# Distribution of the equilibrium positions of a shaft and defining the angular speed of the ring in a floating ring bearing

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**Abstract**

Today, rotors of high-speed turbomachines are commonly supported by hydrodynamic journal bearings. Like other types of fluid bearings, rotors supported by floating ring bearings may become unstable with increasing speed of rotation due to self-excited vibrations. In order to study the stability of rotor, we have to define the angular speed of the ring in the floating ring bearing since the speed of the ring appears in the formulas of the nonlinear bearing forces, which are modeled by applying the short bearing theory for both fluid firms with the considering of the allowance of the lubrication hydrodynamics and the centrifugal force. Additionally, the analytical results are obtained from the condition of the equality of the torques acting on the ring under the allowance of the both fluid firms.

## 1 Introduction

The theory of short bearing researches the rotation of the shaft in the bearing, which is fixed in space. The gap between the rigid bodies is covered by incompressible fluid (commonly known as Newton fluid). In the scope of this article, a more difficult model of the floating bearing that is proposed in the research includes 3 parts (as in the figure 1): the first part is the floating shaft rotating with angular velocity $\omega_1$, the second part is the floating ring rotating with angular velocity $\omega_2$ and the third part is the fixed cylinder (bearing housing). The gap between the rigid bodies is covered by incompressible fluid, where 4–1 and 4–2 are denoted the inner and outer fluid firm.

Dynamic of rotor in the floating bearing have been researched in the classical short -– bearing theory, but all of them use the classical equation to describe the flow in the gap, i.e. Reynolds equations, without the allowance of centrifugal force. As the result, the rotation of the shaft is studied only under the allowance of the hydrodynamic forces [2] or the allowance of the hydrodynamic forces and the friction of fluid [5].

The main point of this article is to define the angular speed of the ring in the floating ring bearing with allowance of lubrication hydrodynamics and centrifugal force, which its influence must be obtained in case of high speed shaft. The dependence of pressure on the position of shaft in the gap is given by the equation describing the flow of the fluid (Reynolds equations). By the short- bearing theory, the expression of the forces and the torques is given by the following tract: the force acting on the shaft is defined by integrating the function of pressure on the surface of shaft and the torques acting on the shaft and the ring are defined by integrating the function of the shearing stress $\tau_{r\varphi}$ in the cylindrical system of coordinates. In this case (as in fig. 1), when defining the torque acting on the

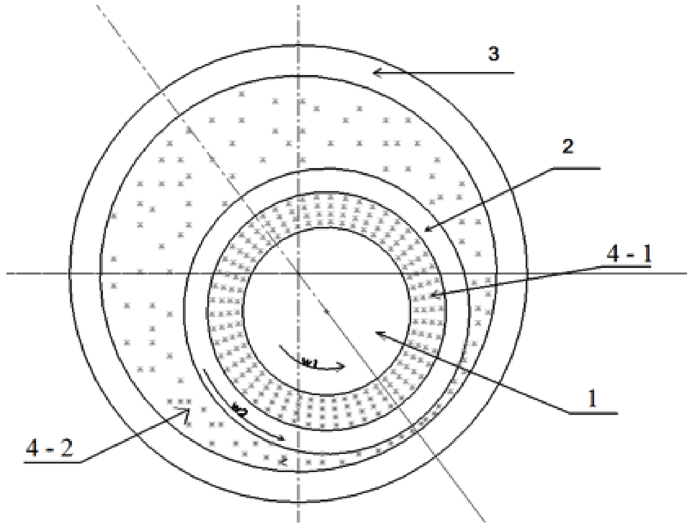


Figure 1: Mechanical model of rotor supported by floating ring bearing: 1 – floating shaft; 2 – floating ring; 3 – fixed bearing housing; 4 – incompressible fluid (4–1 – inner fluid firm, 4–2 outer fluid firm); $\omega_1$, $\omega_2$ are angular velocities of elements 1 and 2.

ring, we have to integrate the function of the shearing stress $\tau_{r\varphi}$ on the both sides of the ring in the floating ring bearing. The angular velocity of the ring $\omega_2$ which is considered as a constant is defined by the condition of the equalization of the torques from both sides of the ring.

## 2 Definition of the border of a lubricant layer

Without the influence of an external field of fluid 4-2, we shall consider only system "the ring – shaft". The system coordinates is fixed in the centre of the ring. Then in this system, incompressible oil 4-1 is in a gap between the ring $(O_2, R_2)$ that its axis of rotation is fixed and a rotating floating shaft$(O_1, R_1)$, see fig. 2. The ring rotates with angular speed $\omega_2$, and the shaft rotates with angular speed $\omega_1$.

It is noted: $h_{01} = R_2 - R_1$ is a nominal gap, $e_1 = e_{P1}(t)$ is eccentricity of the centre of a floating shaft and $\gamma_1 = \gamma_1(t)$ is an angle describing the position of a line of centre of the floating shaft and the centre of the rotating ring. The motion of a shaft on a lubricant layer is non-stationary, i.e. position and speed of its centre depend on time so the external loading and the reaction of the lubricant layer also depend on time.

In work [7] the expression for width of the gap (thickness of a lubricant layer) is received:

$$h_1(\theta_1, t) = h_{01} - e_1(t).\cos\theta_1. \tag{1}$$

And force on unit of the length, acting on the shaft:

$$q_{01} = \frac{1}{L}\int_{-L/2}^{L/2} \left(p_{01} - p_{01}^{*}\right) dz = \frac{\mu L^2(\omega_1 + \omega_2)}{2h_{01}^2}\,\overline{q}_{01}, \tag{2}$$

where

$$\overline{q}_{01} = \frac{\left(\frac{2\dot{\gamma}_1}{\omega_1+\omega_2} - 1\right)\varepsilon_1 \sin\theta_1 + \frac{2\dot{\varepsilon}_1}{\omega_1+\omega_2}\cos\theta_1}{\left(1 - \varepsilon_1\cos\theta_1\right)^3}\,,\ \varepsilon_1 = \frac{e_1}{h_{01}} \tag{3}$$

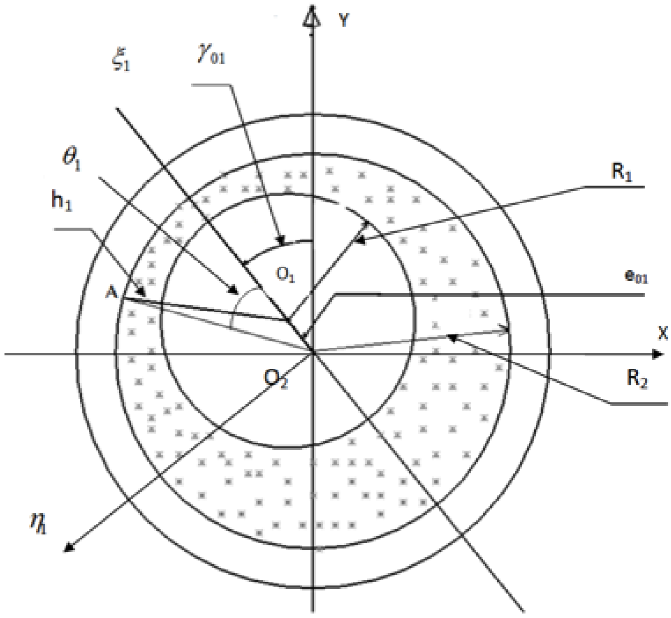


Figure 2: The denoting schema for system "the ring - shaft".

and $p_{01} = p_{01}(r, \varphi, z, t)$ is the function of the pressure in the internal firm 4-1, $p_{01}^* = p_{01}^*(r, \varphi, t)$ is the function of pressure on the ends of the bearing, $\mu$, $L$ are the accordingly dynamic viscosity of the fluid and the length of the bearing.

In a local system of coordinates$(O_2\xi_1, O_2\eta_1)$, where the direction$O_2\xi_1$ corresponds$\theta_1 = 0$ $(\varphi = \gamma_{P1})$, projections of the force$F_1^P$acting on a shaft from a lubricant layer are written as the following form:

$$F_{1\xi}^P = L \int\limits_{\overline{\theta_1}}^{\overline{\theta_2}} (R_2 - h_1) q_{01} \cos\theta_1 d\theta_1, \qquad F_{1\eta}^P = L \int\limits_{\overline{\theta_1}}^{\overline{\theta_2}} (R_2 - h_1) q_{01} \sin\theta_1 d\theta_1. \tag{4}$$

The question on the borders of a lubricant layer now is unsolved, despite of a significant amount of works on this question. In the theory of the dynamic loaded bearings usually use one of the two following hypotheses:

1. The value of angle that is used to define the beginning and the end of the lubricant layer is determined at the positions where the pressure is equal to zero, i.e. in the positions where the thickness of a gap is narrowest or widest; often being denoted $\overline{\theta}_1 = 0$, $\overline{\theta_2} = \pi$, i.e. only half of the gap [5], [6] are accepted.

2. The value of angle $\overline{\theta_1} = 0$, $\overline{\theta_2} = 2\pi$, i.e. a lubricant layer surrounds all the shaft. According to this hypothesis, there is a negative pressure that equal to the positive pressure on the absolute value.

In the present work the second hypothesis is used, i.e. the lubricant layer is full of the gap.

# 3 Distribution of equilibrium positions of the shaft and the ring in the lubricant layer in the floating ring bearing

As a result of integrating in the formulas (4), we receive the projections of the force$F_1^P$ acting on a shaft from a lubricant layer:

$$F_{1\xi}^P = \frac{2\mu L^3 \dot{\varepsilon}_1}{h_{01}^2}\left[\int_0^\pi \frac{R_1.\cos^2\theta_1}{(1-\varepsilon_1\cos\theta_1)^3}d\theta_1 + \int_0^\pi \frac{e_1.\cos^3\theta_1}{(1-\varepsilon_1\cos\theta_1)^3}d\theta_1\right],$$
$$F_{1\eta}^P = \frac{\mu L^3(\omega_1+\omega_2)\varepsilon_1}{h_{01}^2}\left(\frac{2\dot{\gamma}_1}{\omega_1+\omega_2}-1\right)\left[\int_0^\pi \frac{R_1.\sin^2\theta_1}{(1-\varepsilon_1\cos\theta_1)^3}d\theta_1 + \int_0^\pi \frac{e_1.\cos\theta_1.\sin^2\theta_1}{(1-\varepsilon_1\cos\theta_1)^3}d\theta_1\right]. \quad (5)$$

Let the shaft $(O_1, R_1)$ loaded with a constant external force$\overrightarrow{Q^P}$. In case $(\varepsilon_1^*, \gamma_1^*)$denotes the coordinates of equilibrium position of the shaft in the bearing then$\dot{\varepsilon}_1^* = 0$, $\dot{\gamma}_1^* = 0$. It is assumed that the external force $\overrightarrow{Q^P}$ is directed vertically downwards, the condition of the equality forces shows that $\left|\overrightarrow{Q^P}\right|$ is proportional to angular speed of the shaft$\omega_1$. External loading$\overrightarrow{Q^P}$, for example, can be the gravity of the shaft. The set of equilibrium positions of the centre of the shaft in the lubricant layer is a horizontal line segment $O_2M$ (on fig. 3 it is marked bold), in which $\varepsilon_1^* \in [0, 1]$ depends on the external loading $\overrightarrow{Q^P}$ on the shaft. If we use the first hypothesis, the curve of equilibrium positions of the centre of the shaft $O_1$ is a half of a circle, on fig. 3 this curve is marked as a dotted line. Here we notice that$O_2M = O_2M' = h_{01} = R_2 - R_1$.

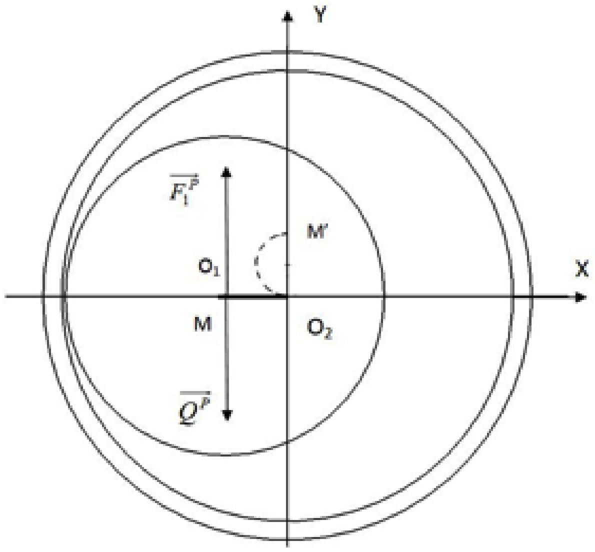


Figure 3: A curve of equilibrium positions of the centre of the shaft.

Up to now we only considered system "the ring - shaft" and neglected the influence of an external field of fluid 4-2. Now we shall consider the bearing which consists of three rigid elements, as shown in figure 1. Similarly we receive the projections of the reaction $F_2^B$ acting on the ring$(O_2, R_2)$from the external field of fluid 4-2 in local coordinate system$(O\xi_2, O\eta_2)$, where the direction $O\xi_2$ corresponds$\theta_2 = 0$ $(\varphi = \gamma_{B2}(t_0))$, see the figure 4:

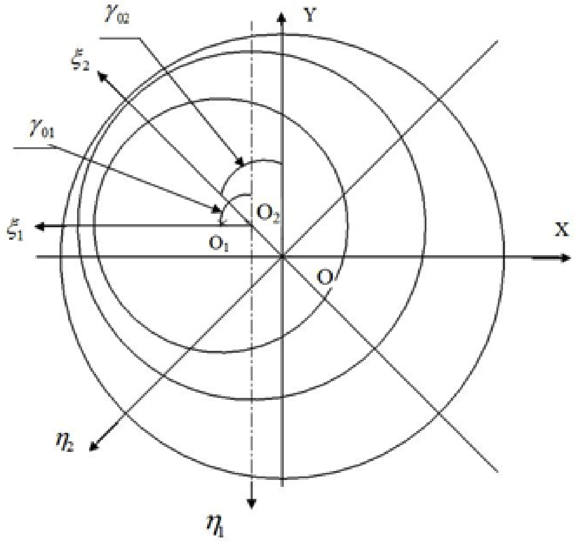


Figure 4: Schematic of designations for the floating ring bearing.

$$F^B_{2\xi} = \frac{2\mu L^3 \overset{\bullet}{\varepsilon}_2}{h^2_{02}} \left[ \int_0^{\pi} \frac{R_2.\cos^2\theta_2}{(1-\varepsilon_2\cos\theta_2)^3} d\theta_2 + \int_0^{\pi} \frac{e_2.\cos^3\theta_2}{(1-\varepsilon_2\cos\theta_2)^3} d\theta_2 \right],$$
$$F^B_{2\eta} = \frac{\mu L^3 \omega_2 \varepsilon_2}{h^2_{02}} \left( \frac{2\overset{\bullet}{\gamma}_2}{\omega_2} - 1 \right) \left[ \int_0^{\pi} \frac{R_2.\sin^2\theta_2}{(1-\varepsilon_2\cos\theta_2)^3} d\theta_2 + \int_0^{\pi} \frac{e_2.\cos\theta_2.\sin^2\theta_2}{(1-\varepsilon_2\cos\theta_2)^3} d\theta_2 \right] \quad (6)$$

Here it is denoted:$h_{02} = R - R_2$ - a nominal gap, $e_2 = e_2(t)$ - eccentricity of the centre of a floating ring and $\gamma_2 = \gamma_2(t)$ - a angle describing position of a line of centre of bearing – housing and the centre of rotating floating ring. Similarly we receive the projections of the reaction$F^B_1$ acting on the floating ring$(O_2, R_2)$ from the internal field of fluid 4-1 in local coordinate system$(O\xi_2, O\eta_2)$:

$$F^B_{1\xi} = -\frac{\mu L^3(\omega_1+\omega_2)R_2}{2h^2_{01}} \int_0^{2\pi} \frac{\sin\theta_1.\cos\left(\theta_1+\gamma^*_1-\gamma^*_2\right)}{\left(1-\varepsilon^*_1\cos\theta_1\right)^3} d\theta_1,$$
$$F^B_{1\eta} = -\frac{\mu L^3(\omega_1+\omega_2)R_2}{2h^2_{01}} \int_0^{2\pi} \frac{\sin\theta_1.\sin\left(\theta_1+\gamma^*_1-\gamma^*_2\right)}{\left(1-\varepsilon^*_1\cos\theta_1\right)^3} d\theta_1. \quad (7)$$

Where $(\varepsilon^*_1, \gamma^*_1)$ and $(\varepsilon^*_2, \gamma^*_2)$accordingly denote the coordinates of equilibrium position of the rotating shaft and the floating ring. When the centre of the rotating shaft and the centre of the floating ring are in the equilibrium position then$\gamma^*_1 = \frac{\pi}{2}$, $\varepsilon^*_1 \in [0,1]$. Each equilibrium position of the shaft $(\varepsilon^*_1, \gamma^*_1)$ in the gap corresponds to a curve of equilibrium positions of the centre of the floating ring, which is defined by the equation:

$$tg\gamma^*_2 = -\frac{F^B_{1\eta}(\varepsilon^*_1, \frac{\pi}{2}) + F^B_{2\eta}(\varepsilon^*_2, \gamma^*_2)}{F^B_{1\xi}(\varepsilon^*_1, \frac{\pi}{2}) + F^B_{2\xi}(\varepsilon^*_2, \gamma^*_2)}. \quad (8)$$

# 4 Definition of constant speed of rotation of the ring in the floating ring bearing

Let the shaft $(O_1, R_1)$ rotates with the given constant angular speed $\omega_1$, and the floating ring $(O_2, R_2)$ rotates with unknown constant angular speed $\omega_2$. This angular speed $\omega_2$ is included into expressions for all forces acting on the shaft and the floating ring. To define the motions of elements of the bearing, it is necessary to investigate the dependence of speed of rotation of the floating ring $\omega_2$ on the speed of rotation of the shaft $\omega_1$. Up to now, the speed of rotation of the floating ring $\omega_2$ was determined only by an experimental method [9], [10]. In this work we shall receive an analytical dependence of $\omega_2$ on $\omega_1$. We notice that the moment of friction only exists in the field of positive pressure of the fluid. At the equilibrium position $\dot{\varepsilon}_1^* = 0$, $\dot{\gamma}_1^* = 0$, $\dot{\varepsilon}_2^* = 0$, $\dot{\gamma}_2^* = 0$, from the equations (2), (3) it is received that the pressure is positive in the area $\theta_1 \in [\pi, 2\pi]$ and similar in the area $\theta_2 \in [\pi, 2\pi]$.

In the article [7], the expression for the torque acting on the floating ring from the internal firm of the fluid in the short bearings theory is given:

$$M_1 = \frac{\mu(\omega_1+\omega_2)\varepsilon_1^* L^3 R_2}{8h_{01}^2} \int_\pi^{2\pi} T_1 \frac{\partial Q_1^*}{\partial \theta_1}\, d\theta_1 - R_2 L \int_\pi^{2\pi} S_1\, d\theta_1, \tag{9}$$

where $T_1 = 1 - \frac{2\ln\left(\frac{R_2}{R_2-h_1}\right)}{\left(\frac{R_2}{R_2-h_1}\right)^2-1}$, $S_1 = \frac{2\mu(\omega_1-\omega_2)}{\left(\frac{R_2}{R_2-h_1}\right)^2-1}$, $Q_1^* = \frac{\sin\theta_1}{\left[1-\varepsilon_1^*\cos\theta_1\right]^3}$.

The expression for the torque acting on the floating ring from the external firm of the fluid in the short bearings theory:

$$M_2 = \frac{\mu\omega_2\varepsilon_2^* L^3}{8h_{02}^2} \int_\pi^{2\pi} (R-h_2)\, T_2 \frac{\partial Q_2^*}{\partial \theta_2}\, d\theta_2 - L \int_\pi^{2\pi} S_2 (R-h_2)\, d\theta_2, \tag{10}$$

where $T_2 = 1 - \frac{2\ln\left(\frac{R}{R-h_2}\right)}{1-\left(\frac{R-h_2}{R}\right)^2}$, $S_2 = \frac{2\mu\omega_2}{1-\left(\frac{R-h_2}{R}\right)^2}$, $Q_2^* = \frac{\sin\theta_2}{\left[1-\varepsilon_2^*\cos\theta_2\right]^3}$.

Rotation of the floating ring around its axis is submitted by the below equation:

$$M_1^B - M_2^B = J_B \dot{\omega}_2.$$

In case we assume the floating ring rotates with a constant angular speed $\omega_2$ then $\dot{\omega}_2 = 0$, so we get:

$$M_1^B - M_2^B = 0. \tag{11}$$

The equations (9), (10) and (11) lead to the expression for definition of angular speed of the ring $\omega_2$. In order to making a diagram, we take a numerical calculation with the following parameters: radius of the bearing housing $R = 0,05$, radius of the floating ring $R_2 = 0,048$, radius of the floating shaft $R_1 = 0,046$.

This paper is devoted to define the speed of rotation of the ring in the floating ring bearing with the considering of the allowance of lubrication hydrodynamics and centrifugal force. By integration a component of tensor stress on a surface of the gaps, we received obvious expressions for forces and the torques acting on the shaft and on the ring of the floating ring bearing. Distribution of the equilibrium positions of the shaft in lubricant layer in mobile coordinate system is researched. The dynamic analysis for the floating ring is defined by analogy. Other parts of this article are devoted to define the speed of rotation

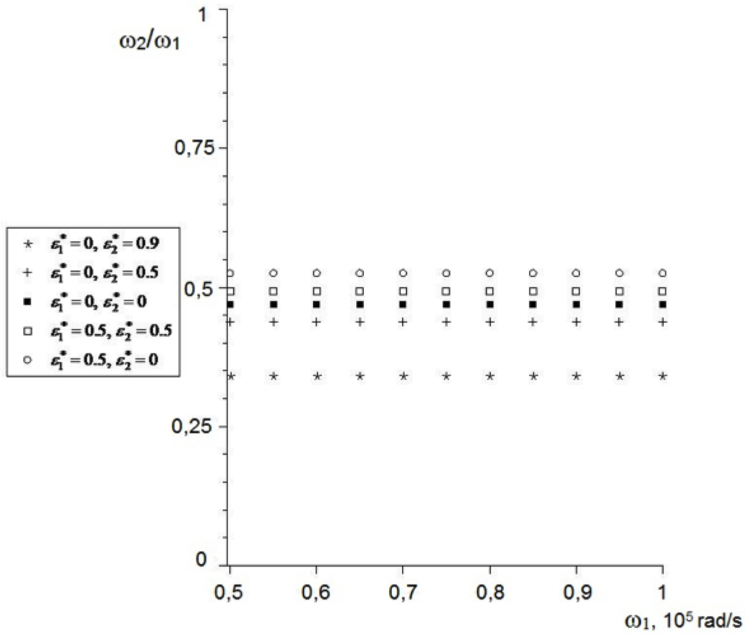


Figure 5: Diagrams of the dependence of the relation of angular speed of the floating ring to angular speed of the shaft $\frac{\omega_2}{\omega_1}$ from equilibrium position of the shaft and the floating ring in the bearing at different values of angular speed of the shaft $\omega_1$.

of the ring in the floating ring bearing. The given problem is important for definition of the forces acting on the shaft as they depend on angular speed of rotation of the floating ring. Angular speed of rotation of the ring in the floating ring is received within the framework of the short bearing theory from a condition of equality of the torque acting on the floating ring from the fluid firms both outside and inside. Numerical calculation has shown that angular speed of rotation of the floating ring in the floating ring bearing can vary in enough wide limits.

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