Influence of the dispersion in theory of continuous mechanics

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Abstract

For continuous mechanics formulation of equilibrium of angular momentum conditions are suggested. In present time formulation of equilibrium force conditions are used. These give us symmetric pressure tensor and disturbance of continuous medium. In this work this question is discussed. Conditions of the existence of A.N. Kolmogorov inertia interval is established.

Keywords: Angular momentum, conservation laws, nonsymmetrical stress tensor, Boltzmann equations, Chapman-Enskog method, conjugate problem the Navie-Stokes, the molecular dynamics method.

1 Introduction

Many experimental facts tell us about the importance of gradients of physical values (density, linear momentum, energy). In the previous studies, the problem of influence of dispersion on the models and equations of continuum mechanics was considered carefully for various applications [1-4]. In those papers one can find also historical facts concerning different approaches to this problem, as well as some examples: in particular, modified Navier-Stokes equations, connection to kinetic theory, boundary layer, shock waves, numerical solutions, asymptotical methods, etc. So, there is no need to repeat all the details and discuss the importance of these equations.

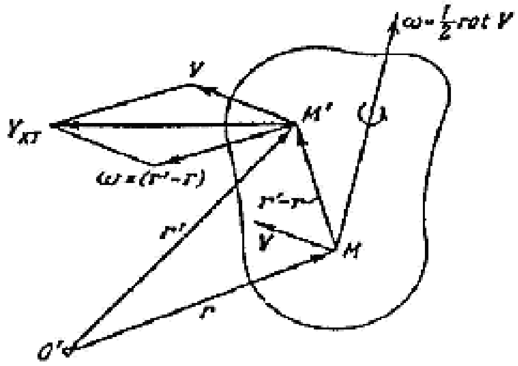

Figure 1: Velocity distribution near the point M

Presented in previous papers the equations of motion, energy and angular momentum were obtained before, but the use of force equilibrium conditions didn't require the calculation of angular momentum. When choosing the equilibrium conditions since the last equation to determine the degree of asymmetry of the stress tensor. The issue arose when writing the law of conservation of density. Try to get it out of the equation by using phenomenological principle.

2 Equation for the density

The modified equation for the density was received from the kinetic theory in the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(x_i \frac{\partial \rho u_i}{\partial x_i}\right) = 0.$$

where u_i - velocity, ρ - dencity, x_i - position.

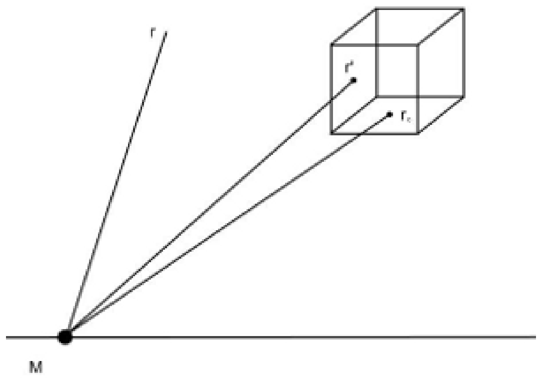

Figure 2: The influence of rotation of the elementary volume

The figure shows that the linear velocity v=$\omega \times \left(r' - r\right)$ is the velocity with respect to M for quasi-solid movement around axis without $\operatorname{div}(\rho u)$. However, the point M can be very involved in the rotation. Consequently, the cross product will appear if the axis of rotation is chosen $r' - r$. The point M may itself be involved in the rotation. For an elementary volume v=$\omega \times \left(r' - r\right)$ formula means a rotation around the axis of the velocity at centre of inertia but axis of moving of elementary volume can be lie outside it. So we have for twisting an elementary volume. Consider the conclusion of the last term of the equation of continuity.

$$\int_{(s)} \nabla \rho \mathbf{u} \left(\mathbf{r}' - \mathbf{r_c}\right) \mathrm{ds} = \int_{(s)} \operatorname{div}\left(\nabla \rho \mathbf{u}\right) \left(\mathbf{r}' - \mathbf{r_c}\right) \mathrm{dv}$$

$\mathbf{r}$ - axis of rotation,$\mathbf{r_c}$ - center of gravity. After integration, the mention expression was received.

3 Infinite plate

The Blasius problem was considered by numerical and analytical. Some results for infinite plate will be formulated here. The equation for this case is

$$\frac{d}{dy}\left(\mu \frac{du}{dy}\right) + \frac{d}{dy}\left(\mu y \frac{d^2 u}{dy^2}\right) = 0.$$

Boundary conditions are

$$u = 0, \qquad \mu \frac{du}{dy} = \tau_w, \qquad y = 0, \qquad u = U_\infty, \qquad y \to \infty.$$

Integrating gives

$$\mu \frac{du}{dy} + \mu y \frac{d^2 u}{dy^2} = Const = \tau_w.$$

Where y-coordinate, ρ-density, u-velocity, μ-viscosity. Index "w" is relative to surface. From boundary condition we have $Const = \tau_w$. τ_w is skin friction. Integral of the equation is

$$u = C \ln y + \frac{\tau_w}{\mu} y + Const.$$

Possible variant to satisfy boundary conditions is that under the $y = \frac{\nu}{v_*}$, where $\nu = \frac{\mu}{\rho}$, $v_* = \left(\frac{\tau_w}{\rho_w}\right)^{1/2}$ we have ln $= 0$. Later on diminution velocity takes place up zero, derivative can be very large but zero velocity observes between surface and y. So layer of the rest liquid is formed. Thickness of this layer is 10^{-3}cm. We have not reliable measurements there. Probably for laminar layer there is no layer with zero velocity. Near the edge the gradient of the velocity tends to work. It works near the rebuilding region too. Far from edge friction strives to zero. It does not follow from the theory for semi-infinite plate that the value of the friction is finite but if we suggest zero friction in the first integral we can get the Karman formula for the mixture length. Equality $\tau_w = 0$ provides $u = 0$ as $y = 0$ and $u = U_\infty$ as $y \to \infty$ and leads to rebuilding of the flow. The profile of the velocity becomes more completed than near the edge. The region with $\tau_w = 0$ formulated the inertial layer(N.A. Kolmogorov). This case relies to logarithm profile for boundary layer. It is interesting that asymptotic friction for half- infinite plate has not the value for infinite plate. In my opinion we have similar situation for tubes.

4 Conclusion

We consider influence the angular momentum variation in an elementary volume near the surface and influence the cross flows through the sides of an elementary volume for great gradients of the physical values. Some examples are investigated.

References

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