Cyclic strength of metallic materials and thin walled structures under the attack of corrosive media

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Abstract

To describe the corrosion fatigue crack growth, the modified Paris-Erdogan equation is presented. When formulating this equation the following consideration was taken into account. The involvement of an aggressive environment in fatigue crack growth depends on a complex interaction between chemical, mechanical and metallurgical factors. The total crack extension rate under corrosion fatigue conditions is approximated by a simple superposition of the crack growth rate in an inert atmosphere and the crack extension rate due to aggressive environment. The Paris-Erdogan equation is applied to describe the crack growth rate in an inert atmosphere. To formulate the crack extension rate due to aggressive environment it was assumed that the corrosive degradation of material is described by the first order chemical equation, where the stress intensity factor is considered to control the chemical reaction. The modified Paris-Erdogan equation was solved for different values of material parameters. The received crack propagation relations were used to formulate the fatigue strength criterions for metallic specimen with a crack and thin walled structures. Any engineering structure contains a certain numbers of initial micro cracks. Under the cyclic stress and corrosion environment they start to develop. Two periods of crack propagation can be distinguished: the initial, prolonged period, when the size of a crack reaches the critical value, and the period of rapid progressing. This is the way how the dominating crack is formed. However, it is not possible to indicate the time and place of its formation. So the strength of the structure can't be estimated by the traditional methods of design, so the probabilistic methods must be applied. In this paper the probabilistic corrosive fatigue failure criterion, based on corresponding crack growth rate, is developed. The crack growth and fatigue failure curves according to the presented theoretical relations are constructed. They are in good agreement with the basic experimental results of the response of metallic materials to the attack of corrosive media.

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The fatigue crack propagation law in inert environments is defined by the Paris-Erdogan equation [1]

$$\frac{dl}{dN} = C\left(\Delta K\right)^m, \tag{1}$$

where ΔK is stress intensity factor range, C, m are material variables, l is current value of the crack length, N - the loading cycles.

Equation (1) covers the linear (in $\log(dl/dN) - \log(\Delta K)$ coordinates) part of full fatigue crack propagation diagram (curve 1 in Fig. 1).

When formulating the kinetic equation for the fatigue corrosive crack growth, the following consideration was taken into account [2, 3]. The involvement of an aggressive environment in fatigue crack growth depends on a complex interaction between chemical,

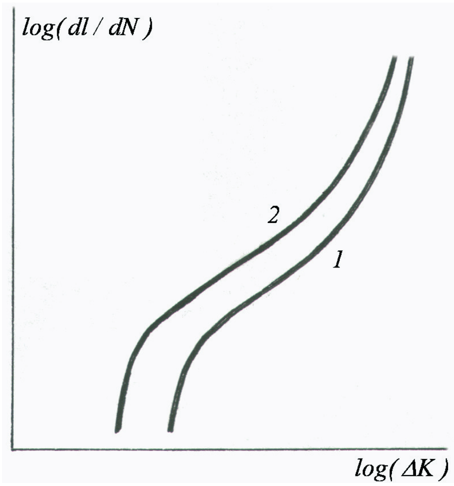

Figure 1: Kinetic diagrams of fatigue failure: for inert media (curve 1), for corrosive media (curve 2).

mechanical and metallurgical factors which leads to the intensification of the crack growth rate as it is shown in Fig. 1 (curve 2). Compare the curves 1 and 2 in Fig. 1 we can see that for the large values of ΔK the influence of corrosive media on the crack growth rate is negligible. Opposite, for the small values of ΔK the influence of corrosive media on the crack growth rate is significant. To describe these experimental effects the total crack extension rate under corrosion fatigue conditions is approximated by a simple superposition [3, 4] of the crack growth rate in an inert atmosphere and the crack extension rate due to aggressive environment

$$\frac{dl}{dN} = C\,(\Delta K)^m + \frac{d\gamma}{dN}, \tag{2}$$

where γ is the current depth of crack due to corrosive material degradation at the tip of a crack.

To formulate the crack extension rate due to aggressive environment it was assumed that the corrosive degradation of material is described by the first order chemical equation, where the stress intensity factor is considered to control the chemical reaction

$$\frac{d\gamma}{dN} = F(\Delta K)\,N^{\beta}, \tag{3}$$

where F is a function of ΔK and β is a constant. Further we will consider the power relation for the function F: $F(\Delta K) = K_1(\Delta K)^{\alpha}$ (K_1 and α are constants).

Taking $\alpha = m$ and introducing (3) into (2), we will receive the following kinetic equation

$$\frac{dl}{dN} = (\Delta K)^m \left(C + K_1 N^{\beta}\right). \tag{4}$$

Further, we will consider the propagation of a small through thickness crack in a large plane specimen, for which $\Delta K = \Delta\sigma\sqrt{\pi\, l}$ ($\Delta\sigma$ is stress range). Introducing this value of stress intensity factor range into (4) we will have

$$\frac{dl}{dN} = (\Delta\sigma)^m \pi^{m/2} l^{m/2} \left(C + K_1 N^{\beta}\right). \tag{5}$$

For $\Delta\sigma = Const$, $l = l_0$ at $N = 0$ the solution of equation (5) is

$$l = \left[\frac{2-m}{2} (\Delta\sigma)^m \pi^{m/2} \left(CN + K_1 \frac{N^{\beta+1}}{\beta+1} \right) + l_0^{\frac{2-m}{2}} \right]^{\frac{2}{2-m}}. \quad (6)$$

If $K_1 = 0$ from (6) follows the Paris-Erdogan relation for the propagation of fatigue crack in inert environments

$$l = \left[\frac{2-m}{2} (\Delta\sigma)^m \pi^{m/2} CN + l_0^{\frac{2-m}{2}} \right]^{\frac{2}{2-m}}. \quad (7)$$

Compare the relations (6) and (7), one can see that the crack grows more intensive in corrosive media than in inert media. In Fig. 2 the theoretical crack growth curves according to the relations (7) (curve 1) and (6) (curve 2) are presented.

Let's consider the fatigue strength criterions for a metallic specimen with a crack in corrosive and inert media. Taking $l = l_*$ (l_* is final crack size at failure) in (7) and (6) we will receive the corresponding failure criterion for the inert media

$$(\Delta\sigma)^m CN = \frac{2\left(l_*^{\frac{2-m}{2}} - l_0^{\frac{2-m}{2}} \right)}{(2-m)\pi^{m/2}}. \quad (8)$$

and for the corrosive media

$$(\Delta\sigma)^m \left(CN + K_1 \frac{N^{\beta+1}}{\beta+1} \right) = \frac{2\left(l_*^{\frac{2-m}{2}} - l_0^{\frac{2-m}{2}} \right)}{(2-m)\pi^{m/2}}. \quad (9)$$

In Fig. 3 in $\log(\Delta\sigma) - \log N$ coordinates are presented the theoretical fatigue fracture curves according to criterions (8) (curve 1) and (9) (curve 2). As it follows from this figure for the relatively large values of $\Delta\sigma$ the influence of corrosive media on the fatigue failure is negligible, while for the small values of $\Delta\sigma$ the influence of corrosive media on the fatigue failure is significant.

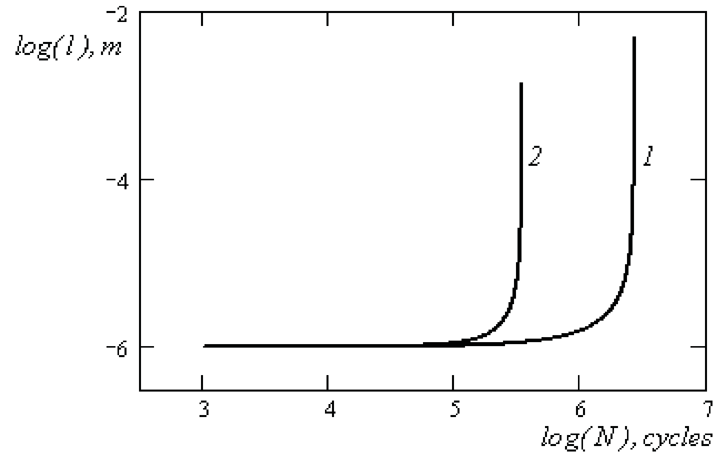

Figure 2: The crack growth curves according to the relations (7) (curve 1) and (6) (curve 2), $\Delta\sigma = 150\ MPa$.

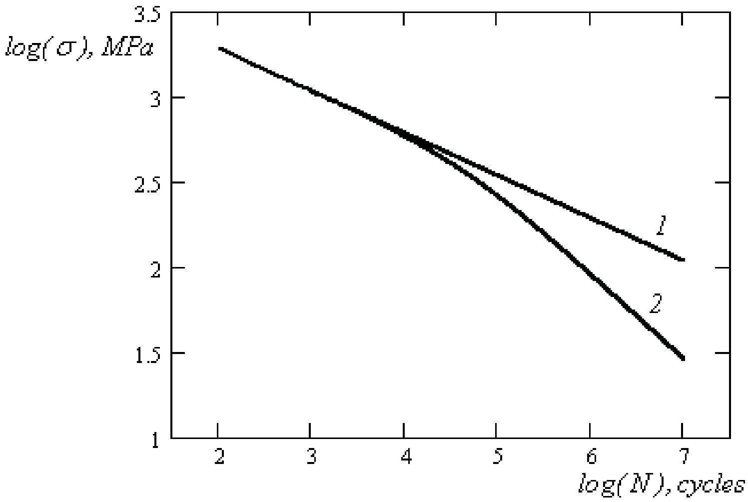

Figure 3: The theoretical fatigue failure curves according to criterions (8) (curve 1) and (9) (curve 2).

When formulating the probability fatigue fracture model we will take into account the following assumptions [5, 6]. An engineering structure is considered as a statistical system containing n initial micro cracks l_{0i} ($l_{0i} \le l_i \le l_*$, $i = \overline{1,\ n}$).

Let's assume that the cracks size distribution is random and is defined by the Poisson's law

$$G(l) = \frac{e^{-\lambda\, l_{0i}} - e^{-\lambda\, l_i(N)}}{e^{-\lambda\, l_{0i}} - e^{-\lambda\, l_*}}, \tag{10}$$

where λ is a constant.

We assume also that the number of cycles to failure N is random one and defined by the Poisson's law. It follows from (10) by introducing into it the value of crack length l defined by the relation (6). Under the cyclic loading and corrosive media cracks are developed and the failure of structure follows when the size of a crack reaches the critical value. The corresponding number of cycles $N = \min(N_i)$ (where N_i is the number of cycles to failure for the individual i-th crack). So we have the minimal value distribution problem for the random variable N_i. Such distribution was considered by Gumbel [7]

$$H(N) = 1 - \left[1 - G(N)\right]^n . \tag{11}$$

For the great number of cracks instead of formula (11) the following asymptotic relation can be used [7, 8]

$$H(N) \approx 1 - \exp\left[-nG(N)\right] . \tag{12}$$

Going to the reliability function $R(N)$ we will have

$$R(N) = 1 - H(N) = \exp\left\{ -n\frac{e^{-\lambda\, l_{0i}} - e^{-\lambda\, l_i(N)}}{e^{-\lambda\, l_{0i}} - e^{-\lambda\, l_*}} \right\} . \tag{13}$$

For the given reliability level R_* the following failure criterion can be derived from formula (8)

$$(\Delta\sigma)^m \left(CN + K_1 \frac{N^{\beta+1}}{\beta+1} \right) = \frac{2}{(2-m)\pi^{m/2}} \left[\left(\frac{1}{\lambda} \ln\left(\frac{1}{B}\right) \right)^{\frac{2-m}{2}} - l_{0i}^{\frac{2-m}{2}} \right], \tag{14}$$

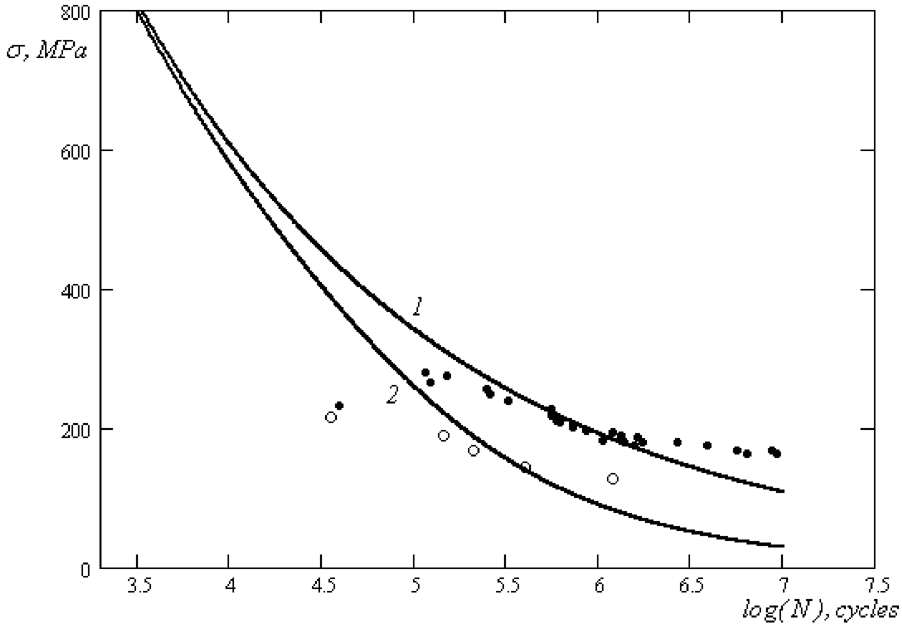

Figure 4: Theoretical fatigue failure curves according to the criterions (15) (curve 1) and (14) (curve 2). By points are shown the experimental results [9].

where $B = e^{-\lambda l_{0i}} + \dfrac{e^{-\lambda l_{0i}} - e^{-\lambda l_*}}{n} \ln R_*$.

Taking $K_1 = 0$ in (14), we will have the failure criterion for the inert media

$$(\Delta\sigma)^m CN = \frac{2}{(2-m)\pi^{m/2}} \left[\left(\frac{1}{\lambda} \ln \left(\frac{1}{B} \right) \right)^{\frac{2-m}{2}} - l_{0i}^{\frac{2-m}{2}} \right], \tag{15}$$

In Fig. 4 in $\sigma - \log(N)$ coordinates are presented the theoretical fatigue failure curves according to criterions (15) (curve 1) and (14) (curve 2). The points correspond to the experimental results [9].

To draw the theoretical curves in Fig. 2-4 the following values of parameters were used: $C = 7,5 \cdot 10^{-11}[m \cdot cycles]^{-1} \cdot [MPa]^{-4}$, $l_* = 10^{-1}m$, $l_{0i} = 10^{-6}m$, $m = 4$, $\lambda = 5[m]^{-1}$, $\beta = 1$, $K_1 = 3 \cdot 10^{-15}[m]^{-1} \cdot [cycles]^{-2} \cdot [MPa]^{-4}$, $n = 20$, $R_* = 0,8$.

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