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SUPERCOMPUTING ANALYSIS OF FAN-SHAPED WAVES IN THE EARTH'S CRUST AT THE DEPTH OF SEISMIC ACTIVITY

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Abstract. The high-speed process of tectonic faults formation in zones of seismic activity of the Earth's crust is analyzed in a plane strain state based on the model of elastic blocks interacting through a domino-structure under conditions of strong hydrostatic compression. Numerical simulation of the dynamics of emerging fan-shaped waves is performed by means of the developed computational algorithm and computer program for multiprocessor supercomputers of cluster architecture.
Keywords: shear rupture, extremely high speed, Tarasov's fan-shaped mechanism, edge dislocation, dynamics, elasticity, high-performance computations

1. Introduction

Along with slowly growing cracks, in hard rocks at great depths shear ruptures are observed, which grow with an abnormally high speed, comparable to or even exceeding the velocity of transverse waves. This phenomenon is widely discussed in contemporary seismology and is the subject of a number of theoretical and experimental studies [1-5]. To explain a high-speed motion of shear ruptures, Professor B.G. Tarasov from the University of Western Australia proposed an original mechanism [6]. According to his ideas, at the rupture tip a fan-shaped structure, consisting of domino-slabs, is formed. These slabs rotate when relatively low tangential stresses appear in the surrounding rock mass and, what is most important, preserve their integrity during rotation. Schematically, the process of formation and propagation of a fan-structure is shown in Fig. 1. The slabs exfoliate from the hard rock because of the high confining pressure, tending to change the direction of propagation of the main crack. At the same time, an abnormally low friction is created at the rupture head due to the weak resistance to separation of the domino-slabs, continuously supplementing the fan-shaped system, and the fan moves like a wave extremely fast.

To illustrate the fan-shaped mechanism, Tarasov created a laboratory model simulating the motion of a fan in a system of elastically bonded slabs on a flat surface [6]. Corresponding mathematical model was developed, the analysis of which with using numerical and analytical methods makes possible to describe main qualitative features of the phenomenon under consideration [7,8]. On the basis of the mathematical model, the dependence of fan speed on the resistance of separation was studied, the influence of viscous and dry friction was analyzed, the multiplicative power interaction of the slabs in a fan-shaped system was shown, which contribute to the fast motion of the fan.

The static problem of the equilibrium of two infinite elastic blocks interacting through a thin interlayer (a fan-structure) in the plane formulation was studied in [9]. As a result, the fields of stresses and displacements were obtained in the blocks in the vicinity of a formed fan, which play the role of initial data under solution of dynamic problem.

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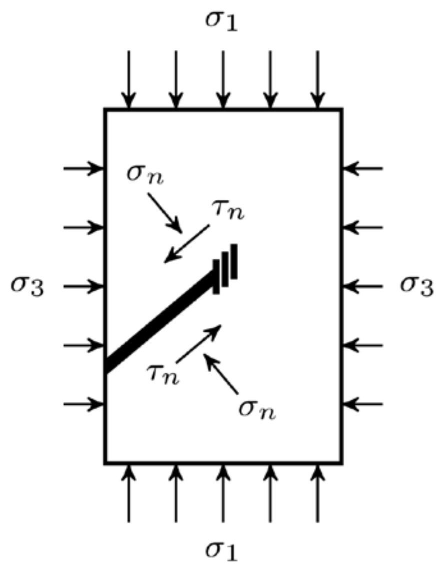


Fig. 1. The scheme of formation of a fan domino-structure

In the present paper, the process of starting and subsequent motion of a fan-shaped system between two elastic blocks is analyzed on the basis of dynamic equations of a plane strain state by use of supercomputers of cluster architecture.

2. Mathematical model

Analysis of the static solution shows that a fan-structure, formed under the action of natural or technogenic processes, with the slabs height of 0.1 m significantly changes the hydrostatic stress state in the surrounding rock. These changes extend over a distance of the order of the fan's length, which is tens of meters, in the direction of the rupture and over a distance of 5 – 10 m in the normal direction. For direct numerical simulation of the dynamic processes taking place, the dimensions of each of two interacting blocks must be at least 5 m · 50 m, and the step of squared difference grid must be approximately 0.01 m, which is ten times smaller than the height of slabs. Thus, we obtain a computationally complex problem that requires resources of RAM and runtime that are characteristic for a supercomputer of average performance of a cluster or hybrid architecture. For solution of this problem, the technology of parallelization on the basis of decomposition of computational domain is applied. Blocks interacting through a fan are uniformly distributed between the cluster nodes, and necessary data exchange over the boundaries takes place. We use previously developed author's program code [10] for the analysis of dynamic processes in a multi-blocky medium with thin interlayers.

Equations of a blocky medium. The plane strain state of blocks (rectangles with sides parallel to the axes of a Cartesian coordinate system) is described by the system of equations of a homogeneous isotropic elastic medium:

$$\begin{aligned} &\rho \dot{v}_1 = \sigma_{11,1} + \sigma_{12,2}, \qquad \rho \dot{v}_2 = \sigma_{12,1} + \sigma_{22,2}, \qquad \dot{\sigma}_{12} = \rho c_2^2 (v_{2,1} + v_{1,2}), \\ &\dot{\sigma}_{11} = \rho c_1^2 \, (v_{1,1} + v_{2,2}) - 2\rho c_2^2 \, v_{2,2}, \qquad \dot{\sigma}_{22} = \rho c_1^2 (v_{1,1} + v_{2,2}) - 2\rho c_2^2 \, v_{1,1}. \end{aligned} \tag{1}$$

Here ρ is the density, c_1 and c_2 are the velocities of longitudinal and transverse elastic waves, respectively. The dot over a symbol denotes partial derivative with respect to time and the indices after a comma denote partial derivatives with respect to spatial variables. The conventional notations of tensor analysis are used. Internal boundary conditions at the interblock artificial boundaries between computational nodes are the conditions of continuity of the velocity vector v and the stress vector f on respective areas.

For obtaining numerical solution of the system (1) with given initial data and boundary conditions, a parallel computational algorithm was developed, in which a two-cyclic splitting method with respect to spatial variables is realized [11]. Systems of 1D equations in blocks are solved on the basis of implicit finite-difference scheme constructed in [12]. The algorithm proposed in this paper makes it possible to perform computations with large time steps, exceeding the maximum permissible value of the step according to the Courant–Friedrichs–Lewy condition for explicit schemes by many times. It can be applied both in the simplest case of the contact interaction between blocks of the type of continuity condition, and in the case of nonlinear internal boundary conditions of a sufficiently general form.

Modeling of a fan. When modeling the dynamics of a fan-shaped system, one of the horizontal interlayers in a blocky massif is assumed to be a tectonic fault – an extended rectilinear zone of small thickness with a fan-structure. A separate fragment of such structure, in accordance with the Tarasov model, can be considered as an absolutely rigid slab with the height a of a unit cross section. Position of the slab relative to horizontal axis at the initial time moment is given by the angle φ_0. Formation and further propagation of the fan is accompanied by rotation of the slab with a change of angle from φ_0 to $\varphi_1 = \pi - \varphi_0$.

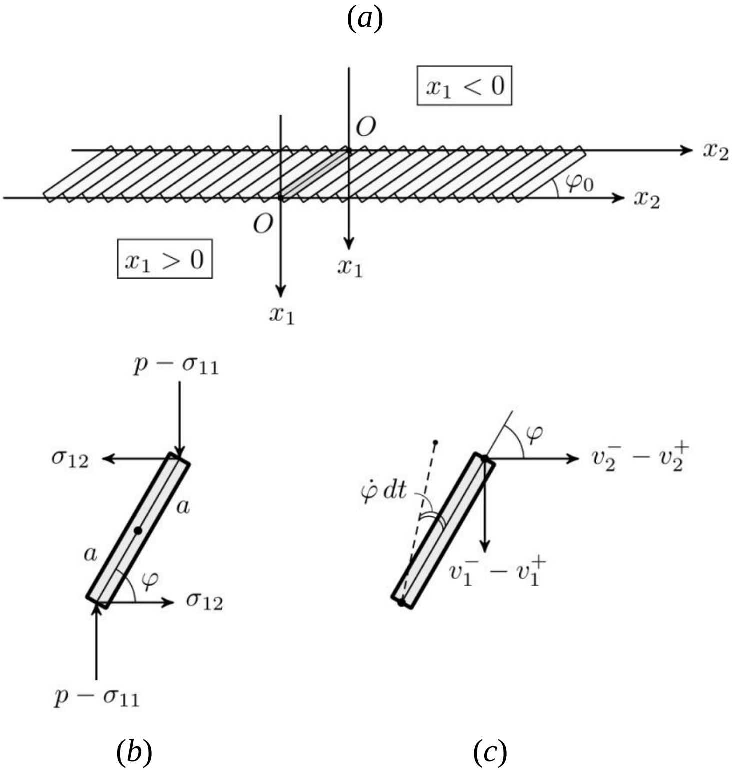


Fig. 2. Coordinate systems in the blocks (a), scheme of loading for a slab (b), and kinematic scheme (c)

Under formulation of boundary conditions in the fan zone, the coordinate systems in blocks, shown in Fig. 2 a, are identified. The loading scheme for the slab oriented at the angle φ ($\varphi_0 \le \varphi \le \varphi_1$) is represented in Fig. 2 b. It includes the normal and tangential stresses acting from the side of upper and lower adjacent blocks, and the rotational moment due to the contact interaction with neighboring slabs. Under the assumption of the smallness of inertial forces and moments, the equilibrium conditions are valid:

$$\sigma_{11}^{+} = \sigma_{11}^{-} \equiv \sigma_{11}, \qquad \sigma_{12}^{+} = \sigma_{12}^{-} \equiv \sigma_{12}, \qquad M + \sigma_{11} a \cos\varphi + \sigma_{12} a \sin\varphi = 0, \tag{2}$$

where superscripts "±" are related to the boundaries of interacting blocks. In accordance with the kinematic scheme in Fig. 2 c, the following equations are satisfied:

$$v_1^{+} - v_1^{-} = a\,\dot\varphi \cos\varphi, \qquad v_2^{+} - v_2^{-} = a\,\dot\varphi \sin\varphi. \tag{3}$$

Contact interaction of fragments is described by the variational inequality:

$$\left(M + \eta\,\dot\varphi\right)\left(\tilde\varphi - \varphi\right) \ge 0, \qquad \varphi_0 \le \tilde\varphi \le \varphi_1. \tag{4}$$

Here $\tilde\varphi$ is an arbitrary admissible variation of the angle, η is the coefficient of viscous friction. If $\varphi = \varphi_0$, then from the variational inequality it follows that the moment M is nonnegative, and if $\varphi = \varphi_1$, then it is nonpositive. If $\varphi_0 < \varphi < \varphi_1$, then, because of the arbitrariness of variation, it follows from (4) that M is equal to the moment of viscous friction forces. This corresponds exactly to the loading scheme.

The algorithm of numerical realization of the conditions for interaction of blocks through a fan interlayer, based on the relationships (2) – (4), was implemented by means of the equations for predictor step obtained in [12]. These equations are approximations of the equations on characteristics for 1D systems in adjacent cells of the neighboring blocks. According to them

$$z_1^{+} v_1^{+} + \sigma_{11}^{+} = P^{+}, \qquad z_2^{+} v_2^{+} + \sigma_{12}^{+} = S^{+}, \qquad z_1^{-} v_1^{-} - \sigma_{11}^{-} = P^{-}, \qquad z_2^{-} v_2^{-} - \sigma_{12}^{-} = S^{-}, \tag{5}$$

where $z_1^{\pm}$ and $z_2^{\pm}$ are the difference analogues for the acoustic impedances of longitudinal and transverse waves, $P^{\pm}$ and $S^{\pm}$ are the analogues of Riemann invariants corresponding to these waves.

After simple transformations, the inequality (4) takes the following form:

$$\left((A+\eta)\,\dot\varphi - B\right)(\tilde\varphi - \varphi) \ge 0, \qquad \varphi_0 \le \tilde\varphi \le \varphi_1,$$

$$A = z_1\, a^2 \cos^2\varphi + z_2\, a^2 \sin^2\varphi, \quad B = (P^{+} - P^{-})\, a \cos\varphi + (S^{+} - S^{-})\, a \sin\varphi, \quad 1/z_k = 1/z_k^{+} + 1/z_k^{-}.$$

Approximation of the derivative by time leads to the discrete variational inequality:

$$(\hat\varphi - \bar\varphi)(\tilde\varphi - \hat\varphi) \ge 0, \qquad \bar\varphi = \varphi + \Delta t\, B/(A+\eta),$$

the solution of which $\hat\varphi$ ($\varphi_0 \le \hat\varphi \le \varphi_1$), related to a new time layer, is defined as the projection $\bar\varphi$ onto constraints: $\hat\varphi = \varphi_0$ if $\bar\varphi < \varphi_0$, or $\hat\varphi = \bar\varphi$ if $\varphi_0 \le \bar\varphi \le \varphi_1$, or $\hat\varphi = \varphi_1$ if $\varphi_1 < \bar\varphi$. Further, the angular velocity is calculated by the formula $\dot\varphi = (\hat\varphi - \varphi)/\Delta t$, and using equations (2), (3), (5) the velocities and stresses at the interlayer boundaries are found. These values are necessary for the subsequent realization of the algorithm for calculating velocities and stresses in adjacent blocks.

3. Results of computations

By means of the described algorithm the computations of fan-shaped waves in an interlayer between two homogeneous blocks of hard rock were performed. It was assumed that at the initial time moment these blocks are in equilibrium under the action of confining pressure $\sigma_{11} = \sigma_{22} = -p$ and static stresses, which are formed around the fan. Motion occurs due to the additional tangential stress $\sigma_{12} = \tau$, increasing monotonically with time or being applied abruptly.

In the initial state the fan was set near the left boundary. To trace its motion over a long distance, the length of both blocks was taken as 80 m, and the step of a grid – as 0.01 m. Computations were performed on 80 nodes of the MVS–1000 supercomputer of the Institute of Computational Modeling SB RAS. The size of computational domain for each node of a cluster was 10 m × 1 m. The acting pressure $p = 250$ MPa and the maximum value of tangential stress $\tau = 100$ MPa were chosen based on the model representations about geological processes occurring at the depth about 10 km [6]. Elasticity parameters of the blocks: $\rho = 2700$ kg/m^3, $c_1 = 6000$ and $c_2 = 3300$ m/s.

Starting of a fan-shaped system. The motion of a fan system is counteracted by frictional forces and, first of all, by the force of resistance to separation of slabs in the head of the fan. Therefore, the fan starts not immediately, but only when a certain limit of tangential stress τ_* is reached in the blocks. In order to characterize the dependence of a limit stress on friction forces, the computations of the problem, in which all domino-slabs are forcibly held in a fixed state, were made. Calculated velocities in this case must be zero, and the stress fields should not depend on time. The obtained results fully satisfy this criterion. Figure 3 a shows the level surfaces of tangential stress around the forcibly held fan, which are consistent with the results of [9].

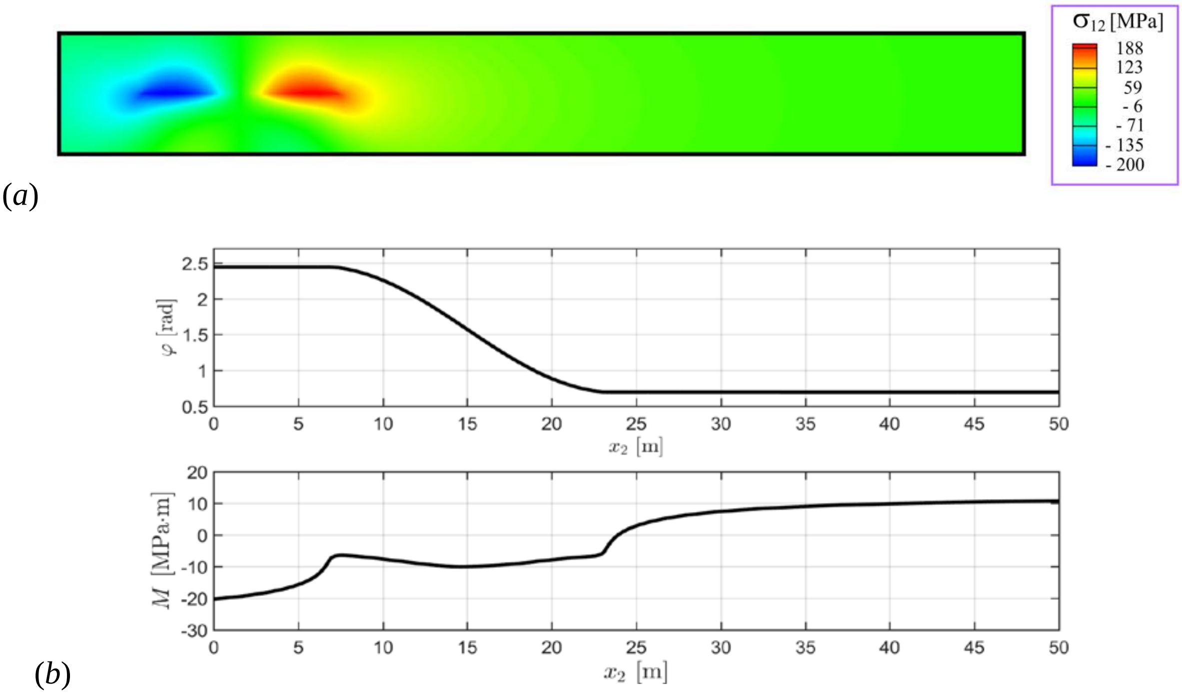


Fig. 3. Fixed fan: Level curves of tangential stress (a), distribution of the angle of rotation of slabs and distribution of the moment of restraining forces (b)

In Fig. 3 b the distributions of the angle of rotation of slabs along the fan-system and the moment of restraining forces are represented. An approximate computation of the integral

$$A = \frac{1}{2}\int_0^l M(x_2)\,d\varphi(x_2),$$

where l is the length of blocks, by the formula of rectangles allows to estimate the specific work of restraining forces, when a fan moves per unit distance. If $\tau_* = 100$ MPa, then $A = 7.66$ MJ/m^2. Because of the linearity of the problem with a fixed fan, the restraining forces are linearly dependent on the value of τ_*. Therefore, the work corresponding to other values of the limit stress can be obtained by simple recalculation.

Figure 4 shows similar results for another variant of the problem, in which only the most right slab in the head of the fan is held. In this case a dynamic process develops, the fan contracts, trying to tear off the fixed slab, which leads to a steady-state redistribution of stresses in the blocks after some time.

Fan wave motion. When modeling a moving fan, viscous friction with a coefficient $\eta = 1$ MPa·m·s, stabilizing computational results, and resistance to separation of the head slab, equivalent to the moment of separation $M_* = 0.75$ MPa·m, are introduced into the model. The value of moment is lower than the limit value, which can be determined from the graph in Fig. 4 b, therefore, the fan starts and moves.

In the considered variant of the problem, the tangential stress increases monotonically with time according to the equation $\tau = \tau_* \sin^2 \pi t / t_0$ within the loading time t_0, and after the end of this time the stress becomes constant. Boundary conditions of the problem are set in terms of the velocities for the state of simple shear, corresponding to the tangential stress reached at the given time moment.

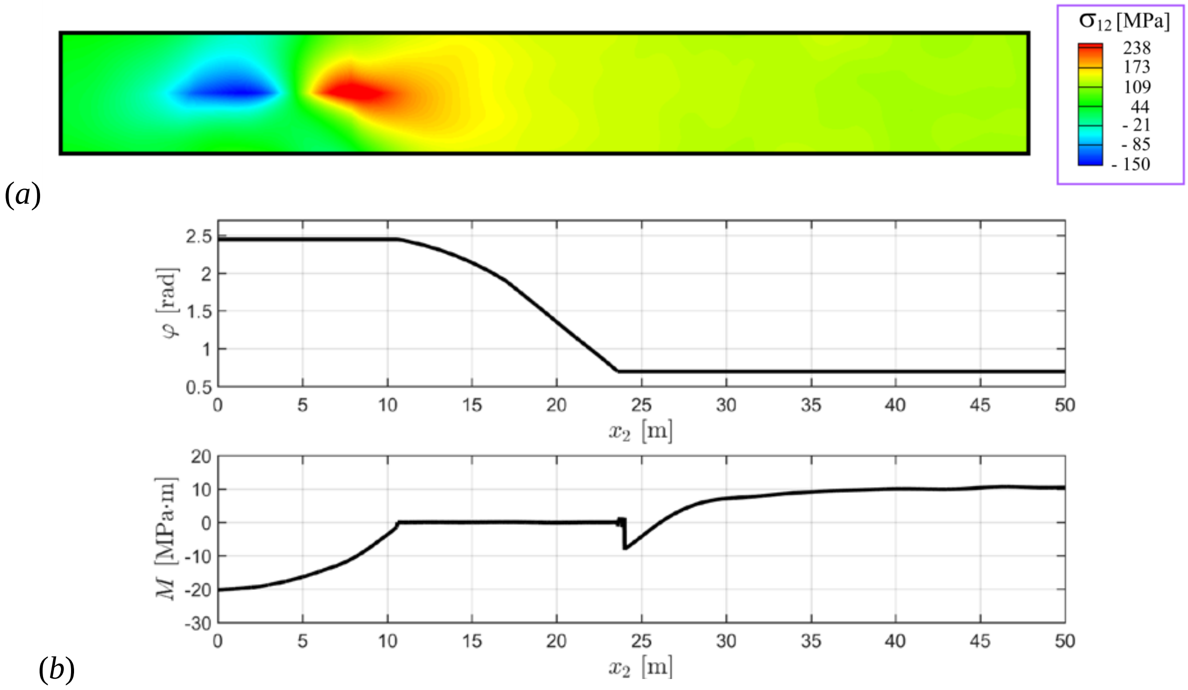


Fig. 4. Stopped fan: Level curves of tangential stress (*a*), distribution of the angle of rotation of slabs and distribution of the moment of restraining forces (*b*)

Figure 5 shows the diagrams of the angle of rotation of slabs in a fan-system successively through 0.02 s after the start, and the diagrams of the rotational moment for the loading time 0.1 s. From this figure, it can be seen that the motion of the fan is essentially nonuniform.

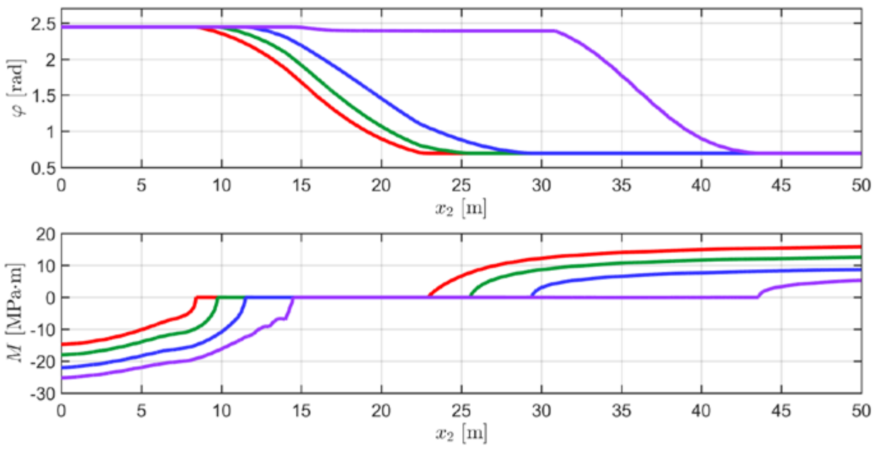


Fig. 5. Distribution of the angle of rotation of slabs and distribution of the rotational moment in a moving fan-system

Level surfaces of tangential stress for the moving fan at different time moments are represented in Fig. 6.

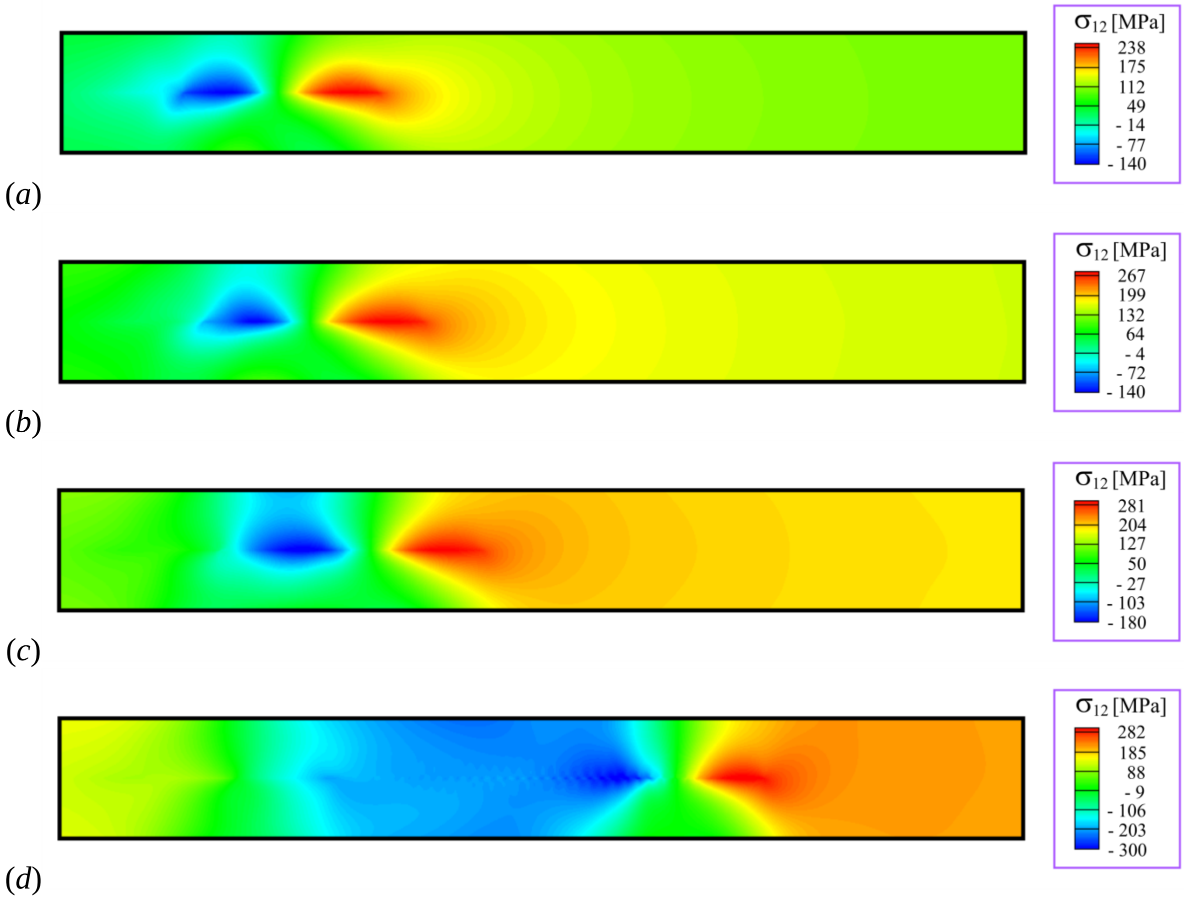
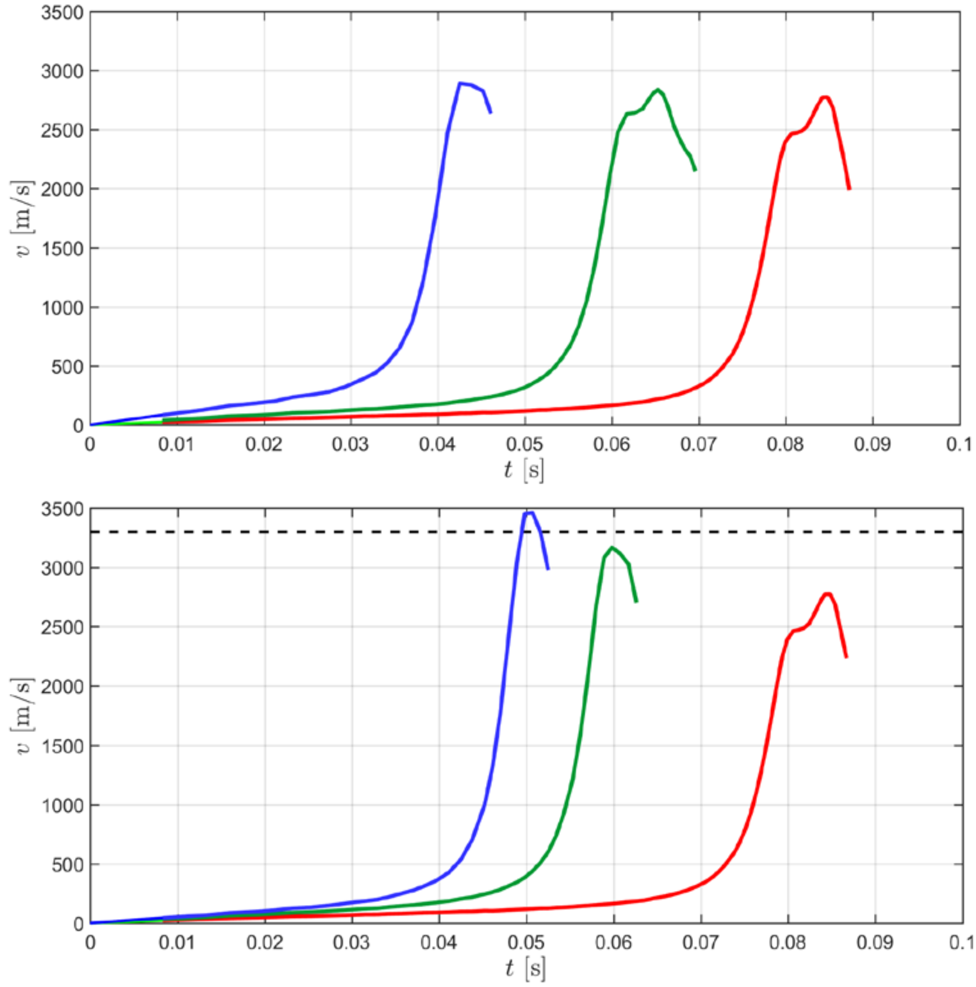


Fig. 6. Level curves of tangential stress in the process of fan motion

As computations have shown, the fan speed depends weakly on the loading time. The maximum speed is 2770 m/s at $t_0 = 0.1$ s and the maximum speed is 2890 m/s at $t_0 = 0.05$ s, both below the velocity of transverse elastic waves. For three values of the loading time $t_0 = 0.05$, 0.075 and 0.1 s the dependences of the fan speed on time are represented in Fig. 7 a. In Fig. 7 b similar dependences are given for three values of the additional tangential stress $\tau = 100$, 150 and 200 MPa. Analysis shows that the speed of a fan depends essentially on the value of tangential stress, and that at $\tau = 200$ MPa it exceeds the velocity of transverse waves, which level corresponds to the dashed line.

Note that the decrease in the fan speed after reaching the maximum value is due to the specifics of the boundary conditions on the right-hand border of the solution domain. In all variants of computations, the velocities of particles of a medium at this border after the time of loading are assumed to be zero, which leads to the effect of locking of the fan. It stops and moves in the opposite direction until the moment of a similar reflection from the left-hand border. The question of how to formulate the conditions for an unhindered passing of a fan across the border of computational domain remains open.

Computations were carried out with a variable step of the finite-difference scheme by time variable. At the beginning, the time step parameter was greater than the characteristic value of Courant–Friedrichs–Lewy $\Delta t = \Delta x / c_1$ by 25 times. With time, it was decreased, practically, to this characteristic value, in order to avoid the appearance of unphysical effects in the fan structure.



(*a*)

(*b*)

Fig. 7. Dependence of the fan velocity on the loading time (a) and on the additional tangential stress (b)

Average speed of the fan system at a given time was calculated by the estimated formula:

$$V = \frac{1}{\varphi_1 - \varphi_0} \int_0^l \dot{\varphi}(x_2)\, dx_2, \tag{6}$$

which follows from the equation of a traveling wave: $\varphi = \varphi(x_2 - Vt)$. Simple transformations of this equation give:

$$\dot{\varphi} = -V\varphi, \quad \dot{\varphi}\, dx_2 = -V\, d\varphi, \quad \int_0^l \dot{\varphi}\, dx_2 = -V \int_{\varphi_1}^{\varphi_0} d\varphi = V(\varphi_1 - \varphi_0).$$

Formula (6) takes into account that the angular velocity of domino-slabs is equal to zero outside the zone of the fan.

The range of loading times, investigated during the computations, corresponds to the passage through the fan system of low-frequency transverse waves with a frequency of 10 – 20 Hz and a wavelength in the range of 150 – 300 m. To study a more interesting case of quasistatic loading with extremely long waves of the order of several kilometers, a much more extensive domain of solution of the problem must be considered. This is possible only using a more powerful supercomputer with 1000 or more computational nodes. Current computations were fulfilled on the cluster MVS–1000 of average performance of the Institute of Computational Modeling SB RAS and on a small cluster of the Siberian Federal University having about a hundred computational nodes.

4. Conclusions

A computational technology is developed including the mathematical model, numerical algorithm and computer program for multiprocessor supercomputers, which allows further study of the Tarasov's fan-shaped mechanism, describing the high-speed process of tectonic fracture propagation in zones of seismic activity of the Earth's crust. Based on this technology, numerical experiments simulating the fan motion excited by elastic shear waves of low frequency, of the order of tens Hertz, in the surrounding rock were performed. Obtained results characterize the fan-system as an exceptionally mobile physically unstable structure. It was shown that in the case of intensive loading the fan speed can exceed the velocity of elastic transverse waves. To analyze the motion of a fan over long distances, of the order of a kilometer, under the action of a static system of tangential stresses, it is necessary to go over to the stationary model of a running fan. Using the considered here dynamic model to solve such problem with the time of loading of the order of hours or even minutes, would require extremely large computational resources, which currently have only the world's largest supercomputer centers.

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