Convection and heat transfer in liquid with internal heat release in a rotating horizontal cylinder

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Abstract

The convection of heat-generating fluid in a rotating horizontal cylinder is experimentally investigated. The thresholds of thermal convection excitation, heat transfer in the cylinder and the structure of convective flows depending on the heat capacity, viscosity and relative length of the cavity are studied. Different modes of convection are revealed. In case of relatively rapid rotation due to the centrifugal force of inertia the temperature distribution is axially symmetric and has a maximum in the center of the cavity. The heat transfer, additional to the molecular one, in this case is provided by the inertial waves and the Ekman flows near the end walls of the cavity. With decrease of rotation velocity the convective flow arises in a threshold way in the form of vortex cells periodically arranged along the axis. The excitation of mean convection is caused by the action of termovibrational mechanism. In overcritical domain the stationary vortex regime is replaced by the oscillatory one (the fluctuations are associated with the periodic variation of the convective structures). For very slow rotation the quasi-stationary gravitational mechanism predominates.

1 Introduction

During the experiment a cylinder filled with a viscous heat-generating fluid rotates around its horizontal axis in a gravity field. The outer wall of the cylinder is isothermal, its temperature is maintained constant. In the cavity frame the gravity field oscillates. The frequency of oscillations coincides with the frequency of rotation. The averaged convection is caused by the nonisothermal liquid oscillations relative to the cavity caused by the gravity. This "vibrational" mechanism of averaged convection has been theoretically described in [1]. It is also shown that in case of liquid with internal heat release the averaged convection in the cavity rotating around a horizontal axis is characterized by a centrifugal Rayleigh number $Ra = \Omega^2 R^6 \beta q/\nu\chi^2 c_p\rho$, vibrational parameter $R_v = (g\beta R^3 q)^2/2\nu\chi^3 c_p^2\rho^2\Omega^2$ and dimensionless velocity of rotation $\omega = \Omega R^2/\nu$. Here β, ν and χ - coefficients of thermal expansion, kinematic viscosity and thermal diffusivity of fluid, c_p - specific heat of fluid at constant pressure, ρ - fluid density, R - radius of the cylinder, q - volumetric power of internal heat sources, $\Omega \equiv 2\pi n$ - angular velocity of rotation (n - rotational speed of the cavity). The various problems of termovibrational convection in rotating cavities were investigated in [2] [3].

2 Experimental setup and technique

The cavity (Fig. 1) is made of a Plexiglas cylinder *1* with flanges *2* on both end sides. The length of the working part of the cell $l = 170$ mm, the internal diameter $d = 36$ or 44

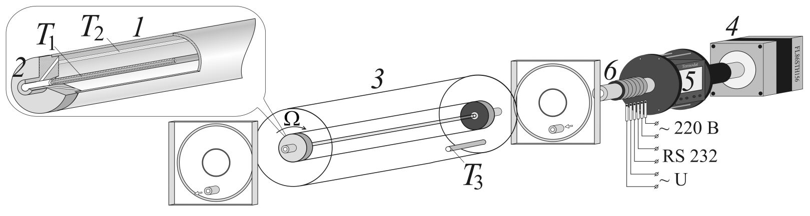

Figure 1: The sketch of the cavity.

mm. The water and glycerol-water solutions of different concentrations are used as working fluids. The internal heat generation is produced by the alternating electric current passing through the liquid. The flanges are equipped with the copper electrodes and the liquid is added with copper sulfate (not more than 5 proc.) for conductivity. The temperature in the cavity is measured with RTDs. The sensor T_1 is located on the axis of the cavity, T_2 - on the cylindrical wall. Both thermometers are made of copper wire and extend along the entire cell length, thus measuring the average temperature along the cell.

To provide the constant temperature of the external wall of the cell the last is placed in a Plexiglas cylinder *3* of a larger diameter, which is also closed with flanges. The water of desired temperature from the thermostat is pumped in the space between the walls of the cylinders. One more sensor T3 controls the temperature of liquid in this water shirt. Due to the system of bearings and seals the working cell can rotate freely. The rotation is given by the stepper motor *4*, the rate ranged from 0.01 to 2.00 revolutions per second.

The sensors data is processed by the Termodat device *5* which rotates with the cavity and transmits the signal to the computer. The connection to the computer is carried out with a multichannel electrical collector *6*. The collector is also used for providing the Termodat with power and internal heating of liquid in the cavity. The walls of the cell are transparent, which allows the visual observation and photography of convective structures. In experiments with photo recording the small amount of light scattering particles of Resine Amberlite is added in liquid. The particles diameter is 50 microns, the density is about the liquid density. The photos are made from the side of the cylinder. The longitudinal vertical light knife is used for illumination. The experiment starts with the relatively rapid cell rotation, electrical current is passed through the liquid. The measurements start after establishing of a stationary temperature distribution in the cavity. During the experiment the rotational velocity of the cylinder is reduced step by step.

3 Results of experiment

The rotation of the cavity around the axis of symmetry produces an axisymmetric field of the centrifugal forces of inertia $\rho\Omega^2\mathbf{r}$. In this case the temperature distribution $\Theta_0 = q(R^2 - r^2)/4\lambda$ corresponds to a stable equilibrium state of the liquid. Here q - volumetric power of internal heat sources, λ - coefficient of thermal conductivity, r - distance from the axis of the cavity. This equation allows to determine the temperature at the center of the cavity relative to the wall $\Theta^{'} = qR^2/4\lambda$. At the beginning of the experiment the temperature in the center of the cavity is higher than the temperature at the cylindrical boundary, which leads to the nonuniformity of the thermophysical properties of fluids. It should be noted that when analyzing the experimental results all parameters are calculated

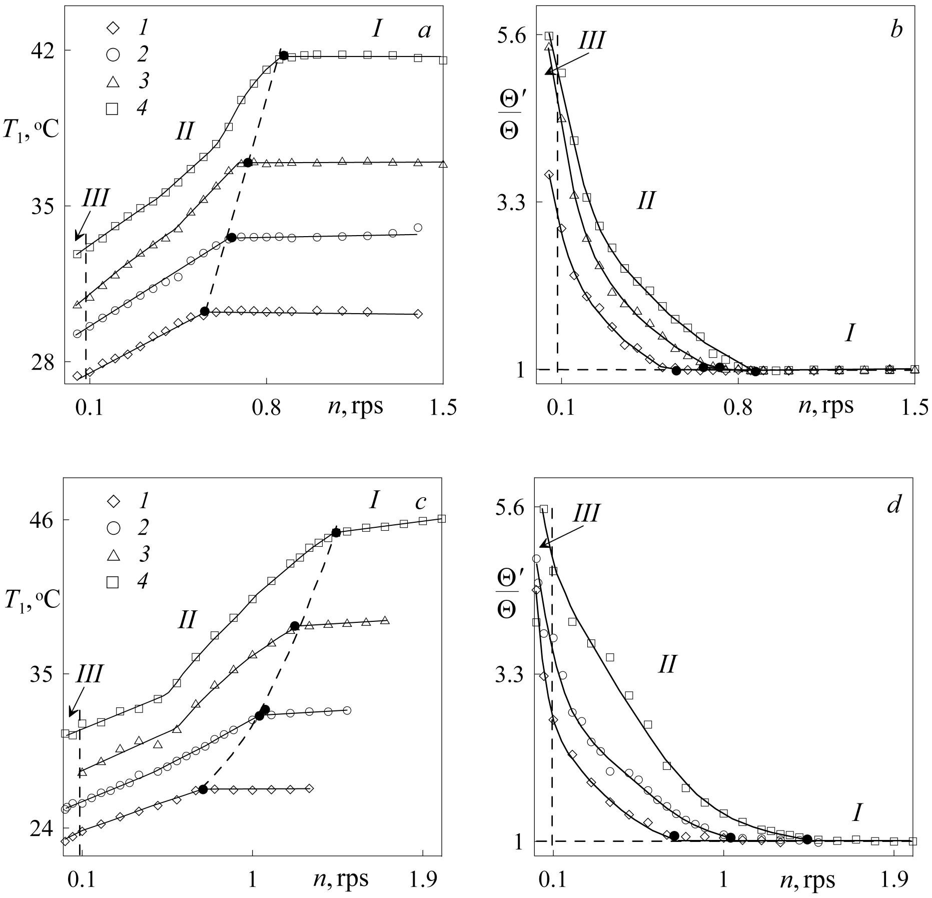

Figure 2: Temperature T_1 and dimensionless parameter $\Theta^{'}/\Theta$ versus the speed of rotation; a, b - water, $R = 1.8$ cm, $q = 0.034$ (*1*), 0.050 (*2*), 0.063 (*3*) and 0.065 W/cm^3 (*4*); c, d - $C = 50$ %, $R = 1.8$ cm, $q = 0.030$ (*1*), 0.051 (*2*), 0.076 (*3*) and 0.103 W/cm^3 (*4*).

using the average temperature in the cavity.

The temperature T_1 variation with n in liquids of different viscosities are presented in (Fig. 2, a and c). The curves of heat transfer corresponding to these experiments are shown in (Fig. 2, b and d). A dimensionless parameter chosen to characterize the heat transfer is $\Theta^{'}/\Theta$, here $\Theta = T_2 - T_1$ - the experimental value of the fluid temperature at the cavity axis relative to the cylindrical wall. One can see a slight change in temperature T_1 with a decrease in speed (field I). With increasing the viscosity of liquid and its internal heating this change becomes more significant. In this case the temperatures Θ and $\Theta^{'}$ are consistent only in the limit of high rotation speed. The flow visualization demonstrates that with the relatively rapid rotation when the heat transfer in the cavity nearly corresponds to the molecular one and the vortical structures in the volume of the cavity are absent, the powder settles on the inner wall of the cavity into thin rings (Fig. 3, a). As it will be shown below this is caused by slow averaged currents that are generated by an inertial wave that occurs near the ends of the cavity [4]. In experiments with a viscous fluid the rotation speed reduction leads to a greater increase in heat transfer. It is indicated by the decrease of the temperature T_1 (Fig. 2, c and d).

With further decreasing the velocity of rotation one observes the change in the regime

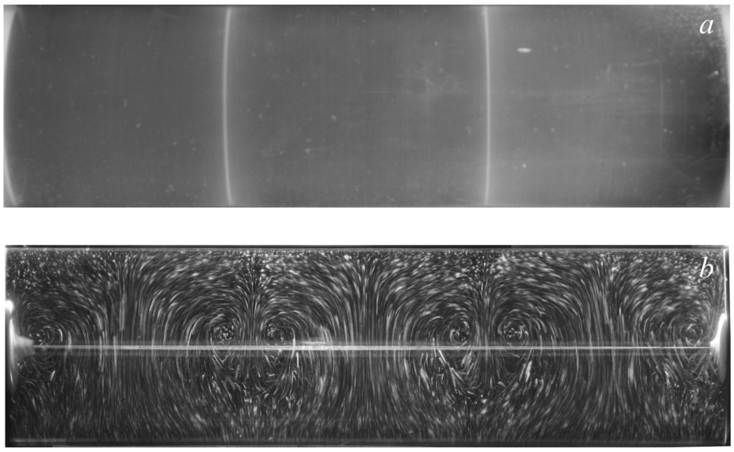

Figure 3: Pictures of structures: water, $R = 1.8$ cm, $q = 0.063$ W/cm^3, $n = 1.1$ (a) and 0.55 rps (b).

of convection, which is accompanied with a critical change in heat transfer (Fig. 2, the boundary of areas I and II). The observations show that the critical temperature change is associated with the appearance of the periodic system of vortex cells (Fig. 3, b). The excitation of the convection is associated with the action of thermovibrational mechanism which is based on the fluctuations of non-isothermal fluid with respect to the cavity caused by the gravity force [1]. The picture shows the overcritical structure of convection, as the flows near the threshold are weak and their visualization is problematic. The structure of the currents in liquids of different viscosity is the same.

Experiments with water in the cell of larger radius show that the reduction in speed also leads to a crisis of heat transfer (Fig. 4, the boundary of areas I and II). When the volumetric power of heat sources is relatively large the averaged convection looks like a system of vortex cells (Fig. 5) as in the experiments with the cavity of smaller radius. Another behavior is observed with decreasing of the rate of heating: one can see two critical transitions in heat transfer (Fig. 4, shaded area). In this range of values of n the variation curves of heat transfer is different from other areas. It was problematic to determine the convective structures in this domain.

With further decrease of rotation speed the different modes of convection are observed. The order of regime change does not depend on the radius of the cell and fluid viscosity. The stationary vortex convection regime is replaced by the oscillatory one. In experiments with the liquids of relatively low viscosity the regular low-frequency oscillations of the temperature T_1 are detected. The observations show that these fluctuations are associated with shifts in convective flows, the vortexes develop and decay periodically. In the liquids of higher viscosity the structures also change periodically but their complete destruction does not occur and the temperature sensor in the center of the cavity captures the oscillations of low intensity.

For very slow rotation (Fig. 2, 4, the boundary of areas II and III) the sensor detects oscillations of temperature T_2. The period of oscillation is equal to the period of rotation of the cavity and the amplitude reaches a few degrees at a very slow rotation. This indicates

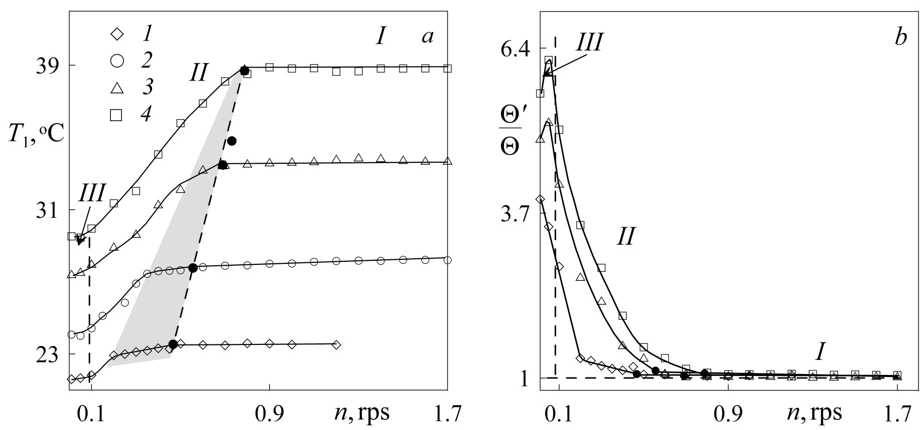

Figure 4: Temperature T_1 and dimensionless parameter $\Theta^{'}/\Theta$ versus the rotation velocity; a, b - water, $R = 2.2$ cm, $q = 0.016$ (*1*), 0.030 (*2*), 0.050 (*3*) and 0.061 W/cm^3 (*4*).

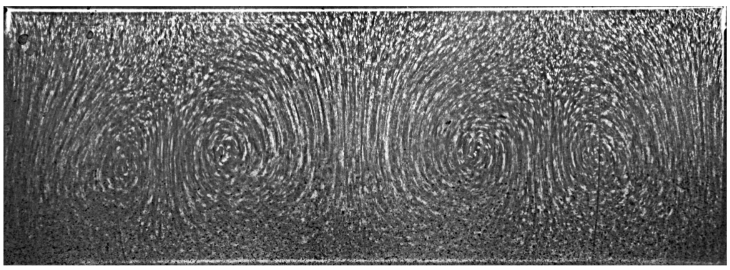

Figure 5: Photo of convective flows: water, $R = 2.2$ cm, $q = 0.065$ W/cm^3, $n = 0.5$ rps.

the predominance of the quasi-stationary gravitational mechanism of convection.

With increase of the heat release and temperature Θ the velocity of rotation corresponding to the threshold of convection excitation increases monotonically (Fig. 6). The instability areas are below the threshold curves (shaded). With increasing of the viscosity the threshold curves are shifted to the higher values of n.

4 Discussion of results

Consider the possible mechanism of formation of ring structures at large n. The rotation of the cavity filled with a nonuniformly heated fluid in a static external force field (gravitational in our case) results in fluctuations of the liquid relative to the cavity. In a rotating and simultaneously oscillating fluid the inertial waves can be excited [4]. Characteristic surfaces with intensive shear flows are specific for them. The angle between the characteristic surfaces and the axis of rotation is defined by the condition $\tan\varphi = (4/N - 1)^{-1/2}$. Here $N \equiv \Omega_{osc}/\Omega$ - dimensionless frequency of liquid oscillation (Ω_{osc} - radian frequency of liquid oscillation, $\Omega = 2\pi n$ - angular velocity of the cavity). In our case the fluctuations of fluid in the cavity frame occur with a frequency of rotation, that is $N = 1$. Thus, the tangent of the angle does not depend on the cell radius R and equals to $1/\sqrt{3}$. The di-

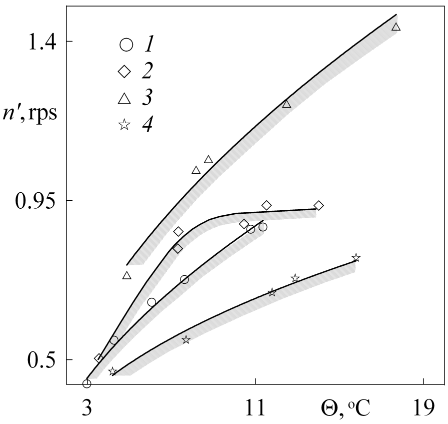

Figure 6: The dependency of the critical velocity on the temperature Θ. $R = 1.8$ cm, water (*1*), $C = 25$ (*2*) и 50 % (*3*); $R = 2.2$ cm, water (*4*).

mensionless spatial period Λ/R is $4\sqrt{3}$. Fig. 7 shows photographs of structures and the possible location of the characteristic surfaces (dashed lines). One can see that the characteristic surfaces determine the position of the rings on the cavity wall. The theoretical value Λ/R agrees well with the experiment. The characteristic surface of excited wave is a cone. Liquid oscillates along the cone. Averaged flow leading to a redistribution of powder on the surface could be generated in the Stokes boundary layers by wave reflection from the cavity wall. In a thin cell (Fig. 7, *a*) the waves from the opposite ends of the cavity reflect from the walls being spatially coherent, so the structures they form are regular. In the cell of bigger radius (Fig. 7, *b*) such agreement is absent: the powder forms two rings at the lines of intersection of the characteristic surfaces of the opposing waves with the boundary of the cavity.

With the rapid rotation a weak radial motion of the fluid can also be generated in the viscous Ekman layers occurring near the end walls of the cylinder in the presence of differential azimuthal rotation of the fluid [4]. Thus the equilibrium state of the liquid subject to the rapid rotation may be disturbed by inertial waves and Ekman flows. This could explain the difference between the temperature field in the cavity from the case of rigid-body rotation of the entire system (Fig. 2 and 4).

In Fig. 8 (*a*) the threshold curves of the convection excitation are presented on the plane Ra,R_v. The area of quasi-equilibrium is marked by hatching. The points obtained for water and glycerin solution of a low concentration of 25 and 35 % agree with each other. With viscosity increase the noticeable discrepancy between the curves appears. This could be explained by the influence of the Coriolis force which is characterized by the dimensionless rotational speed ω. It is known that the Coriolis force has a stabilizing effect on the three-dimensional vortex structures both in gravity [5] and vibrational [2] convection. Only in the case of two-dimensional vortices oriented parallel to the axis of rotation the Coriolis force has no effect. In this case the dependence on the dimensionless frequency is absent, as in [3]. Fig. 8 (*b*) shows the threshold values of vibration parameters R_v versus the dimensionless velocity at a definite value of the centrifugal Rayleigh number $Ra = -4 \cdot 10^4$ (dashed line in Fig. 8, *a*). At large ω (experiments with water) the threshold of thermovibrational convection excitation (critical value of R_v) does not depend on the ω. It can be assumed that the convective structures in the threshold are termovibrational

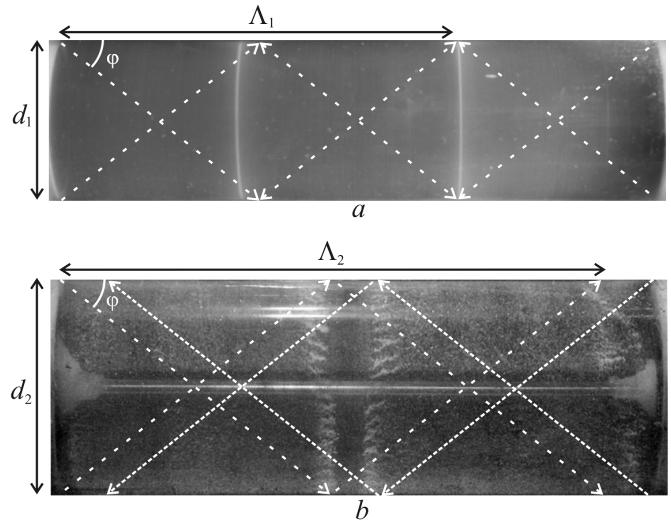

Figure 7: Photos of structures: water, *a* - R = 1.8 cm, q = 0.063 W/cm^3, n = 1.1 rps; *b* - R = 2.2 cm, q = 0.065 W/cm^3, n = 1.5 rps.

rolls elongated along the axis of rotation, similar to ones found in another problem [1].

5 Conclusion

Thermovibrational convection of heat-generating fluid in a horizontal cylinder with rotation is investigated experimentally. The effect of viscosity on the threshold of convection is studied. Heat transfer and the structure of convective flows at different regimes of convection are considered. It is shown that the threshold of the onset of convection is determined

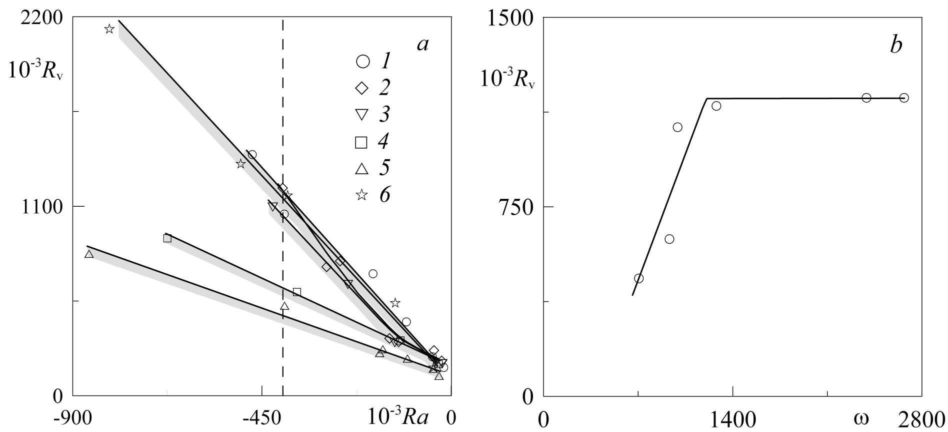

Figure 8: *a* - threshold curves in the plane of control parameters Ra,R_v; R = 1.8 cm, water (*1*), C = 25 (*2*) 35 (*3*), 40 (*4*) and 50 % (*5*); R = 2.2 cm, water (*6*); *b* - the critical vibrational parameter R_v versus the dimensionless rotation velocity ω at Ra = -4·10^4.

by the parameters Ra,R_v, and dimensionless speed of rotation.

Acknowledgements

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References

[1] Kozlov V.G. Thermal vibrational convection in rotating cavities // Fluid Dynamics. 2004. V. 39. N 1. P. 3-11.

[2] Ivanova A.A., Kozlov V.G., Rylova V.V. Thermal convection in a plane layer rotating about a horizontal axis // Fluid Dynamics. 2003. V. 38. N 1. P. 9-17.

[3] Vjatkin A.A., Ivanova A.A., Kozlov V.G. Convective stability of a nonisothermal fluid in a rotating horizontal coaxial gap // Fluid Dynamics. 2010. V. 45. N 1. P. 10-18.

[4] Greenspan H.P. The Theory of Rotating Fluids. Cambridge University Press, New York, 1968. 328 pp.

[5] 5. Gershuni G.Z., Zhukhovitsky E.M. Convective stability of incompressible liquid. M.: Nauka, 1972. 392 p. (Translated in English: Keter Publishing House Jerusalem Ltd., Jerusalem 1976).

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