Influence of vibration on the dynamics of a light spherical body in rotating cavity with liquid

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Abstract

An experimental study is carried of the dynamics of the light spherical body placed in the rotating liquid-filled cavity. The containers used are of cylindrical and spherical shape. The rotation axis is oriented horizontally. The rotation speed is sufficiently high, so that under the action of the centrifugal force the free light body occupies a position near the cavity axis. The system is subject to the translational vibrations, perpendicular to the rotation axis.

In the absence of vibrations, the sphere rotates slower than the cavity (in the laboratory frame). The influence of vibrations is manifested in the resonant excitation of the differential rotation of the sphere. It occurs at coincidence of the inertial oscillations frequency of the sphere and the vibration frequency. Depending on the vibration parameters, the intensive outstripping or lagging rotation of the sphere is excited. The resonant areas position is determined by the ratio of the vibration frequency to the cavity rotation speed $n \equiv \Omega_{vib}/\Omega_{rot}$. Depending on Ω_{rot}, the sphere occupies different steady positions, displacing along the rotation axis. Different steady positions of the sphere are matched by different velocities of its rotation.

It is found that in the liquid contained between the sphere and the cavity end-walls a shear flow appears in the form of the Taylor – Proudman column. At relatively fast differential rotation of the sphere (outstripping or lagging) the column boundary becomes unstable, giving rise to a two-dimensional wave propagating in the azimuthal direction. The wave length decreases with the decrease of the differential rotation speed of the sphere.

1 Introduction

Problems of vibrational hydrodynamics at rotation are actual as they consider the phenomena widely spread in nature and engineering.

If a solid is placed in the liquid-filled rotating cavity, its rotation speed being different from that of the cavity, then in the liquid the shear flow is generated in the form of a column extended along the axis. It is called the "Taylor – Proudman column", its boundary is formed by the shear layer. The fluid particles do not cross the column boundary, as a result it rotates practically as an organic whole with the speed different from that of the surrounding liquid [1].

An important feature of the rotating hydrodynamic systems is their elastic properties determined by the action of the inertia forces on the liquid particles. A remarkable example are the inertial waves propagating on the interface of two centrifuged immiscible liquids of different density (the light liquid forms a cylindrical column in the centre) in consequence of the action of an external periodic force [2].

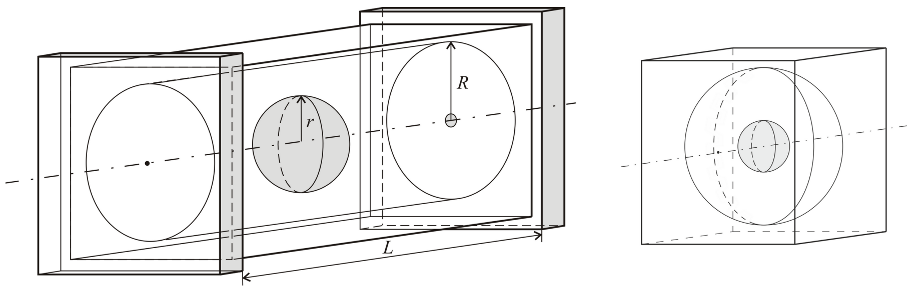

Figure 1: The scheme of the cavities of the cylindrical and spherical shape

In case when a light solid body is placed in the liquid, under the action of the centrifugal force of inertia it occupies the steady position on the cavity axis. An external periodic force, for example due to the transversal vibrations, induces the circular body oscillations. This leads to its differential rotation excitation [3]. In consequence, the liquid Taylor – Proudman column is formed as a geometric continuation of the solid. When the body is considerably shorter than the cavity, in which it is situated, then the liquid column dynamics cannot be neglected. This case is studied in the present work on the light sphere example.

2 Experimental setup and techniques

The experimental setup consists of the cavity in which the light body of the spherical shape is placed. The cavity is filled with liquid (aqueous solutions of glycerin with the kinematic viscosity $\nu = 1 - 10$ cSt) and fixed on the platform of the electrodynamic vibrator, which produces translational vibrations normal to the rotation axis. In experiments the cavities made of plexiglass are used, one has the cylindrical shape (the sizes of cavity: $R = 26.0$ mm, $L = 62.0 - 72.0$ mm) and the other is spherical ($R = 44.5$ mm) (fig. 1). Radius of the sphere $r = 17.7$ mm, average density $\rho_s = 0.17$ g/cm^3.

In the work the spherical body dynamics is studied depending on the cavity rotation speed Ω_{rot} in the absence of vibrations and at the vibration action. The velocity of rotation is always high and the sphere occupies the steady position near the rotation axis under the action of the centrifugal forces. In the experiments the sphere rotation speed Ω_s in the laboratory frame and the distances x_1 and x_2 from the sphere boundaries to the cavity end-faces are measured. Observations are carried out in the stroboscopic illumination. The rotation speed of the cavity and of the body is measured with the strobotachometer with accuracy 0.06 rad/s. The body position in the cavity is found using the photo registration method. The obtained data is used to calculate the relative rotation speed of the sphere $\Delta\Omega = \Omega_{rot} - \Omega_s$ and its position relative to the end-faces of the cavity $x = (x_2 - x_1)/(x_2 + x_1)$.

The cavity rotation speed varies in the interval $\Omega_{rot} = 50 - 300$ rad/s and is set with the accuracy of 0.06 rad/s. Frequency and amplitude of vibrations vary in the intervals $f_{vib} = 20 - 45Hz$, $b_{vib} = 0.1 - 0.5$ mm.

3 Experimental results

In the absence of rotation the light sphere is situated at the cavity boundary in its upper part. With the increase of the cavity rotation speed Ω_{rot} the sphere is entrained by the fluid (in the rotation direction), and on reaching some critical value Ω_{rot} it occupies the steady position on the cavity axis. In the absence of vibrations the sphere in such state always

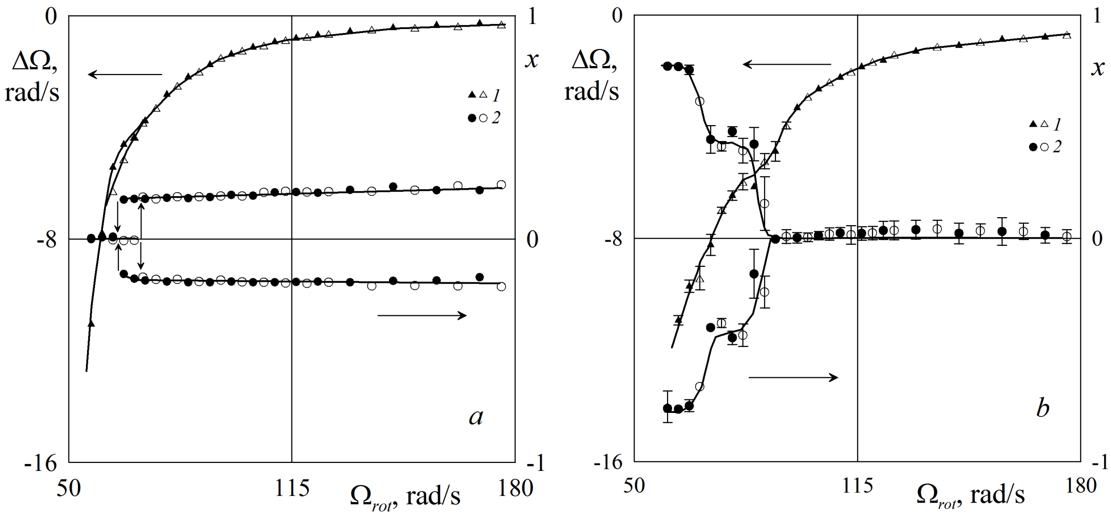

Figure 2: The relative rotation velocity $\Delta\Omega$ (*1*) and the sphere position x (*2*) vs. the cavity rotation speed (a – cylindrical cavity, b – spherical) for $\nu = 5.0$ cSt; here and further light symbols signify the increase of Ω_{rot}, dark – decrease

rotates slower that the cavity, thus $\Delta\Omega < 0$. Such behaviour of the sphere is characteristic for both, cylindrical and spherical cavities (fig. 2, *a*, *b*, points *1*). In process of the Ω_{rot} magnification the body lagging intensity gradually decreases. It occurs until rotation of the whole system does not become almost solid-state. The experimental points obtained at the decrease of Ω_{rot} coincide with the points obtained at its increase. However, on reaching the critical value, at which the sphere transition to the rotation axis occurred, there is no collapse (the transition from the axis to the exterior boundary of the rotating cavity) observed.

Besides the differential rotation speed of the sphere, at the changing of Ω_{rot} there is the changing of the sphere position on the rotation axis relative to the cavity end-walls, characterized by the dimensionless coordinates (fig. 2, *a*, *b*, points *2*). In the cavity of the cylindrical shape ($L = 72.0$ mm) with increase of the velocity Ω_{rot} the sphere position in the center becomes unstable, and the sphere displaces along the rotation axis to one of the end-faces. The new position of the sphere on the axis is stationary. The shift can occur both to the right and to the left. It does not depend on the direction of the cavity rotation. The relative velocity of the body rotation also does not depend on which end-wall it will be moved to.

In the cavity of the spherical shape (fig. 2, *b*, points *2*), the dependence of the sphere position dynamics on Ω_{rot} is opposite. Under the influence of the centrifugal forces at the rotation velocity increase, the sphere transfers to the rotation axis, but not in the cavity center, and to one of its poles. At the further increase of Ω_{rot} the sphere gradually drifts towards the cavity center, this displacement going in steps. When shifting the sphere to the right or left, the curves of dependence $x(\Omega_{rot})$ are mirror-symmetrical, so it is the pitchfork bifurcation.

The change of the fluid viscosity leads only to the change of the thresholds of loss of stability of the sphere symmetrical position, its dynamics does not change qualitatively [4].

The behaviour of the sphere at vibrations significantly changes only in the resonance areas when the frequency of vibration action coincides with the eigenfrequency of inertial oscillations of the sphere. In the region $\Omega_{rot} < \Omega_{vib}$ ($\Omega_{vib} = 2\pi f_{vib}$) the excitation of intensive outstripping rotation of the sphere is observed, which does not exist without

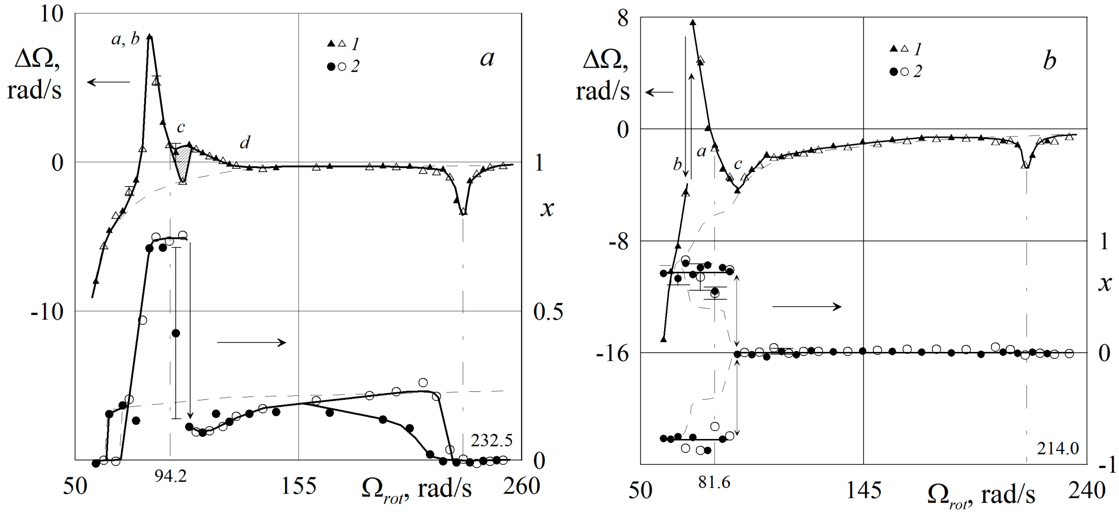

Figure 3: The velocity $\Delta\Omega$ (*1*) and the sphere position x (*2*) vs. the velocity Ω_{rot} of the cylindrical cavity (a) and spherical one (b); f_{vib}= 30 Hz, b_{vib}= 0.27 mm, ν = 5.0 cSt; dashed lines – the experiment in the absence of vibration

vibrations (fig. 3, a, b, points *1*). In the area where the vibration frequency is less than the cyclic frequency of the cavity rotation ($\Omega_{rot} > \Omega_{vib}$), the intensive lagging differential rotation is raised ($\Delta\Omega < 0$). Intensive rotation in resonance areas is accompanied by high-amplitude oscillations of the sphere of circular polarisation with the frequency of driving force Ω_{vib}. Out of the resonance areas the velocity of the sphere rotation at vibrations is not different from the one in their absence.

At slow increase of the cavity rotation speed the outstripping rotation excitation (threshold a) can occur smoothly (fig. 3, a) or abruptly (fig. 3, b) depending on the parameters of vibrations. The finite-amplitude transitions are conventionally shown by vertical arrows. At the subsequent increase of Ω_{rot} the relative velocity starts to decrease sharply. In the cylindrical cavity the rate of the outstripping motion decrease diminishes in the threshold way in the point c, then the vibrational outstripping motion is maintained up to the point d, where the vibrational curve crosses the gravitational one. In the cavity of the spherical shape the threshold d is absent, the intensive outstripping motion stops in the point c. At the decrease of the cavity rotation speed (dark points on fig. 3) the measured data coincide with the results obtained at the increase of Ω_{rot}. The breakdown of the outstripping rotation occurs abruptly in the point b.

Simultaneously with velocity the sphere position changes in the resonance areas (fig. 3, a, b, points *2*) outside of which it practically coincides with the off-vibration case. In the outstripping motion area the sphere shifts to one of the end faces. The maximum displacement is observed in the point c, where the intensive outstripping motion is excited. The position of the sphere in the cavity center ($x = 0$) is stable in the area of the resonant lagging rotation.

Dot-dashed lines in fig. 3 show the position of the resonance frequency on the axis Ω_{rot}. In cavities of various geometry they differ at the same vibration parameters. In the spherical cavity the resonance frequencies have smaller values Ω^{+}_{rot}= 81.6 rad/s and Ω^{-}_{rot} = 214.0 rad/s, for the cylindrical cavity – Ω^{+}_{rot}= 94.2 rad/s and Ω^{-}_{rot} = 232.5 rad/s.

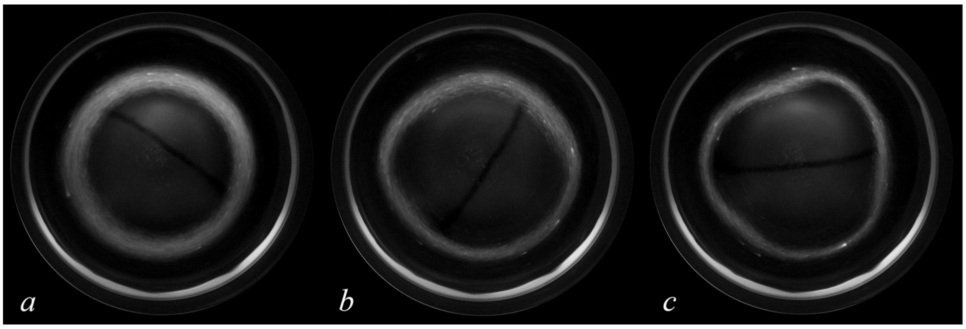

Figure 4: Photos of the Stewartson shear layer in traversal section in the absence of vibrations; $\Omega_{rot} = 100.5$ (a) and 75.5 rad/s (b, c), $\nu = 6.7$ cSt, $L = 62.0$ mm

4 Flow structures

At differential rotation of the sphere, in the fluid volume between the sphere and the cavity end-faces the Stewartson shear layer extended along the rotation axis is formed, delimiting the Taylor – Proudman column. On fig. 4 the flow patterns in the cylindrical cavity in the absence of vibrations are presented. In the centre the sphere is visible, around which white light-scattering particles trace the column boundary.

At slow relative rotation of the sphere the column has the shape of the circular cylinder (fig. 4, a), its traversal size coincides with diameter of the body. With magnification of the difference of velocities between the sphere and the cavity the column boundary becomes instable and takes the form of the polyhedral prism (b). Angular velocity of rotation of the prism (wave phase velocity) in the laboratory frame is less than the velocity of the sphere rotation. With growth of intensity of the differential rotation of the body the length of the azimuthal wave propagating on the liquid column boundary, is incremented (c), the number of bounds of the prism decreases. In the experiments, the patterns with the wave numbers $m = 3 - 6$ are observed. In the spherical fluid shell the similar geostrophic flow in the form of the Taylor – Proudman column is raised.

5 Analysis

In the absence of vibrations, the sphere dynamics is governed by the dimensionless gravitational acceleration $\Gamma = 2g/(\Omega_{rot}^2 d)$, where g is the gravitational acceleration, d – the sphere diameter. With the increase of the cavity rotation speed the sphere rotation intensity decreases, the experimental points displace towards lower $|\Delta\Omega| / \Omega_{rot}$ when the Γ value is decreased (fig. 5). On the graph one can see that the cavity geometry influences only the differential rotation intensity. The relative speed of the sphere varies according to the law $|\Delta\Omega| / \Omega_{rot} \sim \Gamma^2$. This result slightly differs from the one in case of the cylindrical body, where $|\Delta\Omega| / \Omega_{rot} \sim \Gamma^{1.75}$ [5].

The azimuthal wave, propagating on the Taylor – Proudman column boundary (fig. 4), is due to the Stewartson shear layer instability [6, 7]. In difference from the cited works, in our research the sphere is free, and its differential rotation relative to the cavity is not set, but generated by the body circular oscillations under the action of the gravity field and thus entirely determined by the dimensionless frequency $\omega = \Omega_{rot} d^2 / \nu$, i.e. the cavity rotation speed.

The dependence of the dimensionless phase velocity of the azimuthal wave propagation on the Stewartson layer boundary on the sphere differential rotation speed is shown on fig. 6. Here $\Delta\Omega_w$ is calculated relative to the cavity: $\Delta\Omega_w = \Omega_w - \Omega_{rot}$. The decrease of the

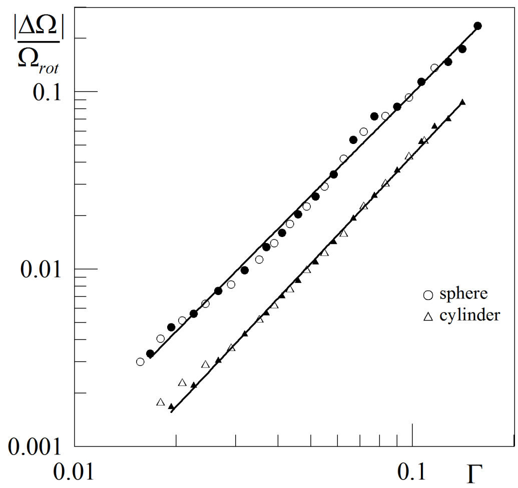

Figure 5: Dependence of the dimensionless rotation speed of the sphere on the dimensionless acceleration Γ, ν =5 cSt

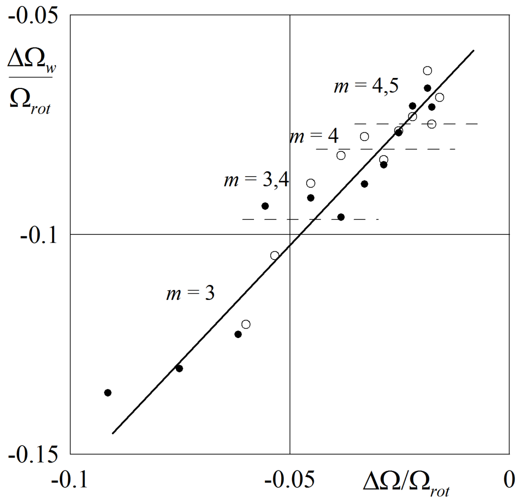

Figure 6: The dimensionless wave velocity vs. the relative rotation speed of the sphere, ν =6.7 cSt

sphere rotation intensity leads to the wave phase velocity increase and the consequent wave pattern change, at the same time the wave length decreases. Thus, at centrifugation the triangular column is formed ($m = 3$), with the cavity rotation speed increase the transition to the state with wave number $m = 4$ is observed, etc. The axisymmetric flow, when the column has the circular cylinder shape (fig. 4, a), is observed at $|\Delta\Omega|/\Omega_{rot} < 0.016$. The boundaries of the column transformation are shown on fig. 6 with the dashed lines.

Comparison of the experimental results with the theoretical ones [7] for the negative differential rotation of the sphere in the spherical liquid layer shows their satisfactory agreement.

The dynamics of the intensive vibrational rotation of the sphere is determined by the dimensionless vibration frequency $n \equiv \Omega_{vib}/\Omega_{rot}$ (fig. 7). The curves obtained for the vibrations of different frequency and equal amplitude are in good agreement on the chosen plane of dimensionless parameters. As in the cylindrical cavity (fig. 7, a), as in the spherical one (b), the values of n are equal for the resonant areas of both, outstripping and lagging motion. This is true for both the differential rotation speed and the sphere position on the axis. The most pronounced sphere displacement from the cavity centre along the rotation axis is shown by the slash-dotted lines. It is remarkable that this one coincides with the transition c, in which the intensive outstripping rotation is excited. The areas of the resonant excitation of the inertial body oscillations (and its intensive differential rotation) in different cavities are only slightly different in the dimensionless vibration frequency. Partially this difference may be due to the different values of the relative body radius r/R [8]. There are no qualitative changes in the sphere dynamics observed as the cavity geometry is changed. The existence of the areas of the intensive lagging and outstripping motion is explained by the coincidence of the vibration frequency with one of the two eigenfrequencies of the sphere inertial oscillations. As a result, the resonant growth of its oscillations amplitude occurs. This leads to the average force generation in the viscous boundary layer on the solid surface of the light body [3], which in its turn excites the body differential rotation.

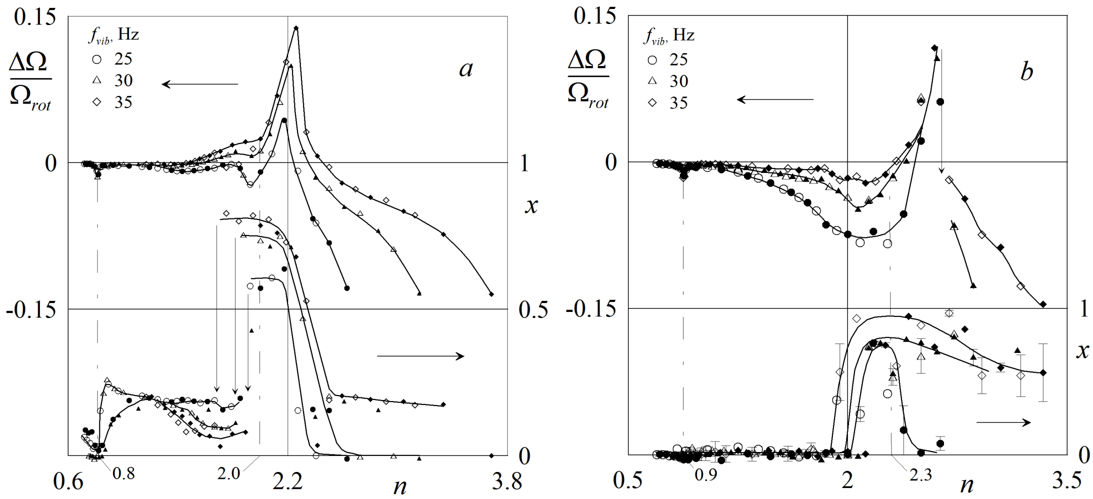

Figure 7: The dimensionless speed of the sphere differential rotation and its position on the axis vs. the dimensionless vibration frequency for the cylindrical cavity (a) and the spherical one (b); $f_{vib} = 30$ Hz, $b_{vib} = 0.27$ mm, $\nu = 5.0$ cSt

6 Conclusion

Was experimentally studied the vibrational dynamics of the light spherical body in the liquid-filled cavity (of cylindrical and spherical shape), rotating around the horizontal axis, at vibrations normal to the rotation axis. The excitation of the intensive differential rotation of the sphere (outstripping or lagging) in the resonant areas, which are determined by the dimensionless vibration frequency $n \equiv \Omega_{vib}/\Omega_{rot}$. The outstripping rotation is excited at $n > 1$, the lagging one – at $n < 1$.

The loss of stability of the sphere position in the cavity centre relative to the end-walls is found. The maximal sphere displacement from the centre is observed at the frequency of excitation of the outstripping rotation. On the contrary, at the intensive lagging vibrational rotation the position of the sphere in the centre becomes stable.

Is found the wave instability on the boundary of the liquid column, which is formed by the Stewartson shear layer.

Acknowledgements

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