

# VALIDATION OF THE MATHEMATICAL MODEL OF ISOTROPIC MATERIAL USING PARAMETRIC OPTIMIZATION OF ITS PHYSICAL AND MECHANICAL CHARACTERISTICS

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**Abstract.** In the article the questions connected with modeling of isotropic material on an example of carbon steel St3ps [1] for subsequent use in analysis of power protective frames of cabins of agricultural machinery in accordance with regulatory requirements are considered [14]. The choice of the Johnson-Cook (JC) material model for performing tasks of this type is substantiated. Full-scale tests of steel specimens were carried out, on the basis of which the mathematical model of material was validated.

**Keywords:** isotropic material; carbon steel; Johnson-Cook material model.

## 1. Introduction

Nowadays in the design of new products there is a growing importance of virtual testing based on the finite element method [5]. In accordance with this, there is an urgent need for more accurate modeling of the static and dynamic behavior of materials used in design models of structures [3, 4, 7]. Generally, the most common materials in engineering industry are isotropic materials with steel as a dominant material.

The most famous model of behavior of steel materials is the JC model [2]. Many works are devoted to the study of the behavior of steel materials on the basis of this model [8-12].

In the study [11] devoted to modeling the behavior of containers in throwing (simulating emergencies) based on the JC model the authors emphasize the need to take into account the temperature component of this material model when performing similar tasks in the nuclear industry. The authors give a comparison of modeling using the JC model and a tabulated model of plasticity.

The paper [9] devoted to the study of the incubation time criterion (in the form of fracture and yielding flow criteria), describing the dynamic effects of the strength behavior in brittle fracture and the yield stress during plastic deformation. The advantage of the approach is in a single macroscopic time parameter, independent of geometry, load mode and phenomenologically related to structural changes in the material at the micro level. The author proposes to interpret the effect of filler, metal fibers on the strength properties of the material in a wide range of external influences (under dynamic loads, the change is most pronounced) on the basis of the concept of incubation time. An important part of the work is devoted to the introduction of the phenomenological model of the deformation curve of elastoplastic materials for various deformation rates on the basis of the concept of incubation time. The author suggests applying the calculated model of the deformation curve to fine-grained and coarse-grained metals and their alloys.

A numerical comparative study of the dynamic properties of metals on the basis of the Kolskiy method and on the basis of the JC model was carried out in paper [10]. The use of the rheological and energy characteristics of impact compression based on the Kolskiy method is proposed to determine the relaxation time of tangential stresses and to find the relationship between the thermal internal energy of a material based on an aluminum alloy.

In work [12] studies on the applicability of the JC model in the modeling of technological stamping processes were carried out. The authors show that it is necessary to take into account the dynamic component of the JC model in simulating such high-speed technological processes by the finite element method, and also to identify the parameters of this model with the Mie-Gruneisen state equation.

Comparative analysis of the JC model with the Cooper-Simonds and Zerilli-Armstrong models was carried out in work [13] using the finite element simulation of the process of volume stamping. The author suggests using the JC model to solve a wide range of plastic deformation problems, as the most simple, satisfactorily describing the curve of plastic yielding flow of a metal in a wide range of changes in basic physical parameters and available in most software packages of numerical analysis.

Thus, the JC material model can be used to simulate the roll-over protective structure (ROPS) of agricultural machinery with the appropriate selection and validation of the material model based on full-scale testing of the specimens. Standard [14] provides for the performance of tests with relatively low (in comparison with the processes of processing metal pressure) speed of impactors and normal temperature conditions.

## 2. Formulation and solving methods

To model the material needed to describe shock phenomena with high strain rates, the LS-Dyna application software [6] often uses the JC material model (\*MAT\_JOHNSON\_COOK). According to this model, the von Mises flow stress  $\sigma$  is calculated by the following formula [2]:

$$\sigma = (a + b\varepsilon^n)(1 + c \ln \dot{\varepsilon}^*)(1 - T^{*m}), \quad (1)$$

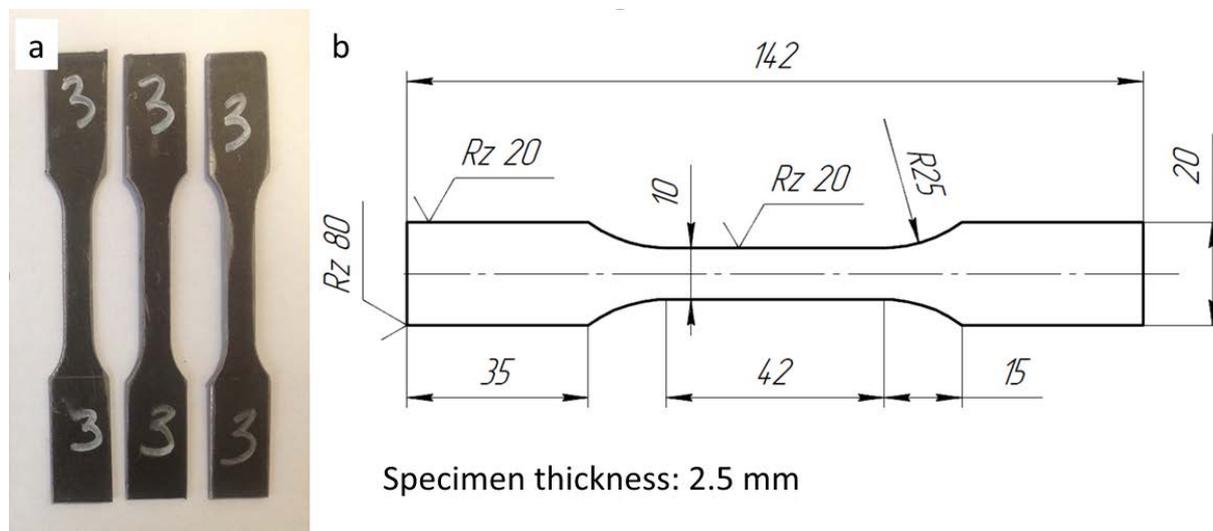
where  $\varepsilon$  – equivalent plastic strain;  $\dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_0$  is the dimensionless plastic strain rate for  $\dot{\varepsilon}_0 = 1.0 \text{ s}^{-1}$ ,  $T^*$  – is the dimensionless temperature, and  $T^* = (T - T_{room}) / (T_{melt} - T_{room})$ ,  $T_{melt}$  is the melting temperature of the material,  $T_{room}$  is the room temperature. The five material constants are  $a$  – yield stress,  $b$  – the hardening modulus,  $c$  – the strain rate sensitivity coefficient,  $n$  – the hardening coefficient,  $m$  – the thermal softening coefficient.

Formula (1) is an equation of flow stress, which is defined as the instantaneous value of stress required to continue the plastic flow of material - to keep the metal flowing. The JC model is purely empirical and it's the most widely used of the strain-rate dependent plasticity models. It makes it possible to take into account the effects of isotropic (static) strengthening, kinematic strengthening, temperature variation and the associated variation in yield strength. This model takes into account both kinematic strengthening and adiabatic heating of the material undergoing strains. Moreover, the JC model is easily implemented in computational codes due to its simplicity.

All the parameters are coupled due to the multiplicative nature of the model. However, difficulties exist with determining them. The problem of determining the numerical values of the parameters of the JC material model can be successfully solved using the parametric optimization tool. However, the accuracy of the description of the deformation curve will largely depend on the objective function used in the optimization process. In practice, the most common objective function is the root-mean-square averaged difference between the values of the experimental and calculated curves. At the same time, the accuracy of the deformation curve description using this objective function is quite high, however, due to modern requirements to the accuracy of computational models, there is a need for more accurate material modeling.

### 3. Results and Discussion

**Gathering of testing data.** For full-scale testing, steel grade St3ps is chosen as the most popular material for the manufacture of power-frame of cabins and base elements of agricultural machinery. Specimens of standard sizes are cut from this material (Fig. 1). To evaluate the reliability of the data obtained is needed to test three specimens.



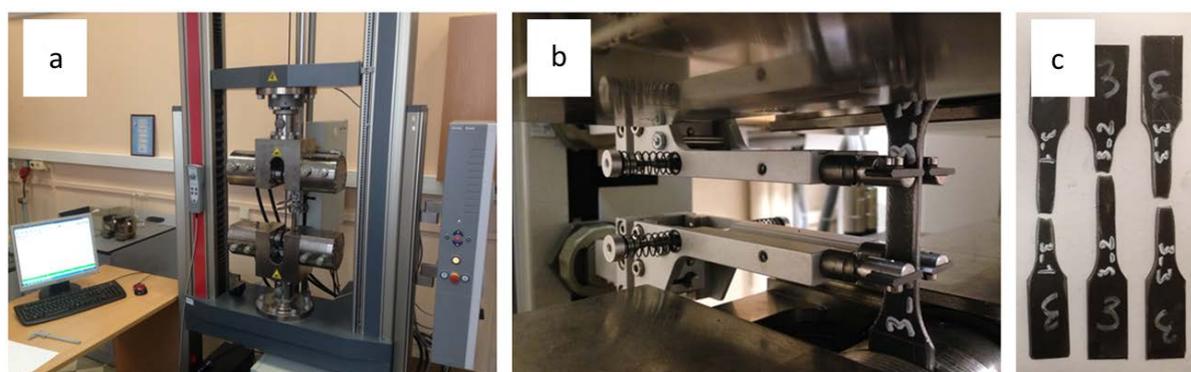
**Fig. 1.** The standard specimen: a – for natural testing, b – drawing.

The chemical composition of the material is described in Table 1.

Table 1. The chemical composition of the material test specimens.

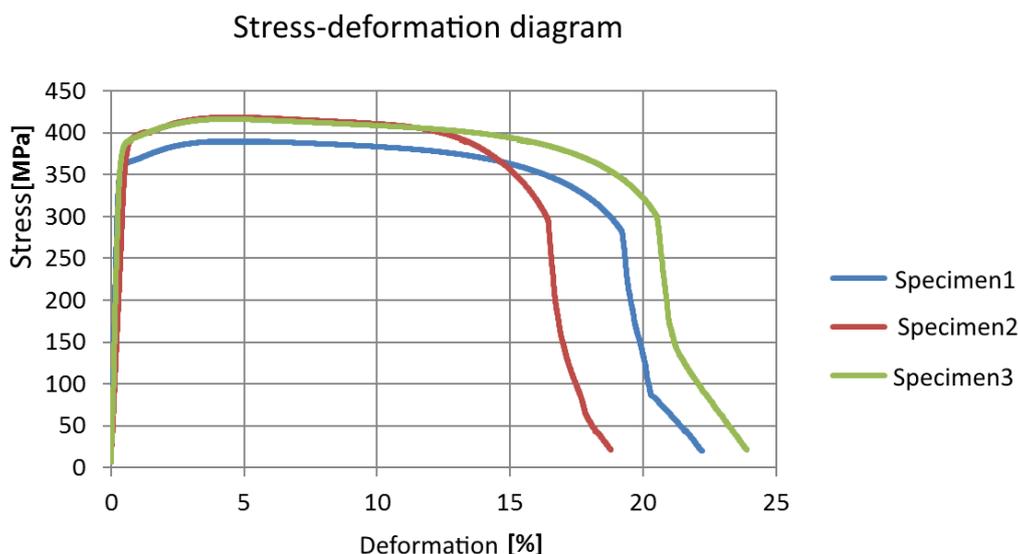
Grade of steel	Mass content of chemical elements		
	Carbon	Manganese	Silicon
St3ps	0.14...0.22	0.40...0.65	0.05...0.15

Full-scale testing of sample materials is carried out on a certified tensile testing machine, which in real time, during the test, records the stresses occurring in the material and the corresponding deformation (Fig. 2).



**Fig. 2.** Testing equipment (a, b) and standard experimental specimens after testing (c).

The results of stress measurements, as a function of deformations for three specimens is shown in Fig. 3. This dependence is presented in engineering units.



**Fig. 3.** Experimentally measured deformation diagrams in engineering dimensions.

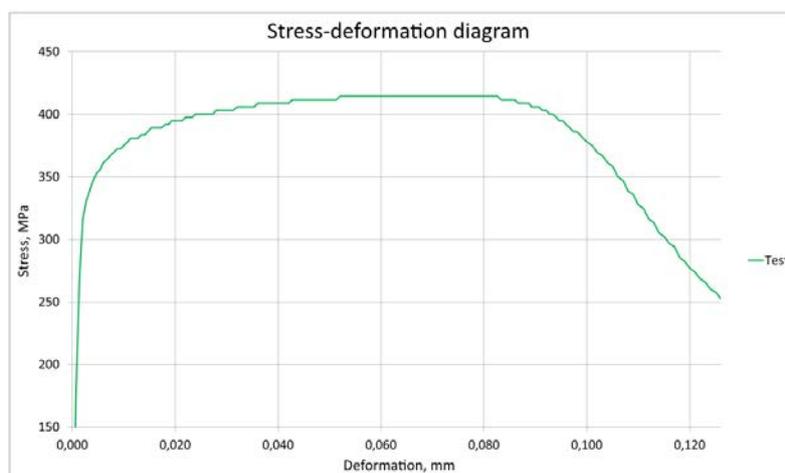
The obtained data are averaged and transferred from engineering ones to true units by the following formulas:

$$\delta = \ln(\varepsilon + 1), \quad (2)$$

$$S = \sigma(\varepsilon + 1), \quad (3)$$

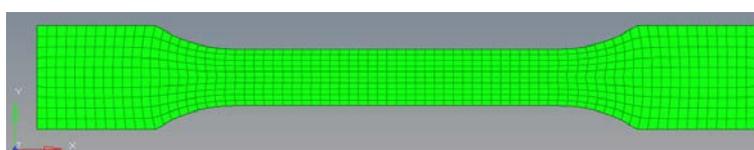
where  $\delta$  – true deformations;  $\varepsilon$  – engineering deformations;  $S$  – true stress;  $\sigma$  – engineering stress.

As a result, the deformation diagram is presented in the form in which it can be used to validate the finite element model (Fig. 4).



**Fig. 4.** The averaged experimental deformation diagram in true dimensions.

**Specimen modeling, parametrization and optimization of the model.** Finite element model of the specimen is modeled by shell elements in accordance with the drawing, which was used to excise the samples during testing (Fig. 5).



**Fig. 5.** Finite element model of standard specimen.

The thickness of the elements is equal to the thickness of the test specimens. The developed model is stretched along the X axis in both directions to repeat the impact test bench. The model without destruction is considered.

The example of mathematical model card of material (\*MAT\_JOHNSON\_COOK) is shown in Fig. 6.

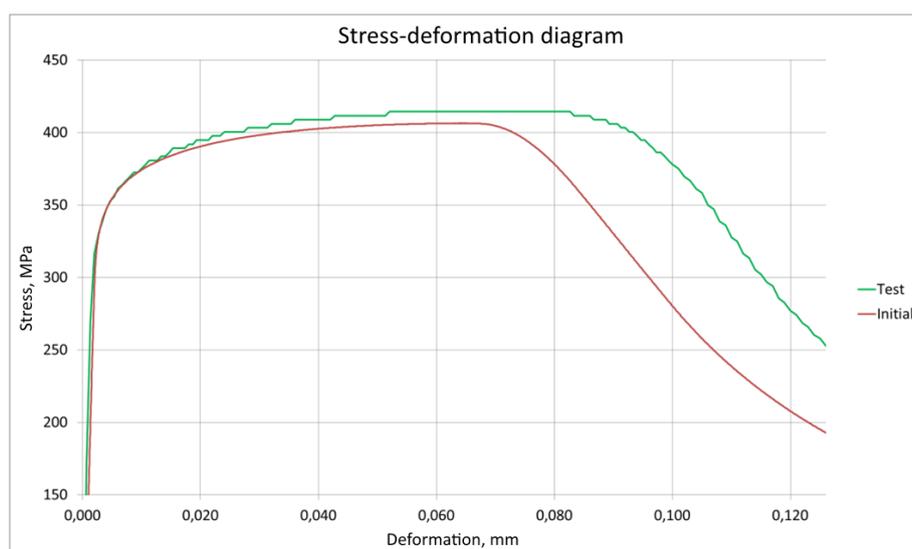
```

*MAT_JOHNSON_COOK
Law_2
$      RHO_I      .0027
$      E          Nu          .33
$      a          b          n          EPS_max          SIG_max0
$      110        120        .15        0          280
$      c          EPS_0      ICC      Fsmooth      F_cut
$      0          0          0          0          0
$      m          T_melt      rhoC_p      T_r
$      0          0          0          0

```

**Fig. 6.** The card of JC material.

As a result, of the modelling, a deformation diagram was obtained, which has a significant discrepancy with the curve obtained from the tests (Fig. 7).



**Fig. 7.** Comparison of experimental and initial stress-deformation diagrams

For accurate reproduction of the test results, it is necessary to optimize the physical and mechanical properties of the material laid down in the JC card. It is necessary to parameterize the model.

Variable parameters include the yield point, hardening module, strengthening index, and the Young's modulus of the material.

Optimization was carried out in the application software HyperStudy developed by Altair. It was proposed to optimize in three ways:

- minimizing the area between curves;
- minimization of the difference between the corresponding values of two graphs averaged by the mean square method;
- adjustment of values in characteristic points of the graph.

Let us consider in detail each of the proposed methods of optimization.

**Minimizing the area between curves.** To set the optimization response, a script was written to calculate the area between the two curves. The text of this script is given below:

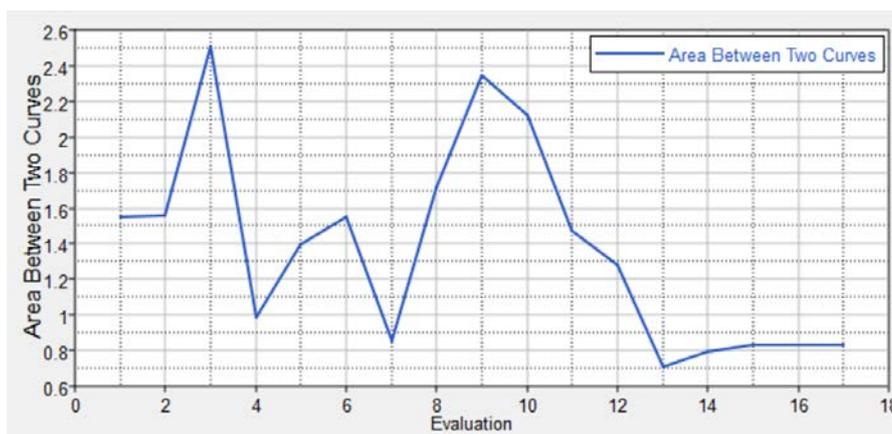
```

function area_between_two_curves(v1x,v2x,v1y,v2y)
{
    newx = sync2(v1x, v2x)           [1]
    newy1 = lininterp(v1x, v1y, newx) [2]
    newy2 = lininterp(v2x, v2y, newx) [3]
    suby = newy1-newy2              [4]
    area_value = absarea(newx, suby)  [5]
    return area_value               [6]
}

```

The first line of code performs synchronization of values along the X-axis of two curves. In the second and third lines, the Y-axis values for the new X values are calculated by linear interpolation. The fourth line determines the difference between the corresponding values along the Y axis. The fifth line calculates the area between the two curves. The sixth line displays the result.

The goal of optimization is to minimize the calculated area between the two curves. The optimization history by iteration is shown in Fig. 8.



**Fig. 8.** The history of optimization by iterations (minimizing of area).

As a result, the area between the curves was reduced more than twice.

**Minimizing the RMS difference.** To set the optimization response, a script was written for the root-mean-square averaging of the differences between the two curves. The text of this script is given below:

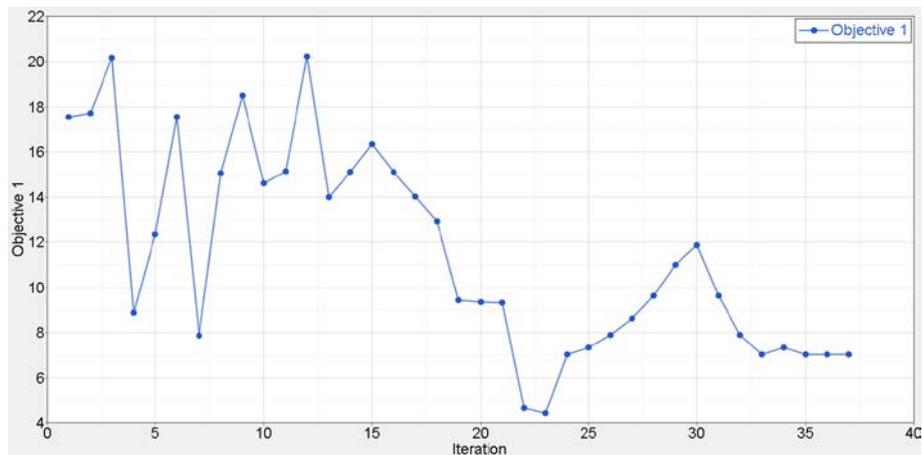
```

function area_between_two_curves(v1x,v2x,v1y,v2y)
{
    newx = sync2(v1x, v2x)           [1]
    newy1 = lininterp(v1x, v1y, newx) [2]
    newy2 = lininterp(v2x, v2y, newx) [3]
    suby = newy1-newy2              [4]
    aver = rms(suby)                 [5]
    return aver                       [6]
}

```

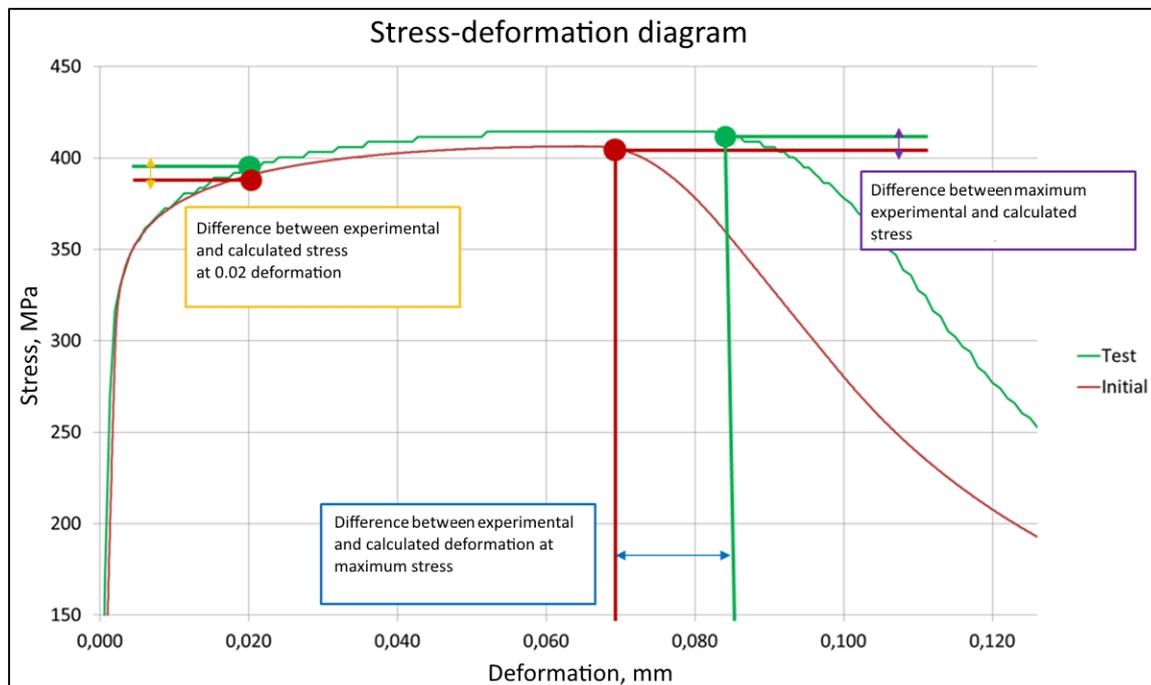
The first line of code performs synchronization of values along the X-axis of two curves. In the second and third lines, the Y-axis values for the new X values are calculated by linear interpolation. The fourth line determines the difference between the corresponding values along the Y axis. The fifth line produces the mean-square averaging of the differences obtained. The sixth line displays the result.

The goal of optimization is to minimize the calculated RMS value. The history of optimization by iteration is shown in Fig. 9.



**Fig. 9.** The history of optimization by iterations (root-mean-square difference).

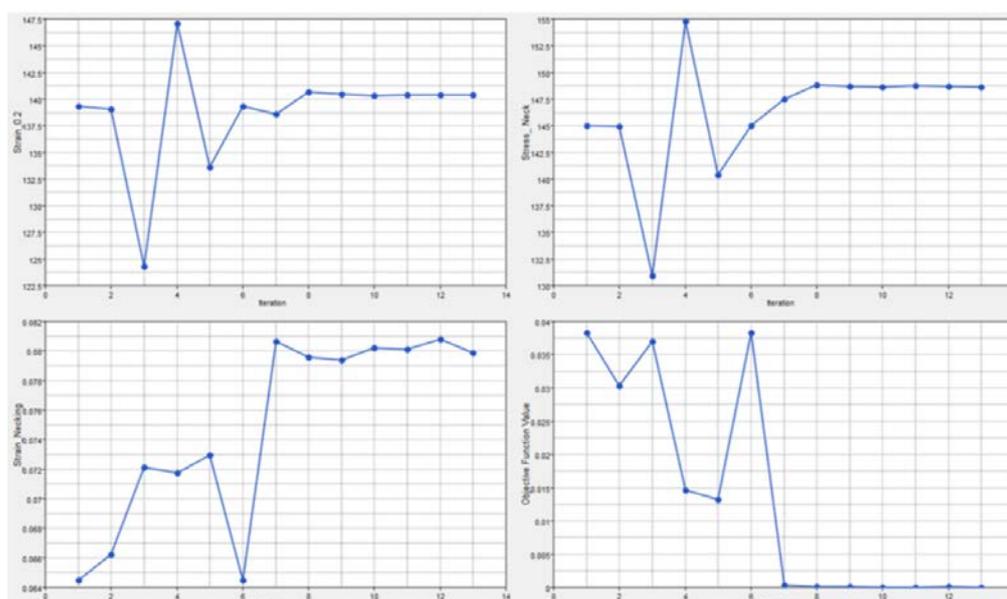
As a result, the root-mean-square difference between the curves was reduced by more than four times.



**Fig. 10.** Characteristic points for optimization.

**Determination of values in characteristic points.** Optimization responses are the values of stresses and strains at characteristic points of the curve, with a deformation of 0.02 mm, and also with the maximum stresses achieved (Fig. 10). Targets for these parameters are taken from the experimental curve; the achievement of these parameters is the goal of the optimization.

The history of optimization by iterations is shown in Fig. 11. As a result, of optimization, all indicators reached their target values.



**Fig. 11.** The history of optimization by iterations (characteristic points).

**Comparison of results.** Optimization for each type of objective functions was carried out using the same convergence conditions. The resulting comparative criterion for estimating the coincidence of the calculated curve with the experimental estimate is the root-mean-square averaged difference in their ordinates. Time of optimization is also one of the important parameters for comparison. Table 2 compares the three optimizations using different objective functions.

Table 2. Comparison of results with the use of various objective functions.

Objective function	RMS average	Number of Iterations
Minimization of area	9,19	17
Minimization of root-mean-square difference	7,51	37
Fitting of values at characteristic points	7,24	13

Comparative analysis shows that minimizing the root-mean-square difference and adjusting the values at characteristic points gives the same results in terms of deviation from the experimental curve. But in terms of time required for optimization, minimizing the root-mean-square difference takes about three times the fitting of values at characteristic points.

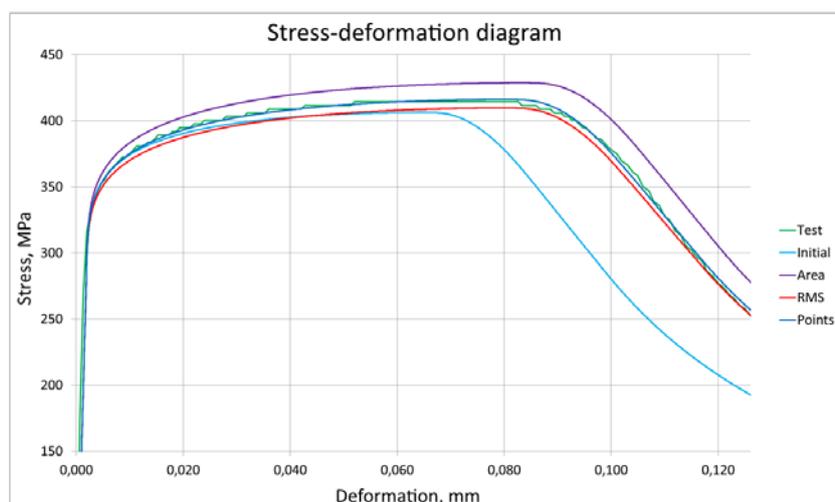
A visual comparison of the stress-deformation diagrams obtained for various optimizations is shown in Fig. 12.

Comparison of the root-mean-square deviation and the visual behavior of the deformation diagram showed that the best objective function for parametric optimization of the physical-mechanical properties of the material is to "adjust values at characteristic points".

#### 4. Conclusions

For the validation of the mathematical model of material used to solve problems on shock phenomena and high-speed deformations, the most effective approach to parametric optimization is the method of characteristic points, which turned out to be more accurate than

the mean-square averaging method. The method of optimization by area has proven to be insufficiently precise, therefore is not recommended for further practical implication.



**Fig. 12.** Comparison of stress-deformation diagrams.

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