Wear and tear of nominally fixed joints affected by vibration and percussive impacts

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Abstract

Vibrational conditions are very common for many of natural and artificial material processing devices such as grinding and rod mills, conical and jaw breakers, screening, conveying and feeding devices and also hydropower, construction and other machines. It is known, that vibration and percussive impact lead to increase of risk of facility parts breakdown and even catastrophes. Decrease in material durability under such impacts referred to as fatigue phenomenon is well investigated. However, wearing of parts of nominally fixed joints referred to as fretting-wear is not intimately examined. Wearing of this kind occurs in bolted-type connections, rotating bearings backs, rod-sleeve joints, leaf springs, gears, sockets and other junctions. This article deals with the fretting-wear effect on facility parts, and its origin consisting in relative micro- and occasionally macro-movability of junctions. The effect is considered in terms of simple physical models. Some recommendations on design and handling of relevant machinery are presented.

1 Introduction

A great number of minerals and waste products processing machines such as ball and rod mills, cone and jaw crushers, screens, conveyors and feeders, operate in presence of vibration and impact stress. It does also hydropower, construction and many other machines. Both the vibration and the impact stress are known to cause a severe risk of parts break-down or even industrial disasters. The phenomenon of material strength deterioration caused by fatigue is well studied. However, the same could not be said about another effect of importance, which is the wear of nominally fixed joints and parts subjected to aforementioned factors – the effect referred to as the fretting wear. Such a wear takes place in bolt connections, flange joints, roller bearing fit surfaces, bushing-shaft connections, leaf springs, gears, couplings and other devices. The byproducts of the wear remaining within the contact clearance increase the wear intensity. In addition the intensity tends to be increased due to water penetration in the contact area and electrochemical corrosion. The aforesaid effect has recently come into the focus of many researchers (see e.g. [1 - 4]). The aim of this paper is to emphasize the influence of the said effect, to study the physical nature of the wear believed to be caused by relative micro- or macromobility of joints, to investigate this effect in terms of simple physical models and to state some recommendations for the aforesaid devices design and exploitation.

2 The effect of vibration and impacts on the effective dry friction coefficients, microslip of adjacent parts

The effective coefficients of dry friction at rest f_1, i.e. the friction coefficients effective in presence of constant or slowly changing forces , tend to decrease under vibration reaching in some cases zero, i.e. the system behaves as if the dry friction disappears.

Let's consider a system consisting of a perfectly rigid body pressed against a rough surface with the force N while a harmonic force $\Phi = \Phi_0 \sin \omega t$ acting independently, the latter being directed either lengthwise or perpendicular or transverse to the surface (Fig.1, a).

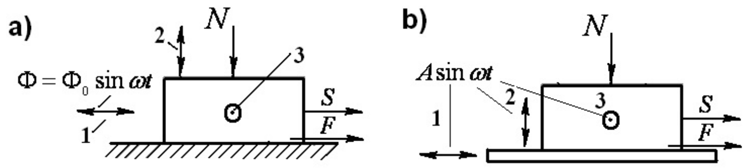

Figure 1: Illustration to definition of effective friction coefficients: a) Disturbing force effect: 1 – lengthwise force, 2 – perpendicular force, 3 – transverse force; b) Surface vibration effect: 1 – lengthwise vibration, 2 – perpendicular vibration, 3 – transverse vibration

Hence the effective coefficients of dry friction in rest, i.e. the friction coefficients in relation to body moving force S, will correspondingly have the following values [5, 6]:

$$f_1^{(=)} = f_1(1 - \frac{w}{f_1}), \qquad f_1^{(\perp)} = f_1(1 - w), \qquad f_1^{(\bullet)} = f_1\sqrt{1 - (w/f_1)^2}, \tag{1}$$

where f_1 – is "usual" coefficients of dry friction at rest , while

$$w = \Phi_0/N \tag{2}$$

–is called "coefficient of overload".

Formulas (1) remain valid for the case when force Φ is absent (Fig.1, b), but the surface will perform harmonic oscillations in corresponding directions according to the law $A \sin \omega t$ (A–amplitude, ω–oscillation frequency). The coefficient of overstress is to be calculated using the formula :

$$w = mA\omega^2/N, \tag{3}$$

where m – is body mass.

Finally, if normal force N represents the weight of the body mg, then

$$w = A\omega^2/g. \tag{4}$$

The formulas (1) are valid only when coefficients $f_1^{(=)}$, $f_1^{(\perp)}$ and $f_1^{(\bullet)}$ are positive. With greater values of the overstress coefficient w an apparent variation of friction mode takes place; in this case one may consider the effective coefficients of dry friction to be equal to zero.

The forecited formulas have been verified through experiments. They explain the effect of microslip in nominally fixed parts contacting by means of dry friction. The effect will be explained below in more details using three simple models.

Regarding the impact stress influenced systems, it was shown by D.M. Tolstoy experiments [7, 8], that even a comparatively slight action of impact stress may result in considerable reduction of the effective coefficients of dry friction $f_1^{(\perp)}$ even though for a very short time. During the experiments a ball of $0.45g$ mass when dropped from $4cm$ height on a body of $1176g$ mass caused diminishing of coefficient $f_1^{(\perp)}$ as compared to coefficient f_1 by 25 %. Theoretical explanation of this effect is given in papers [6, 9].

3 Model 1 – a solid body on a vibrating surface

Let us consider the simplest model representing vibration and impact action on contacting parts interacting through dry friction forces (Fig. 2, a).

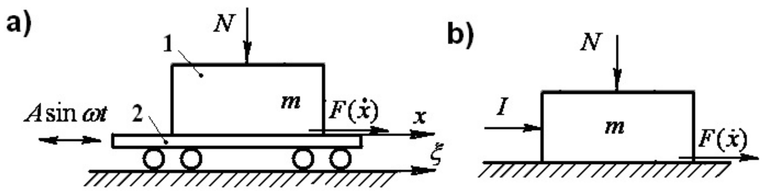

Figure 2: Solid body on a platform: a) – effect of vibrating platform , b) –effect of impact

A solid body 1 with mass m is placed on a rigid platform 2, performing lengthwise oscillations according to the law:

$$\xi = A \sin \omega t, \tag{5}$$

where ξ – is absolute coordinate of the platform, A – amplitude, ω– vibration frequency. A dry friction force F appears between the body and the platform; the body is pressed against the platform by some constant force N, which may include the weight mg of the body. The movement of the body in relation to the platform defined by coordinate x associated with it can be described by the equation:

$$m\ddot{x} = mA\omega^2 \sin \omega t + F(\dot{x}), \tag{6}$$

where

$$\begin{gathered} F(\dot{x}) = \begin{cases} -fN & for \quad \dot{x} > 0 \\ fN & for \quad \dot{x} < 0 \end{cases} \\ -Nf_1 < F(\dot{x}) < Nf_1 \;\; for \;\; \dot{x} = 0, \end{gathered} \tag{7}$$

where f and f_1 are coefficients of sliding and static dry friction correspondingly.

The stable periodic behavior determined by equation (6), were considered in detail in references [5, 6, 10]. Let us represent the results of this solution in a different form.

The mode of movements is assumed to depend on two nondimensional parameters:

$$w = \frac{mA\omega^2}{Nf}, \qquad w_1 = \frac{mA\omega^2}{Nf_1}. \tag{8}$$

Assuming $f = 0.7f_1$, which approximately corresponds to the actual ratio between these two coefficients for many materials, we obtain a single parameter $w = mA\omega^2/N$. Fig. 3 illustrates the body relative oscillation semi-range dependence on this parameter. When w <1/0.7=1.43 the body moves together with the platform – there is no sliding friction between them (area I). When $\frac{1}{0.7} = 1.43 < w < \frac{1}{0.472} = 2.12$ the body slips alternatively forward and backward, stopping its movement for finite time periods after each change in the sliding direction (area II), while with w >1/0.472=2.12 (area III) it slips, changing momentarily the direction of slippage.

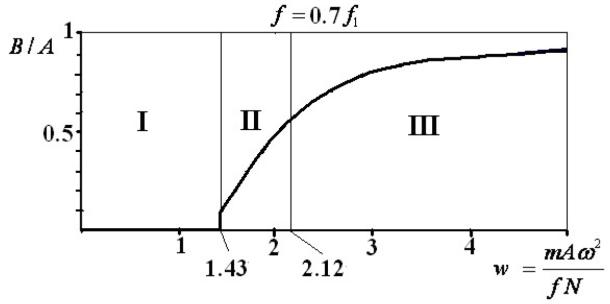

Figure 3: Dependance of the body oscillation half-swing on the overstress parameter

Increase of w leads to increase of the half-swing B, which asymptotically approaches the oscillation amplitude value A when $w \to \infty$.

The condition for no-slipping body behavior could be expressed by the following inequality:

$$f_1N > mA\omega^2 \tag{9}$$

or alternatively, when $f = 0.7f_1$, by the inequality:

$$fN > 0.7mA\omega^2. \tag{10}$$

When the body with a mass m is subjected to an impact with momentum I (Fig. 2, b) it acquires velocity $v = I/m$. It results in a shift of the body to a distance B_1, which may be found from the equality $\frac{1}{2}mv^2 = fNB_1$, hence

$$B_1 = \frac{mv^2}{2fN} = \frac{I^2}{2fNm}. \tag{11}$$

It follows from the forecited expressions that whereas it is possible to avoid interfacial slippage of machine parts in vibration conditions by applying stronger interference N, it is not possible to avoid it in impact conditions even by a very tight interference.

4 Model 2 – a solid body with inner degree of freedom – influence of resonance effects

Let us consider a system extending model 1 by including a second body with mass m_2 placed inside the body with mass m_1, (see Fig. 4,a). The second body is joined to the first one using a flexible member and a damping member, c and β are a coefficient of rigidity and a damping coefficient correspondingly. The body m_1, as it is in the system considered above, is placed on a platform, oscillating according to the law (5), the force $F(\dot{x})$ represents the dry friction arising between the body m_1 and the platform. The body m_2 can move with relatively to the body m_1 along the direction parallel to the plane of contact.

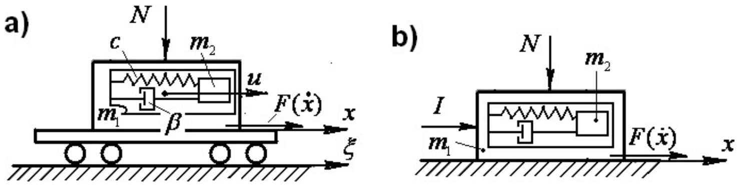

Figure 4: A solid body with inner degree of freedom a) Effect of vibrating platform, b) Effect of impact

The relative to the platform position of body m_1 is described by coordinate x, while the position of the body m_2 relative to the body m_1 is described by coordinate u; the latter is calculated from the position corresponding to that of unstrained flexible member.

Equations describing the motions of the system under consideration could be written in the following form:

$$m_1\ddot{x} = m_1 A\omega^2 \sin\omega t + cu + \beta\dot{u} + F(\dot{x}), \tag{12}$$

$$m_2\ddot{u} = m_2(A\omega^2 \sin\omega t - \ddot{x}) - cu - \beta\dot{u}, \tag{13}$$

where the friction force $F(\dot{x})$ is determined by expressions (7) as shown above.

It is of some difficulties to obtain the exact analytical solution of the non-linear system (12), (13). An approximate solution of a more general system is described in [11], where the object was to investigate vibrational displacement of bodies. In this paper we dwell upon periodic oscillation modes of motion and suggest some solutions based on the other assumptions. For that matter we take as a first approximation the assumption that the motion of the main body m_1 only slightly affects the motion of the body m_2. Then the value $m_2\ddot{x}$ in equation (13) can be neglected in comparison with the other values so that the equation takes the following form:

$$m_2\ddot{u} + \beta\dot{u} + cu = m_2 A\omega^2 \sin\omega t. \tag{14}$$

The solution of this equation corresponding to the forced steady-state oscillations could be described the following way:

$$u = Ak\sin(\omega t + \alpha), \tag{15}$$

where

$$k = \frac{\omega^2}{\sqrt{(\lambda^2-\omega^2)^2 + 4n^2\omega^2}},$$

$$\frac{c}{m_2} = \lambda^2, \quad \frac{\beta}{m_2} = 2n, \quad \sin\alpha = -2\frac{n}{\omega}k, \quad \cos\alpha = \frac{\lambda^2-\omega^2}{\omega^2}k. \tag{16}$$

Hence the equation (12) could be written in the form:

$$m_1\ddot{x} = m_1 A\omega^2 \sin\omega t + cAk\sin(\omega t + \alpha) + \beta\omega Ak\cos(\omega t + \alpha) + F(\dot{x}).$$

This equation can be also reformulated in the following form:

$$m_1\ddot{x} = m_1 A_1 \omega^2 \sin(\omega t + \varepsilon) + F(\dot{x}), \tag{17}$$

where

$$A_1 = A\sqrt{1 + 2k(\frac{\lambda_1^2}{\omega^2}\cos\alpha - 2\frac{n_1}{\omega}\sin\alpha) + k^2(\frac{\lambda_1^4}{\omega^4} + 4\frac{n_1^2}{\omega^2})}, \tag{18}$$

$\lambda_1^2 = \frac{c}{m_1}$, $2n_1 = \frac{\beta}{m_1}$, and ε – some inessential constant which may be always reduced to zero by selecting the initial time point t. The equation (17) is congruent to the equation (6), obtained for one-mass system, so that it is possible to use the solution of the latter. Specifically, assuming $f = 0.7f_1$ it is possible to determine half-swing of oscillations $B = B_2$ using the Fig. 3, it is to be taken into account that the amplitude A value should be replaced with the amplitude A_1 value, which is determined by formula (18).

The solution obtained may be used for calculating an approximate solution. However, there is no special need for it because the solution accuracy can be checked using the software designed for the system (12), (13) equations investigation. Calculations performed using this software proved that the approximate solution provides quite a satisfactory accuracy.

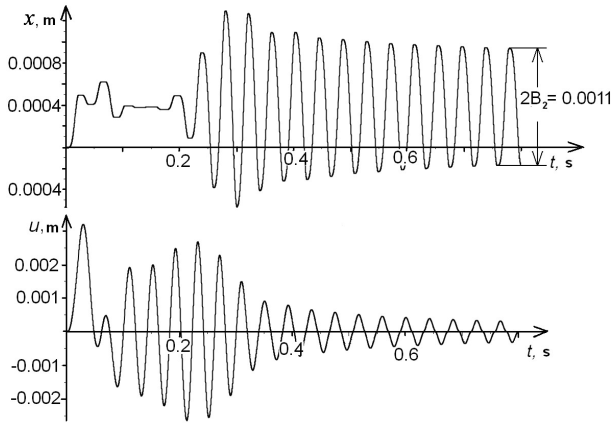

Figure 5: Graphs showing dependence of coordinates$x$ and u on time t (Parameter values: $A = 0.001\ m$; $\omega = 150\ s^{-1}$; $A = 9 \cdot 10^4\ N/m$; $\beta = 30\ N \cdot s/m$; $\lambda_2 = 300\ s^{-1}$; $f = 0.5$; $m_1 = 3\ kg$; $m_2 = 1\ kg$; $N = 100\ N$)

Fig.5 illustrates dependence of the coordinates x and u on time t and Fig. 6 shows oscillation half-swing B_1 versus oscillation frequency ω. One can see that inner freedom degree can significantly enhance influence of vibration on the area of body m_1 sliding along the base plate when oscillation frequency ω approached the frequency of free oscillations $\lambda_2 = \sqrt{c/m_2}$ performed by m_2 body inside m_1 body. The same inference will be obtained when using formula (18) for calculations and Fig. 3 graph.

Formula (11) obtained for the Model 1, remains valid for the calculation of the body m_1 shift resulted from impact influence.

Similarly to the Model 1 case, it is possible to eliminate interfacial slippage of the mass m_1 by application of sufficiently high pressing force N. In impact condition such a microslip cannot be excluded even with a very tight fit.

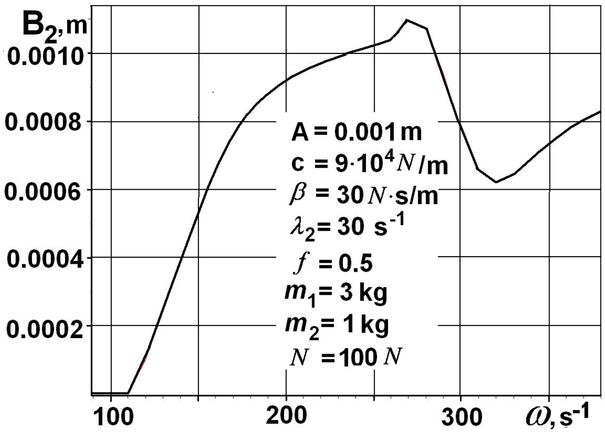

Figure 6: Amplitude of body m_1 oscillations versus induced oscillation frequency ω

5 Model 3 – an elastic rod and a washer planted on it with initial strain

Compression fit is a widespread kind of machinery joints. The behavior of parts of such joints consisting, for example, from a rod and a washer tightly planted on it can be considered on the basis of the model representing a particular case of a so-called Chelomei pendulum. In work [15] it has been shown that in systems of this kind not only microslip could be observed, but also even vibrational translation.

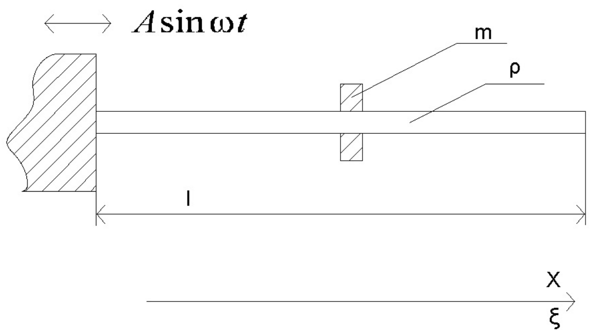

Figure 7: Model configuration

The system under consideration consists of elastic rod of length l and rigid washer of mass m, planted on the rod with initial strain ε^0. The base of the rod is being vibrated, performing oscillations which could be written in fixed coordinates ξ:

$$\xi(t) = A \sin \omega t. \tag{19}$$

The equation of motion of the washer in the rod base bounded (moving) coordinate system could be written in the following form:

$$m\ddot{x} = mA\omega^2 \sin \omega t + F_f(t, x, \dot{x})$$
$$F_f = \begin{cases} -fN(t,x), & \dot{x} > 0 \\ -mA\omega^2 \sin \omega t, & \dot{x} \equiv 0 \\ fN(t,x), & \dot{x} < 0 \end{cases} \tag{20}$$

where x is coordinate of the bar cross-section in the moving frame of reference associated with the oscillating base and at the same time coordinate of the washer near this section, F_f - the friction force that occurs between the washer and the bar, m – the mass of the washer, A – the amplitude of the bar base oscillations, ω - their frequency, f- the friction coefficient, $N(t)$ - the elastic interaction force between the washer and the bar, the change of which is determined by its vibrational excitation. The frictional force is determined by Coulomb's law, not exceeding $|fN(t)|$ in absolute value, and balancing other effects at rest (in this case, the inertia force).

As it was shown in the publication [15], taking into account the initial strain or say the fit, with which the washer is planted on the rod, it is possible to write the friction force in the following form:

$$F_f = \begin{cases} -\eta f S_c E \left(\varepsilon_n^0 - \frac{\rho A\omega^2(l-x)}{2E}\sin\omega t\right), & \dot{x} > 0 \\ -mA\omega^2 \sin\omega t, & \dot{x} \equiv 0 \\ \eta f S_c E \left(\varepsilon_n^0 - \frac{\rho A\omega^2(l-x)}{2E}\sin\omega t\right), & \dot{x} < 0 \end{cases} \tag{21}$$

where S_c is the area of the rod-washer contact, and E is the rod's Young's modulus. It should be required that the following condition for ε^0 is met:

$$\varepsilon^0 \geq \frac{\rho A\omega^2 l}{2E} \tag{22}$$

Otherwise the washer could lose its fit on the rod and the behavior of the washer planted on the rod with a gap could be quite different from that under consideration.

Microslip and vibrational translation could be examined using methods presented in the book [5] for investigation of the problem of motion of a body on a rough inclined vibrating plane.

Considering the problem it is convenient to dissect the time axis into intervals so that the washer planted on the rod relatively resting inside each one of these intervals begins to slide in positive direction I_+, remains in the relative rest I_0, or begins to slide in negative direction I_-.

The motion in positive direction begins under the following conditions:

$$\begin{array}{l} \sin\omega t \;>\; z_+ \\ z_+ = \frac{2\eta f S_c E\varepsilon_n^0}{A\omega^2\left(2m+\eta f\rho S_c(l-x)\right)} \end{array} . \tag{23}$$

Interval of motion in negative direction:

$$\begin{array}{l} \sin\omega t \;<\; z_- \\ z_- = -\frac{2\eta f S_c E\varepsilon_n^0}{A\omega^2\left(2m-\eta f\rho S_c(l-x)\right)} \end{array} . \tag{24}$$

Under the following conditions

$$\begin{cases} \sin\omega t - z_+ < 0 \\ \sin\omega t - z_- > 0 \end{cases} \tag{25}$$

the washer remains in relative rest.

The behavior of the washer is considered to consist of a set of intervals of sliding in positive or negative direction and also relative rest intervals. It should be mentioned that these intervals do not correspond one to one to the aforesaid intervals I. The motion of the washer could be described by the following expressions:

$$\dot{x}(t) = \mp \frac{fS_cE\varepsilon_n^0}{m}(t-t^*) - A\omega\left(1 \pm \frac{fS_c\rho(l-x)}{2m}\right)(\cos\omega t - \cos\omega t^*) + \dot{x}^* \tag{26}$$

$$\begin{aligned} x(t) = \mp \tfrac{fS_cE\varepsilon_n^0}{2m}(t-t^*)^2 + A\omega\left(1 \pm \tfrac{fS_c\rho(l-x)}{2m}\right)\cos\omega t^*\,(t-t^*) - \\ -A\left(1 \pm \tfrac{fS_c\rho(l-x)}{2m}\right)(\sin\omega t - \sin\omega t^*) + \dot{x}^*(t-t^*) \end{aligned} \tag{27}$$

Top marks are for the positive, and bottom marks are for a negative initial velocities, respectively, t^* – is the interval beginning moment and $\dot{x}^*$ parameter is choosen to meet the interval initial conditions. After the stop $\dot{x}(t) = 0$ the washer switches to the mode, corresponding to the interval in which t lies: $I_\pm$ or I_0.

The software developed using the algorithm described above could help also in calculation of the energy consumption or power of dissipative forces which could be useful in estimation of the wear. As described below the wear would be considered to be proportional to the power of dissipative forces. The diagram showing dependence of the power on overload coefficient is presented on the Fig. 8.

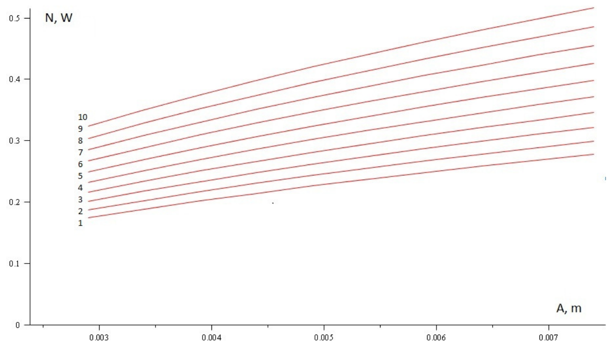

Figure 8: Friction force power versus the amplitude of the exitation dependance, each curve corresponds to fixed overload coefficient (from 10 - curve 1 to 12 - curve 10)

6 On some other Models

Among other models of no pertinence to the wear problem the flexible body models may be referred to [6, 9, 12, and 13]. Flexibility of adjacent bodies was taken into account in the theory of construction hysteresis [14]. If necessary such models can be used in evaluation of the wear caused by vibration.

7 Wear of nominally fixed joints in vibration and impact conditions

The energy consumed in interfacial slip of bodies during one vibration period $T = 2\pi/\omega$, amounts to $E_1 = 4fNB$, while energy consumption per second (i.e. power expense) is:

$$P = E_1/T = \frac{2}{\pi} fNB\omega. \tag{28}$$

Let us suppose that the energy consumed on overcoming friction in interfacial slip of adjacent parts is spent on corruption of these parts materials. To put it otherwise, let us assume that the rate of mass wear W (kg/s), i.e. wear of material mass per second is proportional to the power P:

$$W = \kappa P = \frac{2\kappa}{\pi} fNB\omega. \tag{29}$$

Here the coefficientκ, having dimensionality $\frac{kg}{s}\frac{s}{Nm} = \frac{kg}{N}\frac{1}{m} = \frac{s^2}{m^2}$, represents a mass wear corresponding to $P = 1W$ energy consumption. This assumption is in agreement with so called Energy Theory of Wear [16–18], as well as with solid materials comminution practice (see below) and data on grinding media wear in a barrel mills ([19], p. 302).

Let us denote the area of contacting surface by F and the rate of linear wear by Δ. Then the following could be expressed:

$$W = \rho F\Delta, \qquad \Delta = \frac{W}{\rho F} = \frac{2\kappa f\sigma B\omega}{\pi\rho}(m/s), \tag{30}$$

where $\sigma = N/F$ – normal pressure between the adjacent parts. Knowing the rate of linear wear Δ, the time T_* required for the wear to reach a critical value δ could be readily estimated:

$$T_* = \delta/\Delta \text{ (c)} = \delta/3600 \ \cdot \Delta(h). \tag{31}$$

The value κ in formulas (29), (30) can be treated as an empirically determined coefficient. Unfortunately, we failed to find the magnitude of this coefficient. So below we shall make attempt to estimate this coefficient at least roughly.

The quantity for $\varphi\ (D, d)$, representing the consumption of energy required for destruction of one kilogram of material from initial average size D to a final size d, is well known in the theory of materials comminution [19, 20]. It is obvious that the quantity in consideration is reciprocal to the coefficient κ, i.e.

$$\varphi\,(D, d) = 1/\kappa \ \ (W \cdot s/kg = m^2/s^2) \tag{32}$$

As reported in papers [21, 22] the wear products of mating materials had 40–50 mcm sizes. Grinding ores in disk pulverizers to such size requires approximately about 2000 $kWh/t = 7.2\ 10^6\ W\ s/kg$ of energy φ. Taking into account the imperfect mechanism of treatment taken for the study and allowing for inefficiency of the friction wear we assume the actual consumption of energy to be much higher than it was observed and for that matter take the coefficient φ by two orders higher, namely $\varphi \approx 10^9\ W\ s/kg$.

Supporting $\rho = 7.8 g/cm^3 = 7.8\ 10^3 kg/m^3$, $f = 0.3$, $\sigma = 10N/mm^2 = 10^7 N/m^2$, ω =314 s^{-1}, $B = 1mcm = 10^{-6}$ m and taking into account the formulas (30) and (31) we obtain $\Delta = \frac{2 \cdot 0.3 \cdot 10^7 \cdot 10^{-6} \cdot 314}{3.14 \cdot 10^9 \cdot 7.8 \cdot 10^3} = 0.769 \cdot 10^{-10}$ m/s and at $\delta = 1\ mm = 10^{-3}\ m$. Then we could evaluate $T_* = \delta/\Delta = \frac{10^{-3}}{0.769 \cdot 10^{-10}} = 1.3 \cdot 10^7$ s= $\frac{1.3 \cdot 10^6}{3600 \cdot 24} = 150$ days. In the case of the half-swing of sliding is taken as $B = 0.1\ mm = 10^{-4} m$, the time T_* is reduced by 100 , i.e. will last only 1.5 days.

It is to be noted that in some cases the slip of parts subjected to impacts and vibration could be minimized or even eliminated through construction design or process improvements. For examples, engineering solutions regarding lowering design hysteresis are described in the books [14, 23], and RF Patent [24].

8 Some recommendations concerning design and exploitation of machines operating in vibration and impact conditions

1. It is essential that many of friction joints in mechanisms and structures designed for severe operation in conditions of vibrations and impacts should not be considered as fixed. It relates to bolt connections, flange joints and bushing-shaft connections of ball and rod mills, crushers, screens and power engineering machines.

2. Microslip of adjacent machine parts subjected to impacts and vibration has the effect of producing their wear; a unilateral displacement of parts is possible (for example self-loosening of threaded joints). In inadequate maintenance conditions this may entail the machine's failures and even the plant's shut downs.

3. In contrast to vibrational conditions where some interfacial slip of adjacent machine parts can be excluded by application of press fit, it is not possible to avoid the interfacial microslip in impact conditions even when using a very high pressure fit.

4. The slippage of machine parts caused by vibration and impacts in some cases can be diminished or even eliminated by introducing special design and process innovations.

5. When planning the maintenance schedule for adjacent parts and connections one should take into consideration that in line with the theory suggested here the wear rate of such parts tends to be increased even higher than in proportion to amplitude rise and is likely to be enhanced by higher oscillation frequency.

9 Conclusion

This paper deals with the effect of vibration and impacts on nominally fixed joints of machine parts, such as bolt and flange connections. Under the action of impacts and vibration (or oscillation) such components reveal so called micro mobility in relation to each other and thus are subjected to wear. A number of formulas have been obtained for two virtual models to describe the wear rate, the formulas contain a single empirical coefficient. In its physical sense that coefficient denotes a wear rate per a unit of energy spent on mutual slipping action. Based on the results obtained, some recommendations have been suggested with regard to design engineering and maintenance schedule of the machines in question.

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