EXPERIMENTAL AND NUMERICAL MODELING FOR
POWDER ROLLING

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Abstract: There are multiple nonlinearities during the course of powder rolling and there is a difficulty for constructing its mechanical model and keeping the stability during the numerical calculation process. In addition, mechanical parameters determined by means of numerical simulation of rolling process are great significance both in theory and practical applications for the optimization of the process parameters and the design and manufacture of rolling equipment. Considering Material and Geometrical Nonlinearity during powder rolling, a constitutive model aiming to the powder rolling is constructed based on the Updated Lagrange (U.L.) formulation by which the basic theory of numerical simulation is deduced. The reasons are analyzed based on the experiments of powder rolling which led to the error during the numerical simulation and the effect of the different factors on powder rolling are analyzed. It is shown that the result of simulation is less than that of experiment and the whole result is dependable. The effect of various process factors is analyzed by the simulation of the rolling process and based on visual analysis of the simulation result, the primary and secondary factors affecting the relative density and the rolling force are obtained.

1. INTRODUCTION

The research on theory and process of powder rolling has been developed in two decades. Hajaligol and Deevi [1] etc. made the strip with 24% Al by rolling water spray powder, in which average grain size is 20 μm with good resistance and high heat resistance and it was a kind of very good heat-resistant material. Hirohata and Shima [2] studied such an asynchronous powder rolling process (two rolls of rotating speed), in which the stress state and other powder parameters are different from the traditional rolling method. According to this paper, in the same rolling load, relative density with asynchronous was improved more than 15% compared to traditional methods. Guigon and Simon [3,4] analyzed the effect of powder feeding system for rolling with quality and obtained the function relation between rolling pressure, rolling speed and feeding speed. Zhou [5] comprehensively introduced the principle, usage, method and process parameters of metal powder rolling, and analyzed the main factors affecting the rolled-piece performance. Xu [6] studied the effect of powder feed height on rolling with tail thickness and density, and analyzed the effect of loose density on the critical height. Jiang Yao [7,8] made Ti-Al alloy plate by powder rolling and reaction synthesis, studied the effect of different process parameters on rolling plate quality.

Considering volume compressibility of the powder materials, the yield models in rock mechanics involved the corrected C-M model, Drucker-Prager-Cap model, Cam-Clay model [9-11] were introduced into the numerical simulation of the metal powder pressing process. However the published literature on the numerical simulation of powder rolling is very few. Mori [12] used rigid-plastic finite element method to simulate the plane strain of slight compressible
sintered porous blank in rolling process; Wang [13] simulated the cause of porous aluminum plate hot rolling by finite element method. There are some shortcoming in mathematical model during the practices, such as the limited reference information, the narrow applied scope, big error of calculation results, the unstable calculation result and the low efficiency of calculation. The reasons are mainly that in these models, considering the actual use of feasibility, in some ways, the traits of material and geometrical nonlinearity during powder rolling, were simplified and much more practical process factors were not considered.

The present paper presents a new constitutive relation under a general form of the yield function with an ellipsoidal yield surface for the densification process of iron powder during the rolling compaction. The finite element calculations derived from user-subroutines of Marc are testified by experimental data. The reasons are analyzed based on the experiments of powder rolling which led to the error during the numerical simulation and the effect of the different factors on powder rolling are analyzed.

2. THEORETICAL MODELING [14,15]

2.1. Constitutive relation

Taking account into the influence of porosity and hydrostatic pressure of porous materials, the yield criterion of isotropic material can be formulated by invariants of the stress tensor. A general form of yield criterion used by many investigators is:

\[ f = A J'_2 + B J'_3 - \delta Y'_2 = 0, \] (1)

where \( J'_2 \) and \( J'_3 \) are the second invariant of deviatoric stress tensor and the first invariant of stress tensor, respectively. \( A, B, \) and \( \delta \) are parameters related to hardening of the matrix material and are all functions of the relative density \( \rho \). \( Y'_2 \) is the yield limit of the solid material. According to Kuhn and Doraivelu, parameters \( A \) and \( B \) can be expressed as:

\[
\begin{align*}
A(\rho) &= 2 + \rho^2 \\
B(\rho) &= (1 - \rho^2) / 3.
\end{align*}
\] (2)

According to (1) and (2), the derivative of Eq. (1), the general elasto-plastic formula of the constitutive relationship can be expressed as:

\[ df = \frac{\partial A(\rho)}{\partial \rho} J'_2 d\rho + A(\rho) J'_2 \frac{\partial J'_2}{\partial \sigma_j} d\sigma_j + \frac{\partial B(\rho)}{\partial \rho} J'_3 d\rho + B(\rho) \frac{\partial J'_3}{\partial \sigma_j} d\sigma_j = 0, \] (3)

Let \( P_A = \frac{\partial A(\rho)}{\partial \rho}, \ P_B = \frac{\partial B(\rho)}{\partial \rho}, \) and according to [16],

\[ \frac{d\sigma}{\sigma} = -d\rho/\rho = d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3, \] (4)

where \( \sigma \) is the volume of a micro-body. The final density is independent on the irretrievable plastic strain. There is

\[ d\rho = -\rho d\varepsilon^p. \] (5)

Based on Eq. (5), Eq. (2) can be derived as:

\[ df = -\rho P_A J'_2 d\varepsilon^p + 2A(\rho) J'_2 \frac{\partial A(\rho)}{\partial \sigma_j} d\sigma_j - \rho P_B J'_3 d\varepsilon^p = 0. \] (6)

The associated flow rule is defined as:

\[ d\varepsilon^p = d\lambda \left( \frac{\partial f}{\partial \sigma} \right), \] (7)

where \( d\lambda \) is a coefficient related to the material properties and \( f \) is the yield function. According to the Hooke’s law (elastic constitutive relationship), \( d\sigma \) can be expressed as:

\[ d\sigma_j = D^e_{\varepsilon^p} d\varepsilon^p = D^e_{\varepsilon^p}(d\varepsilon^s - d\varepsilon^p), \]

\[ D^e_{\varepsilon^p} d\varepsilon^p = D^e_{\varepsilon^p} d\varepsilon^s - D^e_{\varepsilon^p} \frac{\partial f}{\partial \sigma_j} d\sigma_j = 0. \] (8)

where \( D^e_{\varepsilon^p} \) is the elastic constitutive tensor, \( \varepsilon^j \) is the total strain tensor, \( \varepsilon^p \) is the elastic strain tensor, and \( \varepsilon^s \) is the plastic strain tensor.

According to Eqs. (6) and (8), \( d\lambda \) can be expressed as:

\[ d\lambda = \frac{\frac{\partial f}{\partial \sigma_j} D^e_{\varepsilon^p}}{\frac{\partial f}{\partial \sigma_j} D^e_{\varepsilon^p} + \rho \left( P_A J'_2 + P_B J'_3 \right) \frac{\partial f}{\partial \sigma_j}} d\varepsilon^p. \] (9)

Based on Eqs. (8) and (9), the elasto-plastic constitutive relationship can be expressed as:

\[ d\sigma_j = \left( D^e_{\varepsilon^p} - D^p_{\varepsilon^p} \right) d\varepsilon^s = D^p_{\varepsilon^p} d\varepsilon^p, \] (10)

where \( D^e_{\varepsilon^p} \) is the plastic constitutive tensor, which can be expressed as:

\[ D^e_{\varepsilon^p} = D^p_{\varepsilon^p} + \frac{\partial f}{\partial \sigma_j} \left( P_A J'_2 + P_B J'_3 \right) \frac{\partial f}{\partial \sigma_j} \left( P_A J'_2 + P_B J'_3 \right) \frac{\partial f}{\partial \sigma_j}. \] (11)
2.2. Constitutive integration [14,15]

It is usually a course with large displacement and strain during powder rolling. In this condition, the effect of the deformation on the equilibrium should be taken into account. The equilibrium equation should be established on the basis of the configuration after the deformation. Therefore, the equilibrium equation and the geometrical relationship are both nonlinear and this is the so-called geometrical nonlinearities.

Generally, Lagrange formulation is used in the solid mechanics and the incremental analysis method is used in the problems concerned about geometrical nonlinearities. In the incremental analysis, the whole analysis procedure is divided into a series of discrete time points, that is, 0, $\Delta t$, $2\Delta t$, $\ldots$, $t$. Therefore, a solution can be performed in a time step $t\rightarrow t+\Delta t$ and each time point is correspondent to a specific configuration. Every determined configuration can be referenced by a solution in a new time step. Practically there are two kinds of Lagrange formulations: one is the so-called Total Lagrange Formulation (T.L. formulation), in which the reference configuration is fixed on the configuration at the initial time 0; the other is the Updated Lagrange Formulation (U.L. formulation), in which the previous configuration is referenced, in which the previous configuration is referenced on the current configuration.

For example, in the solution of the time step in a new time step $t\rightarrow t+\Delta t$, the reference configuration of the solution is the current configuration and each time point is corresponding to a specific configuration. Every determined configuration can be referenced by a solution in the solution of finite deformation problem and it is applied in this paper.

Assuming that the position of a certain material particle at time 0 can be denoted by the position vector $X (i = 1, 2, 3)$, the position of this same particle at time $t$ is $x (i = 1, 2, 3)$. The two positions have the relationship as this form:

$$
\begin{align*}
X_i &= x_i (X, t) \\
x_i &= X_i (x, t).
\end{align*}
$$

In U.L. formulation, referenced the configuration at time $t$, the equilibrium of a body at time $t\rightarrow t+\Delta t$ can be express by the principle of virtual displacement

$$
\int_{V}^{t+\Delta t} S_{ij} \delta_{ij} \delta_{ij} dV = \int_{V}^{t+\Delta t} Q.
$$

where

$$
S_{ij} = \frac{\rho}{\rho} \frac{\partial X_i}{\partial x_j} \frac{\partial X_j}{\partial x_i},
$$

is the Jaumann stress rate tensor, $\sigma_{ij}$ is the strain rate tensor, and $\rho$ is the material density at the referenced configuration and $\rho$ is that of the current configuration.

In the large deformation problem, the relationship between stress and stain tensor is usually expressed by the Jaumann stress rate tensor and strain rate tensor. This rate-form constitutive relationship is

$$
\dot{\sigma}_{ij} = D_{ij} \dot{\varepsilon}_{ij} = D_{ij} \dot{\varepsilon}_{ij},
$$

Where $\dot{\sigma}_{ij}$ is the Jaumann stress rate tensor, $\dot{\varepsilon}_{ij}$ is the strain rate tensor, and $D_{ij}$ is the elasto-plastic constitutive relationship. The Jaumann stress rate tensor and the Cauchy stress tensor have the relationship like

$$
\dot{\sigma}_{ij} = \sigma_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij}.
$$

Note that the stress tensor in (10) is the Cauchy stress tensor. To establish the rate-form constitutive relationship with the 2nd Piola-Kirchhoff stress tensor and Green strain tensor, it is necessary to build the relation between the 2nd Piola-Kirchhoff stress rate tensor and Cauchy stress rate tensor. According to (14), there is

$$
\sigma_{ij} = \frac{\rho}{\rho} \frac{\partial x_i}{\partial x_j} \frac{\partial x_j}{\partial x_i} S_{ij}.
$$

Deduced by the substantive derivative of this equation, the relationship between the 2nd Piola-Kirchhoff stress rate tensor and Cauchy stress rate tensor can be established as

$$
\dot{S}_{ij} = \frac{\rho}{\rho} \frac{\partial X_i}{\partial x_j} \frac{\partial X_j}{\partial x_i} \left( \dot{\sigma}_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij} \right) = \frac{\rho}{\rho} \frac{\partial X_i}{\partial x_j} \frac{\partial X_j}{\partial x_i} \left( D_{ijkl} \dot{\varepsilon}_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij} \right).
$$

Referring the configuration at time $t$, there is $x_i = X_i$. And with the relationship in (14), there is

$$
\dot{S}_{ij} = \frac{\rho}{\rho} \left( D_{ijkl} \dot{\varepsilon}_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij} - \sigma_{ij} \dot{\varepsilon}_{ij} \right).
$$

The incremental form of (19) is

$$
\Delta S_{ij} = \dot{S}_{ij} \Delta t = \frac{\rho}{\rho} \left( D_{ijkl} \Delta \varepsilon_{ij} - \sigma_{ij} \Delta \varepsilon_{ij} - \sigma_{ij} \Delta \varepsilon_{ij} \right) \Delta E_{ij} = \Delta E_{ij} = \Delta E_{ij},
$$

where

$Q$ is the virtual work of the external loads at time $t\rightarrow t+\Delta t$, $S_{ij}^\alpha$ is the 2nd Piola-Kirchhoff stress tensor and $\sigma_{ij}$ is the Cauchy stress (true stress) tensor. $e_{ij}$ is the Green strain tensor, $\rho$ is the material density at the referenced configuration and $\rho$ is that of the current configuration.
Experimental and numerical modeling for powder rolling

Where $\vec{D}_{ijkl}$ is the elasto-plastic constitutive tensor in finite deformation problems. Eq. (20) is the incremental constitutive relationship of large deformation problems.

2.3. Correlative parameters in constitutive model

It can be known from (1) that $\delta$ is a parameters correlative with hardening. There are several assumptions about this parameter. According to the experiment [16], in this work, $\delta$ is assumed as:

$$
\delta = \left( \rho - \rho_c \right) / \left( 1 - \rho_c \right),
$$

(21)

Where $\rho_c$ is the critical relative density at which the compact strength loses.

According to Kuhn’s experiments [16], the Young’s modulus $E$ and the Poisson’s ratio $\mu$ is a function of the relative density, for the cold compaction, which are expressed as:

$$
\begin{align*}
\mu &= 0.5 \rho^{1.22} \\
E &= E_0 \rho
\end{align*}
$$

(22)

where $E_0$ is the Young’s modulus of the solid material.

The relative density $\rho$ during deformation is determined by the irretrievable plastic strain [15]. That is

$$
\rho = \rho_0 \exp \left( -\varepsilon^{\text{pl}} \right),
$$

(23)

where $\rho_0$ is the initial relative density, $\varepsilon^{\text{pl}}$ is the accumulative logarithmic volumetric plastic strain.

The numerical realization of the constitutive relationship is important to the elasto-plastic solution. This implement includes two aspects, one is the stress recovery algorithm, and the other is the iteration algorithm of the nonlinear equation system. The Newton-Raphson method is applied in the nonlinear system solution. And the radial return method is applied in stress recovery procedure. The above model and assumptions of $\delta$ are implemented into user subroutines of MSC.Marc, a commercial finite element analysis software.

3. RESULT ANALYS

According to parameter conditions based on the designed rolling device (shown in Fig. 1), finite element model was established as shown in Fig. 2. The rollers were seen as the rigid in the contact settings definition. In this paper, Al powder is raw material in powder rolling, the feeding thickness is 5 mm and compressibility is 84% (with strip thickness 0.8 mm). Parameters of rolling process and powder materials were shown in Table 1.

3.1. Relative density and mechanical parameters

In Figs. (3) and (4), the Von Mises equivalent stress and the distribution of relative density were shown respectively. Based on the simulation results, from material feeding area to compaction are at output of

Table 1. Process and material parameters used in the rolling compaction.

<table>
<thead>
<tr>
<th>Power Rolling Parameters</th>
<th>Material Parameters (Al)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feeding thickness H1 (mm)</td>
<td>5</td>
</tr>
<tr>
<td>Roller size $D*B$ (mm)</td>
<td>$\phi 150 \times 100$</td>
</tr>
<tr>
<td>Ratio of roll-gap to roll diameter $h/D$</td>
<td>0.0053</td>
</tr>
<tr>
<td>compressibility $\varepsilon$</td>
<td>84%</td>
</tr>
<tr>
<td>rotational speed of roller $n$ (mm/s)</td>
<td>31.8</td>
</tr>
<tr>
<td>friction coefficients $\mu$</td>
<td>0.3</td>
</tr>
</tbody>
</table>
roller, the density of metal powder materials has gradually increased from the initial loose state until stable rolling stage, then the change of powder density is stable and the distribution of density is uniformity. Furthermore, it was shown that at the compaction area between two rollers, the corresponding equivalent stress and relative density is equivalent to the maximum.

The comparison of the relative density between the simulation and the experimental result along the rolling direction was shown in Fig. 5. In Fig. 5, region I was corresponding to feeding area. Region II and III were respectively described as nip area and the compaction and release area. The relative density in region III is about 0.919. There is consistency on the whole between the experimental result and that of simulation. The result of simulation is less than that of the experiment, but the maximum relative error was less than 11%.

In near compaction areas, the density on the area near to two roller surfaces is slightly smaller than that in the roll gap, but the density gradient was small in value, which means that after stable rolling stage, the density distribution of powder materials was uniform along the direction of the thickness.

In Fig. 6, it was shown the comparison of the rolling force between simulation and experimental result. From Fig. 6, after the stable rolling stage, the change of rolling force tends to gently. In the initial stage, there is large difference between the numerical simulation and experimental result. In stable stage, the simulation result is less than the experimental result which was about 31500 N while...
Experimental and numerical modeling for powder rolling

The average value of simulation results was about 28600 N, and there was a consistence between them at stable stage in a whole.

Based on above analysis, it shows that there is consistent between the simulation results obtained by the calculation module based on the constitutive model in this paper and the metal powder rolled experimental results. For the relative density and rolling force, as two of the most important parameters, the simulation results can be accepted, especially the density can achieve higher accuracy, which can be used as the useful reference for actual process. On this basis, this subroutine can be used for comprehensively analyzing rolling process parameters and investigating the effect of different processing conditions on the rolling process.

### Table 2. Process parameters for simulations of rolling compaction.

<table>
<thead>
<tr>
<th>The parameters of rolling</th>
</tr>
</thead>
<tbody>
<tr>
<td>roll diameter $D$</td>
</tr>
<tr>
<td>compressibility $\varepsilon$</td>
</tr>
<tr>
<td>rotational speed of roller $\nu$ (rad/s)</td>
</tr>
<tr>
<td>friction coefficients $\mu$</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>180</td>
</tr>
<tr>
<td>84</td>
</tr>
<tr>
<td>82</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>0.42</td>
</tr>
<tr>
<td>0.35</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.35</td>
</tr>
<tr>
<td>0.3</td>
</tr>
</tbody>
</table>

### Table 3. Results of the orthogonal experiments for simulation.

<table>
<thead>
<tr>
<th>Arrangements of experiments</th>
<th>Factors</th>
<th>The result of simulation</th>
<th>The Minimum of Relative density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>$\nu$</td>
<td>$\mu$</td>
<td>$D$</td>
</tr>
<tr>
<td>1</td>
<td>84%</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>84%</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>84%</td>
<td>0.42</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>82%</td>
<td>0.3</td>
<td>0.35</td>
</tr>
<tr>
<td>5</td>
<td>82%</td>
<td>0.35</td>
<td>0.4</td>
</tr>
<tr>
<td>6</td>
<td>82%</td>
<td>0.42</td>
<td>0.3</td>
</tr>
<tr>
<td>7</td>
<td>80%</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>8</td>
<td>80%</td>
<td>0.35</td>
<td>0.3</td>
</tr>
<tr>
<td>9</td>
<td>80%</td>
<td>0.42</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### Table 4. The result of direct observation.

<table>
<thead>
<tr>
<th>item</th>
<th>The Maximum of rolling force</th>
<th>The Minimum of relative density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>35310.967</td>
<td>25542.933</td>
</tr>
<tr>
<td>$k_2$</td>
<td>27052.033</td>
<td>30241.167</td>
</tr>
<tr>
<td>$k_3$</td>
<td>24602.767</td>
<td>31181.667</td>
</tr>
<tr>
<td>$R$</td>
<td>10708.200</td>
<td>5638.734</td>
</tr>
</tbody>
</table>

Based on above analysis, analysis of process parameters in powder rolling

3.2. The analysis of process parameters in powder rolling

Based the above analysis, the main factors affecting metal powder rolling are physical and mechanical properties of powder materials and the process parameters. By numerical simulation, the effect of the process parameters can be previously considered without the expensive experiments. From Table 1, compressibility $\varepsilon$, rotational speed of roller $\nu$, roll diameter $D$ and friction coefficients $\mu$ are seen as main process parameters in powder rolling. The $L_9(3^4)$ orthogonal experiments are adopted based on these process parameters. The maximum rolling force and the minimum density of strip at stable rolling are two important quality index which are seen as the reference result and by
comprehensive comparison of the simulation result, the degree of effect of each process factors can be judged. The material parameters are still shown in Table 1. The process parameters are shown in Table 2. The simulation results according to orthogonal experiments are shown in Table 3.

The simulation result is shown in Table 3. The result of direct observation is shown in Table 4 in which $K_i$ ($i = 1, 2, 3$) is the mean value at different level of the each factor. According to the equation: $K_i = \sum d_i/m$, where $m$ is the total number of experiments at level $i$, $d_i$ is the density corresponding to factor $j$ at the level $i$. $R_i$ is the extreme difference which is calculated by equation: $R_i = k_{max} - k_{min}$. In Table 4, it shows that the sequence of effect of every factor is: $D > \varepsilon > \mu > v$ and the change of the strip density and the rolling force is in proportion to $D$, $\varepsilon$, $\mu$, and $v$, respectively.

4. CONCLUSIONS

In this paper, based on Continuum mechanics, the mathematical model for powder rolling is constructed. The ellipsoidal yield criterion and elasto-plastic constitutive relationship correspond to the mechanical behaviors of powder compaction were analyzed. Considering the geometrical nonlinearity conditions such as large displacement and large strain, the incremental-form constitutive relationship based on the Updated Lagrange (U.L.) formulation was derived. And the correlative calculating codes was implemented into the user subroutine in the MSC.Marc platform. With these codes, the theoretical and technical basises for three dimensional elasto-plastic Finite Element (FE) simulations of powder rolling were provided.

The reasons were analyzed based on the experiments of powder rolling which led to the error during the numerical simulation and the effect of the different factors on powder rolling were analyzed. It is shown that the result of simulation was less than that of experiment by comparison and the whole result is dependable. The effect of various technological factors is analyzed by the simulation of the rolling process and based on visual analysis of orthogonal experiment. The sequence of factor affecting on the relative density and the rolling force is: $D > \varepsilon > \mu > v$. The permutation of factors is correctly reflects the relationship between rolling force and density.

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