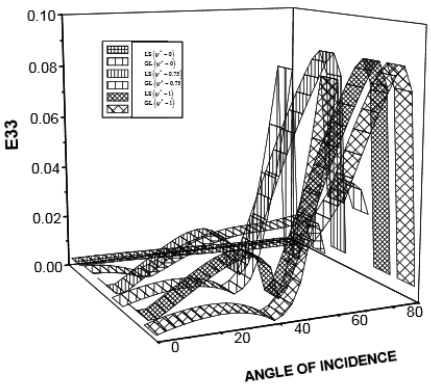


**Fig. 6.** Variation of energy ratio $E_{33}$ w.r.t. angle of incidence P-wave.

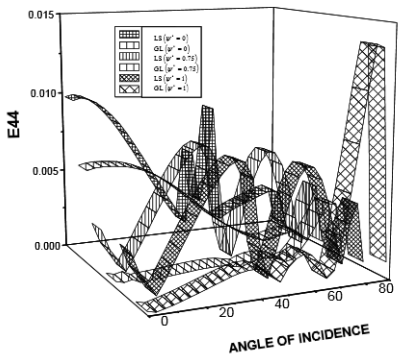


**Fig. 7.** Variation of energy ratio $E_{44}$ w.r.t. angle of incidence P-wave.

Figure 11 represents the variation of energy ratio $E_1$ with $\theta_0$ and it indicates that the behavior of the curves for the values of $E_1$ for both cases LS and GL are similar, however, the corresponding values are different in magnitude.

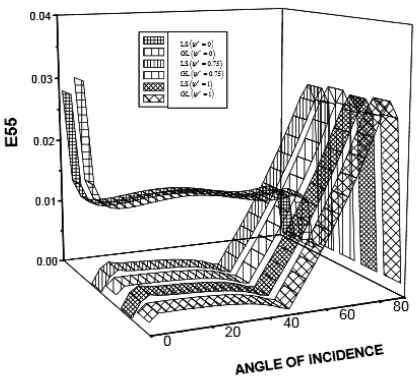


**Fig. 8.** Variation of energy ratio $E_{55}$ w.r.t. angle of incidence P-wave.

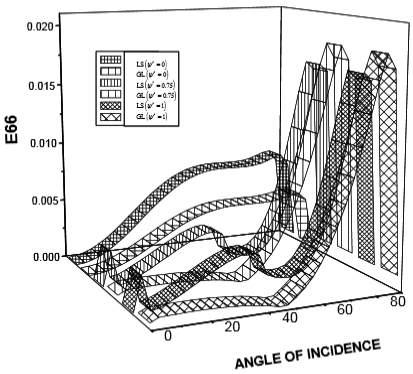


**Fig. 9.** Variation of energy ratio $E_{66}$ w.r.t. angle of incidence P-wave.

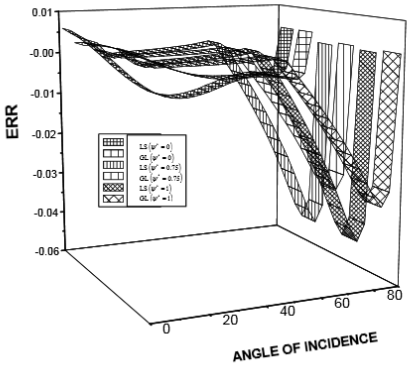


**Fig. 10.** Variation of energy ratio $E_{RR}$ w.r.t. angle of incidence P-wave.

The values of energy ratios increase monotonically for smaller values of $\theta_0$, whereas for higher values of $\theta_0$, the values of $E_1$ decrease and finally become dispersionless.

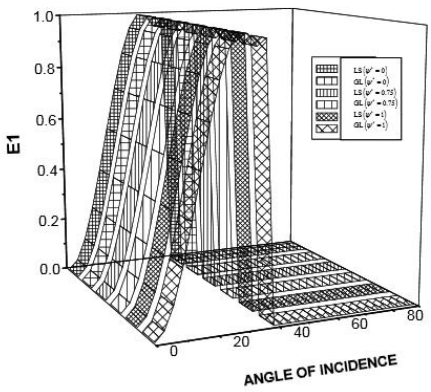


**Fig. 11.** Variation of energy ratio $E_1$ w.r.t. angle of incidence SV-wave.

Figure 12 shows the variation of energy ratio $E_2$ with $\theta_0$.and it indicates that the behavior of the curves is opposite to that of the Fig. 11. In this figure the values for both cases

LS and GL decrease when $0 \le \theta_0 < 20$ which sharply increase for $20 \le \theta_0 < 50$ and for higher values of $\theta_0$ the values $E_2$ become dispersionless. Figure 13 shows that for the whole range of $\theta_0$ the values of $E_{11}$ for different cases LS and GL show an oscillatory behavior with respect to different value of $\psi^*$. For the loosely bonded interface the values of $E_{11}$ in case of LS are higher as compared to GL and the same trend is noticed for smooth and welded interface. Figure 14 exhibits the variation of energy ratio $E_{22}$ with $\theta_0$ and it indicates that the values of $E_{22}$ oscillates for smaller values of $\theta_0$ although for higher values of $\theta_0$, the values of $E_{22}$ become constant. For higher values of $\theta_0$, it is noticed that the values of $E_{22}$ in case of LS remain more for smooth interface in comparison with other cases. Figure 15 depicts that the values of energy ratio $E_{33}$ oscillates for both LS and GL cases for all values of $\theta_0$ and it is noticed that, for higher values of $\theta_0$, the values of $E_{33}$ in case of GL remain more for welded interface in comparison with other cases. Figure 16 shows the variation of energy ratio $E_{44}$ with $\theta_0$ and it is evident that the behavior and variation of $E_{44}$ is similar as $E_{33}$ whereas magnitude values of $E_{44}$ are different.

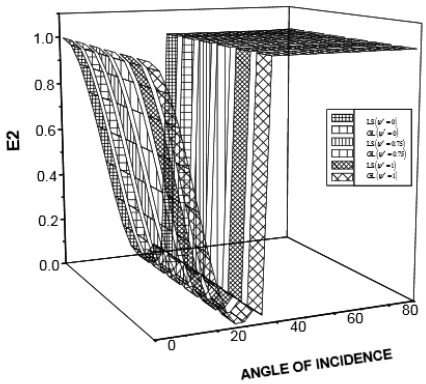


**Fig. 12.** Variation of energy ratio $E_2$ w.r.t. angle of incidence SV-wave.

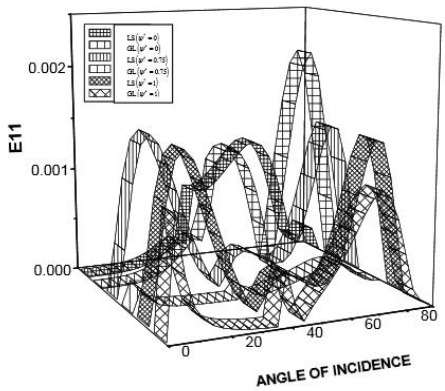


**Fig. 13.** Variation of energy ratio $E_{11}$ w.r.t. angle of incidence SV-wave.

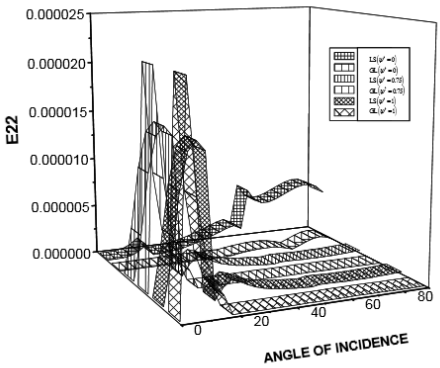


**Fig. 14.** Variation of energy ratio $E_{22}$ w.r.t. angle of incidence SV-wave.

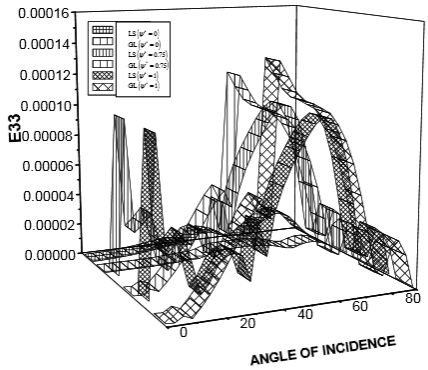


**Fig. 15.** Variation of energy ratio $E_{33}$ w.r.t. angle of incidence SV-wave.

Figures 17-18 show that the values of $E_{55}$ and $E_{66}$ depict an oscillatory behavior for all values of $\theta_0$. A significant effect of loosely bonded interface can be seen on energy ratio $E_{66}$ for higher values of $\theta_0$. Figure 19 represents the variation of $E_{RR}$ with $\theta_0$ and it shows that the values of $E_{RR}$ increases for smaller values of $\theta_0$, whereas for higher values of

$\theta_0$, the values of $E_{RR}$ slightly decreases. For smooth interface, it is noticed in GL case the values of $E_{RR}$ remain more for higher values of $\theta_0$ in comparison with other cases.

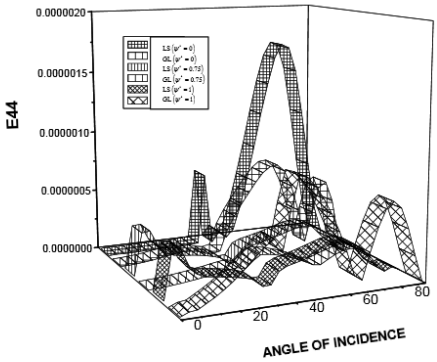


**Fig. 16.** Variation of energy ratio $E_{44}$ w.r.t. angle of incidence SV-wave.

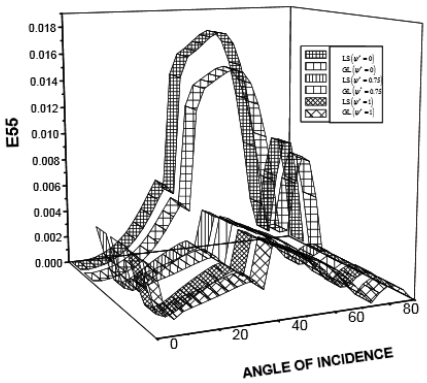


**Fig. 17.** Variation of energy ratio $E_{55}$ w.r.t. angle of incidence SV-wave.

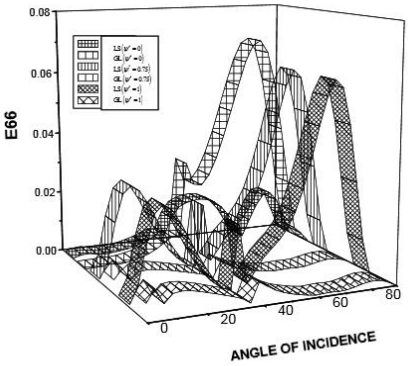


**Fig. 18.** Variation of energy ratio $E_{66}$ w.r.t. angle of incidence SV-wave.

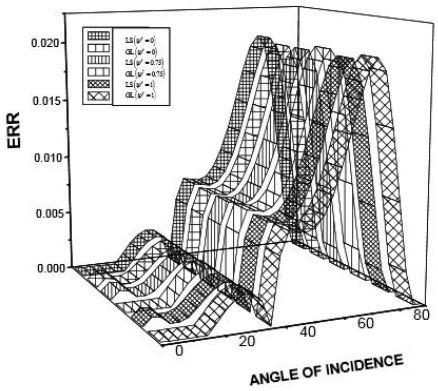


**Fig. 19.** Variation of energy ratio $E_{RR}$ w.r.t. angle of incidence SV-wave.

## Conclusion

The reflection and transmission of elastic waves at the loosely bonded interface between an elastic solid half-space and a microstretch thermoelastic diffusion solid half-space has been investigated. The six waves in microstretch thermoelastic diffusion medium are identified and explained through different wave equations in terms of displacement potentials. The energy ratios of different reflected and transmitted waves to that of incident wave are computed numerically and presented graphically with respect to the angle of incidence. An appreciable effect of looseness of the boundary is observed on energy ratios of various reflected and transmitted waves.

Also, from the numerical results we conclude that, for incident P-wave, the energy ratios $E_{11}$, $E_{22}$, and $E_{44}$ are more influenced due to the effect of loosely boundary and in case of incident SV-wave, a significant effect of loosely boundary is noticed on the energy ratios $E_{11}$, $E_{33}$ and $E_{66}$. Moreover, in majority of cases, the magnitude of energy ratios for G-L theory is more as compared to L-S theory. The sum of all energy ratios of the reflected waves, transmitted waves and interference between transmitted waves is verified to be always unity which ensures the law of conservation of incident energy at the loosely bonded interface.

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