Thermal stresses in a layered cylinder are the result of the process cooling and consolidation of the melt during the formation individual layers

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Abstract

The formation of thermoelastic stresses review in a layered cylinder with a view of phase transitions in separate layers. We formulate an evolutionary boundary value problem in the quasistatic approximation of the kinetics of crystallization and changes in temperature. Numerical solutions were obtained for three-layer cylinder in the case of crystallization of one or two layers.

Modern industrial development is directly linked to the widespread introduction of new materials, among them a special place is given layered composite materials (LCM). Engineering processes for some of layered composite materials, such as bimetals or materials on base glass and metal [1] include the temperature conditions under which in the materials are possible phase transitions of first order. As a result, during the production of level of technological stresses caused by from thermal stresses and crystallization stresses may exceed the ultimate strength of a composite material that gives rise to defects. Therefore, the development and improvement of methods for studying the kinetics of formation of stresses, taking into account the phase transitions are modern problems in the mechanics of deformable solids.

The main results of studies of domestic scientists of phase transitions in the framework of continuum mechanics in recent years are reflected in the works [2-5]. In this paper we consider the problem of determining the stress-deformation condition (SDC) at the last stage of formation LCM - cooling, during which the possible consolidation of the individual layers. Numerically solved the problems for of crystallization from the liquid phase of the inner layer or two outer layers for LCM cylindrical form which made of three different materials.

Mathematical model of this problem can be written using methods mechanics of growing coats (bodies) [6], where of crystallized material is represented as growing out from the liquid phase. Model is carried out under the following assumptions: the contribution of the dissipative stresses is insignificant and does not have effect on heat transfer and course processes of phase transformations; rheological processes are absent, it means maintaining the elasticity of the solid and liquid phase, a quasistatic equilibrium of the liquid phase.

These assumptions make it possible to divide the problem into two independent:

- the problem of determining the temperature fields and the movement of frontier the phase transition of first order, if any are possible in the material, the numerical method for solving this problem is proposed in [7];
- boundary value problem of determining the SDC for a layered material with thermal and structural homogeneities.

One of ways constructs a boundary value problem with moving boundary; it constructs equations of continuum mechanics and the boundary conditions in the velocities. Then mathematical model will represent the system of equations for the unknowns $\dot{\sigma_{ij}}$, $\dot{\varepsilon_{ij}}$, $\dot{u_i}$ containing for each layer: the equilibrium equation, the equation of state (like the Duhamel-Neumann relations), the conditions of Cauchy, boundary conditions on the outer surface of the LCM, conditions of pairing for different materials and equation at the phase interface. In fact, the model will differ from the classical model of the mechanics of growing bodies [6], only the last equation, so we stop for consider in detail this condition. We define the conditions a complete mechanical at the boundary of the growing body (growing body will be solid phase) and liquid phase. This condition of continuity vectors of the displacement and the stress.

$$[u_i]\Big|_{\Gamma^*_{kl}} = 0 \tag{1}$$

$$[\sigma_{ij} n_j]\Big|_{\Gamma^*_{kl}} = 0 \tag{2}$$

here n_j are the direction cosines of the outward normal to the Γ^*_{kl} phase interface in the k layer from the solution of the problem is known function of the temperature surface phase transition boundary $(\Phi(M,t) = 0)$ for each point in time.

As for the layered materials in the case of crystallization of the inner layers have to take into account the pressure arising from side the liquid phase, the stress of the liquid phase is determined from the equation of state

$$\sigma_{ij} n_j\Big|_{\Gamma^*_{kl}+0} = n_j \int_0^{t^*} 3K(\dot{\varepsilon} - \beta\dot{T})\delta_{ij}dt = n_i \int_0^{t^*} 3K(\dot{\varepsilon} - \beta\dot{T})dt,$$

where $\dot{\varepsilon} = \dot{\varepsilon_{ii}}$, $t^*(M)$ is time of accession element to a growing body and also it‘s the time the birth of element of the point $M(x_1, x_2, x_3)$. Unknown vector of external forces determined by the pressure of the fluid on boundary of growing the solid phase.

At the same time, to the deformation element, acquired in a liquid state, we add the structural deformation associated with the change of aggregation state of the element and the elastic deformation as resulting from the pressure of the liquid phase

$$\varepsilon^*_{ij} = \varepsilon_{ij}(t^* - 0) + \varepsilon^s_{ij} + \varepsilon^e_{ij}$$

where $\varepsilon^s_{ij}-$ structural deformations, $\varepsilon^e_{ij}-$ elastic deformations, $\varepsilon_{ij}(t^* - 0)-$initial of deformations of the solid phase will define as follows

$$\varepsilon_{ij}(t^* - 0) = \int_0^{t^*} \dot{\varepsilon_{ij}} dt.$$

Structural deformations appear at the time of accession in the arisen solid phase, it can be explained the change of aggregation state. If the arisen solid phase has got isotropic nature, then the correct formula

$$\varepsilon^s_{ij} = \frac{\delta_{ij}}{3}\Big(\frac{\rho^+}{\rho^-} - 1\Big),$$

where ρ^+, ρ^-– density of liquid and solid phases, respectively.

The stresses in the accession element will be consisting from stresses accumulated in the liquid phase and some stresses as result of deformation as part of a growing body:

$$\sigma^*_{ij} = \int_0^{t^*} \dot{\sigma_{ij}} dt + \Delta\sigma_{ij}.$$

It is obvious, the value of the additional stresses and strains are related by Hooke's law:

$$\Delta\sigma_{ij} = E_{ijkl}\varepsilon^e_{kl}.$$

In order to write the junction conditions of the liquid and solid phases in the rates of stresses and displacements, we differentiate with respect to time of (1) and (2), we obtain

$$\frac{d}{dt}\Big[u_i(x_1,x_2,x_3,t)\Big]\Big|_{\Gamma^*_{kl}} = \Big[\frac{\partial}{\partial t}u_i(x_1,x_2,x_3,t) + \frac{\partial}{\partial x_j}u_i(x_1,x_2,x_3,t)\frac{dx_j}{dt}\Big]\Big|_{\Gamma^*_{kl}}. \tag{3}$$

It is obvious that $\left(\frac{dx_1}{dt}, \frac{dx_2}{dt}, \frac{dx_3}{dt}\right)$ are the components of the velocity vector $\underline{\vartheta}(t)$, if we take into account that the boundary of the phase transition is moving on normal vector, then the speed of moving boundary we will define by the following expression

$$\underline{\vartheta}(t) = -\frac{\partial\Phi}{\partial t}\frac{\underline{grad}\Phi}{|\underline{grad}\Phi|^2},$$

and condition (3) takes the form

$$\Big[\frac{\partial u_i}{\partial t}\Big]\Big|_{\Gamma^*_{kl}} = -\vartheta_m\Big[\frac{\partial u_i}{\partial x_m}\Big]\Big|_{\Gamma^*_{kl}}. \tag{4}$$

Similarly, differentiating with respect to time, the condition (2) and taking into account

$$\frac{d}{dt}\Big([\sigma_{ij}]n_j\Big|_{\Gamma^*_{kl}}\Big) = \Big(\frac{\partial([\sigma_{ij}]n_j)}{\partial t} + \vartheta_m\frac{\partial([\sigma_{ij}]n_j)}{\partial x_m}\Big)\Big|_{\Gamma^*_{kl}},$$

we get

$$\Big(\frac{\partial[\sigma_{ij}]}{\partial t}n_j + \frac{\partial n_j}{\partial t}[\sigma_{ij}] + \vartheta_m\frac{\partial[\sigma_{ij}]}{\partial x_m}n_j + \vartheta_m\frac{\partial n_j}{\partial x_m}[\sigma_{ij}]\Big)\Big|_{\Gamma^*_{kl}} = 0. \tag{5}$$

For the components of the normal vector we have the following equality

$$n_j = \frac{\partial\Phi}{\partial x_j}\Big(\frac{\partial\Phi}{\partial x_m}\frac{\partial\Phi}{\partial x_m}\Big)^{-\frac{1}{2}}.$$

If we differentiate to time this, we obtain the connection between of components the velocity of normal and the equation of surface

$$\frac{\partial n_j}{\partial t} = \Big(\frac{\partial^2\Phi}{\partial t\partial x_j}\Big)\cdot\frac{1}{|\underline{\nabla}\Phi|} + \frac{|\vartheta|}{|\underline{\nabla}\Phi|}n_j.$$

Conditions (4) - (5) close the boundary value problem of elasticity theory for a layered material with consolidating the layers.

Using the proposed model was obtained by numerical solution of the problem of determining the thermal stresses in the process of formation:

(1) layered rod, glass-aluminum-steel, aggregation state changes in the middle layer for the elastic approximation (Fig. 1 a), b), c));

(2) three-layer cylindrical shell of aluminum -glass-aluminum, aggregation state changes in outer layers for the elastic approximation (Fig. 1 d), e), f)).

Fig . 1 (a) shows the position boundary of the phase transition in time. Fig. 1 (b) shows the plots of stress intensity from time in the middle layer at the border contacts, it shows that the interval of time, when the layer is crystallized, the stress intensity grows much faster, that may be explained the existence structural deformation during crystallization. Fig. 1 (c) shows the plots of stress intensity of upper graph and stresses lower graph from the radial coordinate at the finite of moment time. Similar graphs were made and show on Fig. 1 (d), (e), (f). But on some initial moment of time two boundaries of the phase transition are in the presence of, this is reflected in the graph Fig. 1 (d).

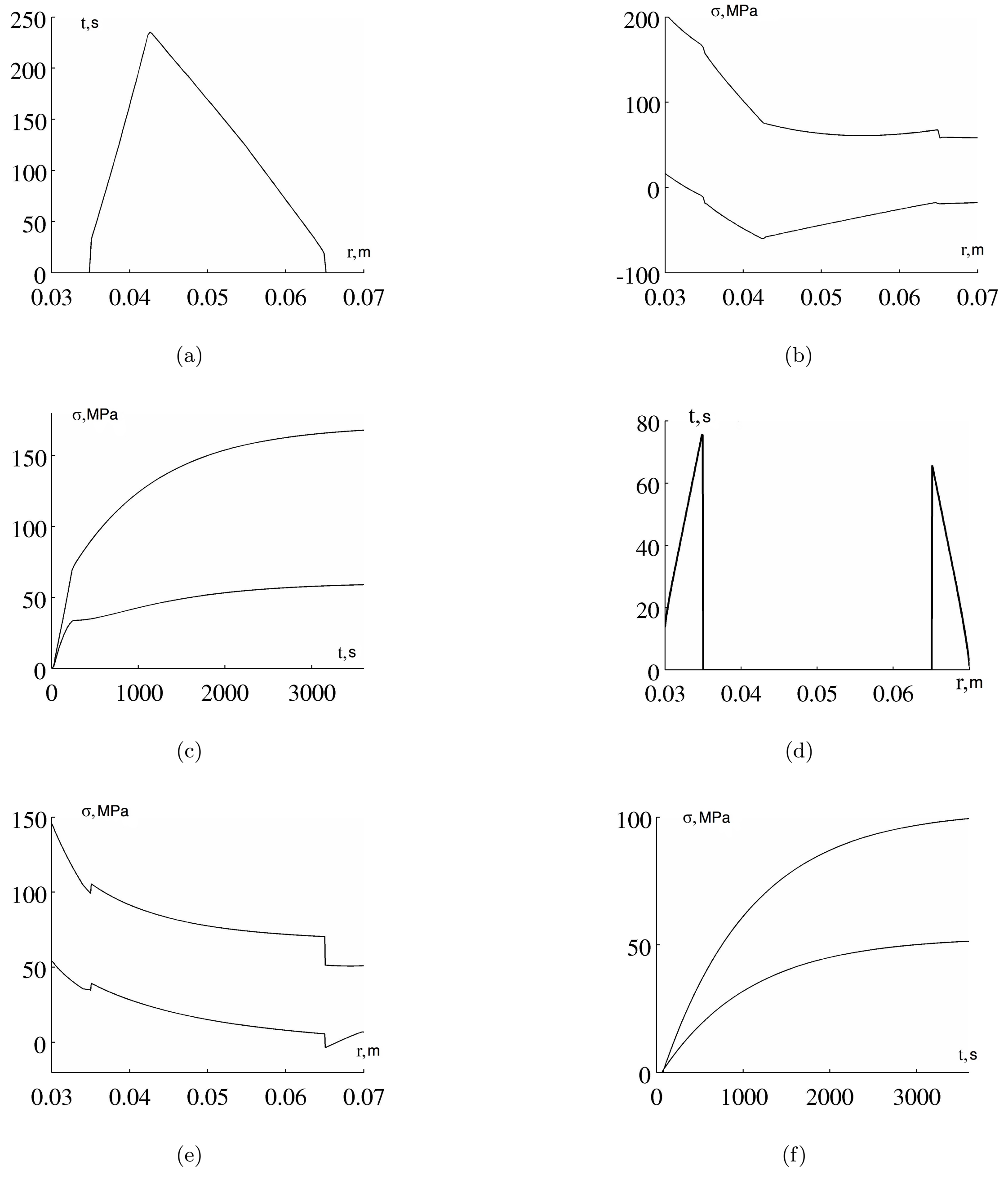

Figure 1: The results of numerical solution for composites glass-aluminum-steel and aluminum-glass-aluminum in the elastic approximation

The results are qualitatively consistent about mechanical representations of the deformation process, taking into account the crystallization.

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