Application of hybrid algorithms to computational diagnostic problems for hydromechanical systems

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Abstract

Reliable mathematical modeling in hydromechanical system dynamics is very important for studies related to the safety of significant objects in the nuclear power industry. Consideration is being given to problems of identification of anomalies in the phase constitution of the coolant circulating throw the reactor primary circuit. Main dynamical characteristics of the object under diagnosing are considered as continuous functions of the bounded set of control variables. Possible occurrence of anomalies in the phase constitution of the coolant can be detected owing to changes in dynamical characteristics of the two-phase flow. As the normal state of the coolant and anomalous one are characterized by different spectra, then it is necessary to minimize simultaneously individual differences between spectral components. So, the diagnostic problem is formulated as the inverse spectral problem. Two novel hybrid algorithms for solving the corresponding global minimization problem are proposed. The first algorithm M-PCALMS combines the stochastic Multi-Particle Collision Algorithm (scanning of the search space) and deterministic gradient algorithm (local minimization) with smoothing approximations of the error function. The second algorithm M-PCASFC implements the local search procedure using the space-filling curve method. Results of successful computational experiments are presented to illustrate the efficiency of the approach.

1 Introduction

Modern methods for solving practical problems relating to trouble free, efficient and prolonged operation of complex systems are presumed the application of computational diagnostics [1]. The investigations are concerned with mechanical and hydromechanical systems which are constituents of nuclear power plants, aerospace structures, chemical process equipment. Computational diagnostic methods make it possible to study properties of the object under diagnosing on base of circumstantial measured data [2–4]. Input data for diagnosing usually contain the results of experimental measurements of the system certain investigatory characteristics; among them may be registered parameters of oscillatory motion or impact process. It needs to evaluate causal characteristics like the dynamic equation coefficients, boundary conditions, geometrical and other characteristics. The diagnostic procedure is founded on the solution of the corresponding inverse spectral problem; the problem in many cases may be reduced to a minimization of an appropriate error criterion. Eigenvalues from the direct problem for the mathematical model and useful measured data for the system are used in order to construct the criterion. When solving these inverse problems, consideration must be given to following special features: the error criterion may be represented by nondifferentiable and multiextremal function. In the general case it is necessary to solve the problem using inaccurate incomplete experimental data.

Possible availability of repeated or very close eigenvalues one should take into account. It is well known that the above problem is often ill posed, so that small perturbations in the data can result in large changes of the solution. Regularization technique is a standard method of transforming the initial problem into well-posed form [5, 6]. Implementation of special procedures is required for finding regularization parameters. Incompleteness of the spectral data results in the error crterion being non-convex. As the error function has numerous local minima, it is necessary to use global optimization methods.

Vibration monitoring methods are cost-effective and reliable tools for early failure detection and preventive maintenance in the up-to-date nuclear power industry. One of the most severe accidents in nuclear power generation is loss of coolant, where the re-circulating coolant of the pressurized water reactor may flash into steam [7, 8]. The standard reactor instrumentation can register signals caused by pressure fluctuation of the coolant. The problem at hand is an interpretation of the registered spectra and useful data extraction for diagnostics. Methods of solving the problem for computational models of the VVER-1000 primary circuit are under consideration. Possible occurrence of anomalies in the phase constitution of the coolant can be detected owing to changes in dynamical characteristics of the two-phase flow. Mathematical models for numerical analysis of acoustical oscillations in two-phase coolant have been developed [9]. As the normal state of the coolant and anomalous one are characterized by different spectra, then it is necessary to minimize simultaneously individual differences between spectral components. These differences can be described by an appropriate error function. The goal is to find a set of controlling variables, which will minimize the error function and determine current phase constitution of the coolant. So, the diagnostics problem is formulated as the inverse spectral problem.

Implementation of the approach with the use of hybrid global optimization algorithms is discussed. As usual hybrid algorithms combine stochastic and deterministic algorithms in order to achieve better computational properties. Two novel stochastic algorithms have been proposed recently: the Particle Collision Algorithm PCA [10] and the Multi-Particle Collision Algorithm M-PCA [11]. The NMPCA is a good example of the hybridization method [12]. The hybrid NMPCA performs as follows: first, a wide search in the solution space is carried out using a stochastic optimization algorithm (the PCA), and then scanning the promising areas is made with a deterministic local search technique (Nelder-Mead Simplex). This searching is performed iteratively until a certain number of fitness function evaluations is reached. However, it is well-known that the convergence theory for Nelder-Mead simplex method is far from completion; so the method can fail to converge or converge to non-stationary points [13]. As an alternative to the NMPCA two novel hybrid algorithms are introduced. These algorithms combine the M-PCA and the deterministic gradient techniques with smoothing approximations for fitness functions or the space-filling method throw the local search.

The plan of the remainder of this paper is as follows. The section following contains statement of the diagnostics problem. Section 3 provides brief descriptions of the hybrid global optimization algorithms. In section 4 successful computational experiments for identifying anomalies of the VVER-1000 equipment components and the phase constitution of the coolant in the primary circuit are presented to illustrate peculiarities of the approaches. Section 5 gives conclusions and discussion on further work.

2 Formulation of the problem

It is supposed that a set of performance index values associated with a computational model to be updated is defined by a set of controlling variables. Experimental spectral data

registered by permanent instrumentation may be incomplete. So the goal is to determine vectors of controlling variables using only measured data on natural frequencies of the object. The standard approach is to set the inverse spectral problem and then to solve the corresponding least squares problem

$$\min_{x \in X \subset R^n} f(x), \tag{1}$$

where $f(x) = \sum_{i=1}^{N} w_i(\zeta_i(x) - \zeta_i^*)^2$; x, X — the vector of controlling variables and its feasible domain of the error function $f(x)$ respectively; the w_i stand for weighting factors that reflect the confidence level in the measurements; N is the number of eigenvalues under consideration; $\zeta_i(x)$ and ζ_i^* denote the eigenvalues that correspond to computed (solutions of the direct problem) and to measured natural frequencies respectively;

$$X = \left\{ x_i \,\middle|\, x_i^L \le x_i \le x_i^U;\ i = \overline{1,n} \right\}; \tag{2}$$

here x_i^L, x_i^U — the lower and upper bounds on the ith controlling variable.

As practical observations show, the error function in the considered problem is often multiextremal. Therefore, it is necessary to turn to methods of global optimization. It is clear that if the measured spectral data exactly match to the computational model then the solution of the minimization problem will cause error function to take its global minimum value of zero. Let us suppose that there is a unique solution of the ill-posed inverse spectral problem and that this corresponds to the global minimum of the error function. However, the fact is that the theoretical question of the uniqueness of solutions of the problem may not be relevant to practical applications in which there is the additional complication of accuracy of experimental measurements. Furthermore, some complications may arise due to incompleteness of measured spectral data, influence of the two-phase interference on the flow dynamics, the presence of noise, etc. Within the scope of this work we take it as a convenient and reasonable assumption that global minimization of the error function in the above inverse problem will yield correct model updating for objects under consideration.

3 Hybrid global optimization algorithms

3.1 The Particle Collision Algorithm (PCA)

The modern Particle Collision Algorithm [10] has some essential advantages in relation to well known stochastic global optimization algorithms such as the Genetic Algorithm, Simulated Annealing, Fast Simulated Annealing, etc. Specifically, the PCA does not require any additional parameters other than the number of iterations; the algorithm is extremely easy to implement and can be applied to both continuous and discrete optimization problems. The PCA performs using the analogy with nuclear particle collision reactions, in particular scattering and absorption. So, a particle that hits a high-fitness "nucleus" would be "absorbed" and would explore the boundaries. Otherwise, a particle that hits a low-fitness region would be scattered to another region. This reasoning makes it possible to simulate the exploration of the search space and the exploitation of the most promising areas of the fitness landscape throw successive scattering and absorption collision events.

The original PCA works as follows. First an initial configuration is chosen, then a modification of the old configuration into a new one is implemented. The qualities of the two configurations are compared. A decision then is made on whether the new configuration is acceptable. If it is, the current configuration acts as the old configuration for the

next step. If it is not acceptable, the algorithm proceeds with a new change of the old configuration. It is pertinent to note that acceptance of current trial solution with certain probability may avoid the convergence to local optima.

However, the PCA is in its early stages. In spite of its advantages over Genetic Algorithm and Simulated Annealing in solving test problems, practical application of the PCA is restricted because of solutions remain too expensive. As possible development, the local search procedure in the algorithm could be improved. It seems promising to use gradient methods for local minimization of the error function. But in so doing the problem of non-differentiability of the function should be taken into account.

3.2 The Multi-Particle Collision Algorithm (M-PCA)

The modern Multi-Particle Collision Algorithm is based on the canonical PCA, but a new characteristic is introduced: the use of several particles, instead of only one particle to act over the search space [11]. So, the new outer loop for the particle control has been added to the basic global optimization algorithm. Thanks to use of several particles the M-PCA can better explore the search space, avoiding convergence to a local minimum. Coordination between the particles was achieved throw a blackboard strategy, where the Best_Fitness information is shared among all the particles in the process.

Similar to PCA, M-PCA also has only one parameter to be determined, the number of iterations. But in this case, the total number of iterations is divided by the number of particles which will be used in the process. The division of the task is the great distinction of the M-PCA, which leads to a great reduction of required computing time.

The pseudo code brief description of the M-PCA algorithm is as follows.

```
0 Generate an initial solution Old_Config
Best_Fitness = Fitness (Old_Config)
Update Blackboard
  For n = 0 to # of particles
    For n = 0 to # of iterations
      Update Blackboard
      Perturbation( )
      If Fitness (New_Config) > Fitness (Old_Config)
        If Fitness (New_Config) > Best_Fitness
          Best_Fitness := Fitness (New_Config)
        End If
        Old_Config := New_Config
        Exploration( )
      Else
        Scattering( )
      End If
  End For
End For
2. Exploration( )
  For n = 0 to # of iterations
    Small_Perturbation ( )
    If Fitness (New_Config) > Fitness (Old_Config)
      If Fitness (New_Config) > Best_Fitness
        Best_Fitness := Fitness (New_Config)
      End If
```

```
      Old_ Config := New_ Config
    End If
  End For
Return
3. Scattering( )
  p_scatt = 1−( Fitness (New_Config)) / (Best_ Fitness)
  If p_scatt > random(0, 1)
    Old_ Config := random solution
  Else
    Exploration ( )
  End If
Return
Perturbation( )
  For i = 0 to (Dimension-1)
    Upper = Superior_Limit[i]
    Lower = Inferior_Limit[i]
    Rand = Random(0, 1)
    New_ Config[i] = Old_ Config[i] – ((Upper – Old_ Config[i])*
    Rand) – ((Old_ Config[i] – Lower)*(1 – Rand))
    If (New_ Config[i] > Upper)
      New_ Config[i] = Superior_ Limit[i]
    Else          If (New_ Config[i] < Lower)
        New_ Config[i] = Inferior_ Limit[i]
      End If
    End If
  End For
Return
Small_Perturbation( )
  For i = 0 to (Dimension-1)
    Upper = Random(1.0, 1.2) – Old_ Config[i]
    If (Upper > Superior_ Limit[i])
      Upper = Superior_ Limit[i]
    End If
    Lower = Random(0.8, 1.0) – Old_ Config[i]
    If (Lower > Inferior_ Limit[i])
      Lower = Inferior_ Limit[i]
    End If
    Rand = Random(0, 1)
    New_ Config[i] = Old_ Config[i] – ((Upper – Old_ Config[i])*
    Rand) – ((Old_ Config[i] – Lower)*(1 – Rand))    End For
Return
```

3.3 The smoothing technique for local optimization

Inverse problems are considered to be substantially difficult because of the kinks connected with presence of the multiple frequencies in registered spectra of acoustical oscillations in two-phase coolant. The difficulty motivated the development of algorithms for the solution of the minimization problem via some smooth approximation, which could be minimized by using any of the efficient classical approaches for smooth optimization. Several

approximations to smooth out the kinks may be introduced. One of them results in a continuously differentiable approximate function, whereas another one leads to a twice continuously differentiable approximate function. Two-parametric smoothing approximations were successfully used to solve problems of multi-criterion optimization of mechanical and hydro-mechanical systems with continuous but not everywhere differentiable functions [14].

3.4 The hybrid algorithm M-PCALMS

As an alternative to the NMPCA a novel hybrid algorithm M-PCALMS is introduced. In this new version of the global optimization algorithm the local search mechanism is a standard deterministic linearization method. Inverse problems are considered to be substantially difficult because of the kinks connected with presence of the repeated or very close frequencies in registered spectra for the computational model under updating. The difficulty motivated the development of algorithms for the solution of the minimization problem via some smooth approximation, which could be minimized by using any of the efficient classical approaches for smooth optimization. Several approximations to smooth out the kinks may be introduced. One of them results in a continuously differentiable approximate fitness function, whereas another one leads to a twice continuously differentiable approximate function. These approximations replace the original function in some neighborhoods of directional differentiability points. Moreover, this approach preserves such important property of the original function as its convexity. It is clear that the approach makes it possible to implement efficient gradient techniques in the solution process. In general case the error function is not differentiable everywhere, so the implementation of the smoothing technique may be quite pertinent. Computational experiments show the principal applicability of the proposed hybrid algorithm M- PCALMS for solving the inverse spectral problems.

3.5 The hybrid algorithm PCASFC

Some powerful algorithms for multi-extremal non-convex optimization problem are based on reducing the initial multi-dimensional problem to the equivalent problem of one dimension. This reduction can be executed by applying Peano-type space-filling curves mapping a unit interval on the real axis onto a multi-dimensional hypercube [15, 16].. The Peano curve development maps the segment [0, 1] of the real axis R^1 into the hypercube $X \subset R^1$ determined in (2). Actually, this is the case of continuous single-valued mapping that offers finding point $x(z) = \big(x_1(z), \ldots, , x_n(z)\big)^T \in X$ for each point $z \in [0, 1]$:

$$\min_{x \in X} f(x_1, \ldots, x_n) = \min_{0 \le z \le 1} \phi(z).$$

So, the initial multi-dimensional minimization problem (1) is equivalent to the above one-dimensional problem of finding the global minima of the discontinuous multi-extremal function $\phi(z)$. The Hilbert technique is used here for building the development of the Peano space-filling curve depending on parameter m that stands for the number of subdivision levels.

The approach needs not any derivatives of the function to be minimized with updating parameters. Some disadvantage of this approach is in the fact that one-dimensional problem obtained by the above reduction leaks some information on the closeness of iteration points in the initial multi-dimensional space.

Algorithm SFC: reduction of the problem dimension using the space-filling curve method.

0. Set formally the point x^* of local minimum. Let two small numbers $\gamma_1 > 0, \quad \gamma_2 > 0$, and the vector of constraints $\beta \in R^n$ be given. Let $k = 0$, $m = 1$.

1. The local minimization phase: find new point x^* (after l sub-iterations); set $k = k + l$. If $f\left(x^k\right) = 0$ then go to step 2. If $f\left(x^k\right) < \gamma_1$ and $x_i^k > \beta_i$ for some $i \in I^x, \quad I^x = \{1, 2, ..., n\}$, then build the effective sub-vector x_{eff}^k, reduce the multi-dimensional problem to the one-dimensional form and go to step 2. Else, go to step of stochastic scanning.

2. Set $m = m + 1$. If $m \leq m_{\max}$ then go to step 3. Else, go to step 4.

3. Define the current value z_* using the development of the Peano curve and calculate the current approximation of $\phi(z_*)$. If $\phi(z_*) \geq \gamma_2$ then go to step 2.

4. Reconstruct x^* and $f(x_*)$ using z_* and $\phi(z_*)$.

5. Define the point of local minimum $x^{LM} = x^*$, stop.

The pseudo code brief description of the hybrid M-PCASFC algorithm that combines the PCA and the deterministic space-filling curve method is as follows.

```
0 Generate an initial solution Old_ Config
Best_ Fitness = Fitness (Old_ Config)
Update Blackboard
For n = 0 to # of particles
  For n = 0 to # of iterations
    Update Blackboard
    Perturbation( )
    If Fitness (New_ Config) > Fitness (Old_ Config)
      If Fitness (New_ Config) > Best_ Fitness
        Best_ Fitness := Fitness (New_ Config)
      End If
      Old_ Config := New_ Config
      Exploration( )
    Else
      Scattering( )
    End If
  End For
End For

2. Local search( )
  Apply procedure of local search
  using the Space-Filling Curve Method
Return

3. Scattering( )
  p_scatt = 1−( Fitness (New_Config)) / (Best Fitness)
  If p_scatt > random(0, 1)
    Old_ Config := random solution
  Else
    Exploration ( )
  End If
Return
```

4 Computational results

In this section two numerical examples of hybrid algorithms M-PCALMS and M-PCASFC applications to inverse spectral problems for VVER-1000 nuclear reactor equipment are presented. First example is devoted to the steam pipe finite element model updating. In the second example the updating of the computational model of the coolant two-phase flow dynamics in the primary circuit is carried out.

Example 1. The computations were performed in connection with the problem of identification of the coolant phase constitution in the VVER-1000 primary circuit. Appearance of the second phase is possible: in a coolant heating zone (pressure tank of the pressurizer), in an exit volume of the reactor pressure vessel (RPV), in a core barrel of the RPV, in exit volumes of main circulating pumps. In order to formulate the inverse problems two vectors of relative acoustic velocities in a coolant flowing throw the specified zones are introduced. Let now the anomalous coolant state constitution be characterized by second vector of controlling variables: $x_1^* = 79.0\%$; $x_i^* = 100\%$, $i = \overline{2,\,4}$. The error function is determined using ten lower spectral components. Table 1 displays the known spectral data for the considered model upating problem. Here we have: i — mode number; ω_i — natural ith frequency of the coolant oscillation under normal conditions (without appearance of the second phase in the coolant); ω_i^* — natural ith frequency of the coolant oscillation with the availability of anomalies in coolant phase constitution.

Table 1: Given spectral data for Example 1

i	1	2	3	4	5	6	7	8	9	10
ω_i, Hz	0.89	6.77	9.82	15.44	15.96	18.94	24.57	26.69	27.07	30.52
ω_i^*, Hz	0.84	6.77	9.82	15.44	15.96	18.87	21.44	26.67	27.06	30.52

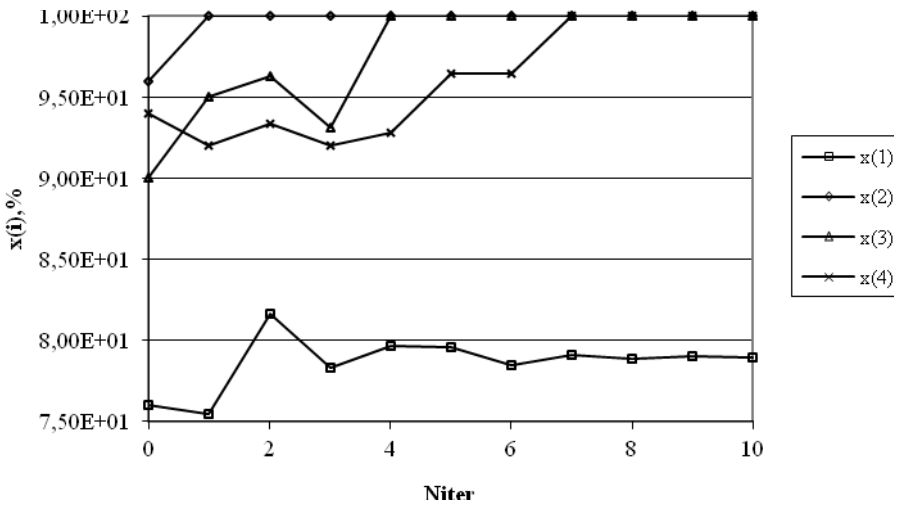

Figure 1: Number of final iterations of the M-PCALMS vs. relative velocities (Example 1)

The approximate solution reached by using the PCAHS algorithm is: $x_1^* \approx 78.9\%$; $x_i^* = 100\%$, $i = \overline{2,\,4}$. Fig. 1 and Fig. 2 illustrate the solution history (final iterations of the hybrid algorithm). The inaccuracy of the relative acoustical velocity computing is about 1%. As follows from the results obtained in this example the coolant phase constitution anomalies are conditioned by boiling process in the coolant heating zone.

Example 2. Let now the anomalous coolant state constitution be characterized by

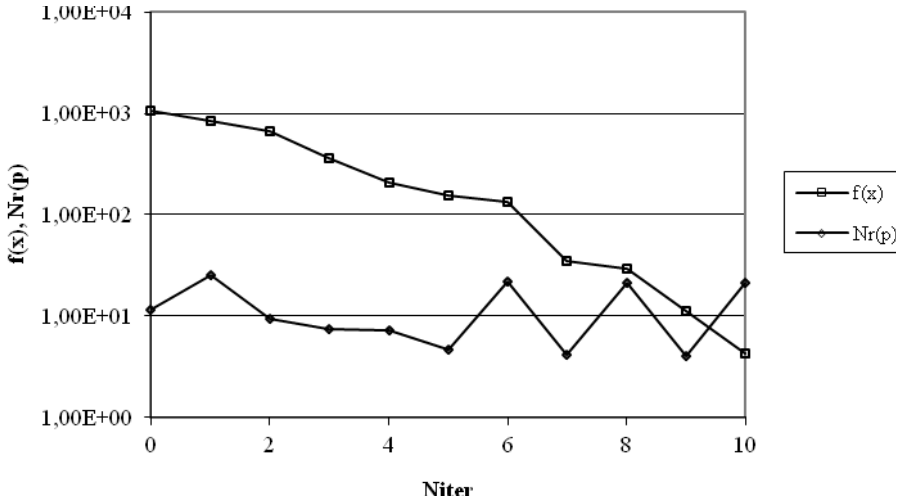

Figure 2: Number of final iterations of the M-PCALMS vs. error function and search gradient norm (Example 1)

second vector of controlling variables: $x_1^* = 77.5\%$; $x_2^* = 88.0\%$; $x_3^* = 82.5\%$; $x_4^* = 100\%$. The error function is determined using ten lower spectral components. Table 2 displays the known spectral data for the considered model diagnostic problem.

Table 2: Given spectral data for Example 2

i	1	2	3	4	5	6	7	8	9	10
ω_i, Hz	0.89	6.77	9.82	15.44	15.96	18.94	24.57	26.69	27.07	30.52
ω_i^*, Hz	0.82	6.77	9.36	15.33	15.96	18.86	21.22	26.67	26.93	29.41

The approximate solution reached by using the PCALMS algorithm is: $x_1^* \approx 77.53\%$; $x_2^* \approx 87.58\%$; $x_3^* \approx 83.23\%$; $x_4^* = 100\%$. Fig. 3 and Fig. 4 illustrate the solution history (final iterations of the hybrid algorithm). The inaccuracy of the relative acoustical velocity computing is about 1%. As follows from the results obtained in this example the coolant phase constitution anomalies are conditioned by boiling process in the coolant heating zone, in the exit volume of the reactor pressure vessel and in the core barrel of the RPV.

5 Conclusions

Two novel global optimization algorithms combining a Metropolis-based stochastic algorithm M-PCA and deterministic gradient technique or space-filling curve method for local search are presented. Smoothing approximations are introduced during the local search that makes it possible to expand the M-PCALMS algorithm on the class of non-differentiable problems. The M-PCASFC algorithm being introduced here does not require any gradient information. Both the algorithms were used for solving inverse spectral problems in connection with computational model updating for the two-phase coolant flow in the nuclear reactor primary circuit and for the steam pipe supporting units. Numerical experiments show the principal applicability of the proposed hybrid algorithms for solving the above model updating problems. The future work will be devoted to increasing the computational efficiency of tools for solution the model updating problems with regard to noisy data.

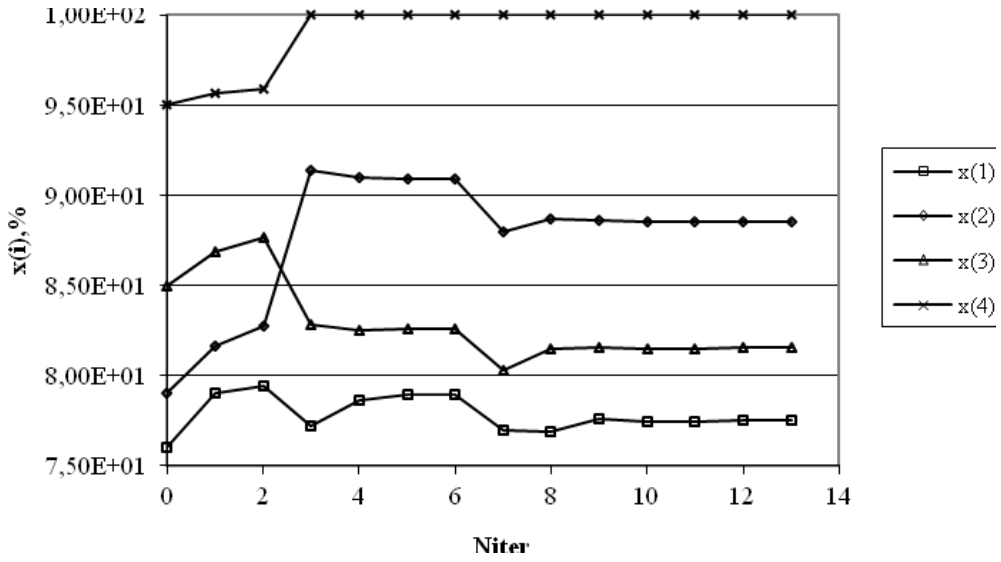

Figure 3: Number of final iterations of the M-PCALMS vs. error function and search gradient norm (Example 2)

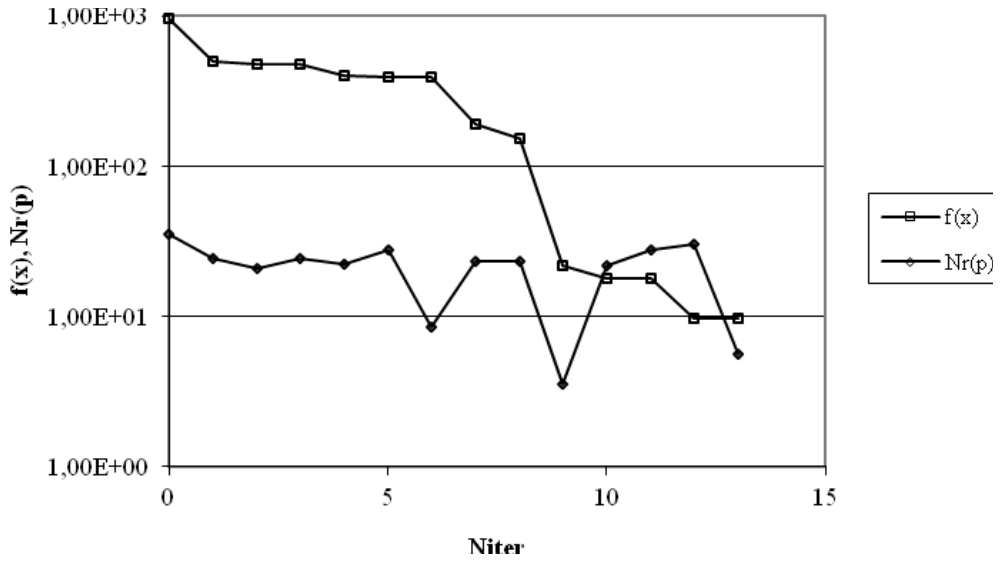

Figure 4: Number of final iterations of the M-PCALMS vs. error function and search gradient norm (Example 2)

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