Vibrational motion of the cavity filled with perfect viscous gas

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Abstract

The mathematical model of the vibrational motion of the cavity filled with viscous gas is defined. The numerical investigation of the heat and mass transfer processes in the cavity as a result of vibrations is performed. The maximum values of temperature and pressure of the gas on the walls of the cavity depending on the frequency of vibration were found. The cavity is filled by perfect viscous gas with air characteristics. The case is considered in a one-dimensional setting. The gas characteristics were chosen like the air characteristics.

1 Introduction. Problem formulation

As the technological expansion, people are more and more contact with the phenomena of vibration. Almost all machines are susceptible to mechanical vibrations in a varying degree. Even with a free flight of the spacecraft there is vibration from the operating equipment [1]. Vibration of machines may be a cause of abnormal operation of machinery and cause serious accidents. It is quite common to situations where there are cracks, pores or voids filled with air. Therefore, it is important to study gas influence to the boundaries of the cavity. We will numerically investigate the effects of the vibrating action with a constant frequency on the rectangular cavity filled with a viscous perfect gas. Due to the medium compressibility there is a forming of acoustic waves, which reinforce the heat and mass transfer. There are two frequencies: the natural frequency of the system and the oscillation frequency caused by the boundary conditions [2]. Specifying high-frequency vibration leads to appearance of shock waves. It is useful to determine the intensity of the impact of the waves on the boundary to avoid possible destructions of the product. Using this data (maximum pressure and maximum temperature near the boundary) we may calculate the maximum frequency of vibration for the other given characteristics.

2 Mathematical model

The cavity of length L filled with a viscous perfect gas with thermal properties of air is considered. At the initial moment the gas in the cavity stayed at rest with a constant temperature T_0 and a constant pressure P_0. The equilibrium state is disbalanced due to the vibrational effects of amplitude A and frequency ω. In the beginning the cavity was in the extreme right position. The boundaries are kept in temperature equal with the initial temperature. The thermal conductivity, the dynamic viscosity and the heat capacity are assumed to be constant.

The motion of the gas under these assumptions is described with the one-dimensional nonstationary system of equation in Cartesian coordinates, consisting of conservation laws of mass, momentum and energy. The Clapeyron ideal gas law is considered as the equation of state for gas. The system is written in the noninertial frame of reference associated with the vibrating cavity. Use the following formulas for the transition:

$$t = t', \tag{1}$$

$$x = x' - A\cos(\omega t), \tag{2}$$

$$u = \frac{\partial x}{\partial t} = u' + A\omega\sin(\omega t) \tag{3}$$

Here, t is time in the noninertial frame of reference, x is coordinate in the noninertial frame of reference, u is the velocity in the noninertial frame of reference, t' is time in the inertial frame of reference, x' is coordinate in the inertial frame of reference, u' is the velocity in the inertial frame of reference.

The system of dimensionless equations describing motion of such gas has the form

$$\frac{\partial\tilde\rho}{\partial\tau} + \frac{\partial\tilde\rho\tilde u}{\partial X} = 0, \tag{4}$$

$$\frac{\partial\tilde\rho\tilde u}{\partial\tau} + \frac{\partial\tilde\rho\tilde u\tilde u}{\partial X} = -\frac{\partial\tilde P}{\partial X} + \frac{4}{3}\frac{1}{Re}\frac{\partial^2\tilde u}{\partial X^2} + \tilde\rho A\Omega^2\cos(\Omega\tau), \tag{5}$$

$$\frac{\partial\tilde\rho\Theta}{\partial\tau} + \frac{\partial\tilde\rho\tilde u\Theta}{\partial X} = \frac{\gamma}{Pe}\frac{\partial^2\Theta}{\partial X^2} - \tilde P\frac{\partial\tilde u}{\partial X} + \frac{4}{3}\frac{1}{Re}(\frac{\partial\tilde u}{\partial X})^2, \tag{6}$$

$$\tilde P = \frac{\tilde\rho(\Theta+1)}{\gamma}, \tag{7}$$

The dimensionless variables are taken as:

$$X = \frac{x}{L},\ \tau = \frac{t\sqrt{\gamma RT_0}}{L},\ \tilde u = \frac{u}{\sqrt{\gamma RT_0}},\ \tilde P = \frac{P}{\gamma P_0},\ \tilde\rho = \frac{\rho}{\rho_0},\ \Theta = \frac{T-T_0}{T_0},$$

Dimensionless parameters:

$Re = \frac{\rho_0 L\sqrt{\gamma RT_0}}{\mu}$ - acoustical Reynolds number

$Pe = \frac{\rho_0 Lc_p\sqrt{\gamma RT_0}}{k}$ - Peclet number

$\gamma = \frac{c_p}{c_v}$ - adiabatic exponent

$\Omega = \frac{\omega L}{\sqrt{\gamma RT_0}}$ - nondimensional vibration frequency

$\tilde A = \frac{A}{L}$ - nondimensional vibration amplitude

$\sqrt{\gamma RT_0}$ is adiabatic speed of sound in the region of the temperature T_0.

Here, ρ is the density, P is the pressure, T is the temperature, R is the gas constant, μ is the coefficient of dynamic viscosity, k is the thermal conductivity coefficient and c_v is the specific heat capacity at constant volume.

The initial conditions are as follows:

$$\tilde u|_{\tau=0} = 0, \Theta|_{\tau=0} = 0, \tilde P|_{\tau=0} = \frac{1}{\gamma}, \tilde\rho|_{\tau=0} = 1 \tag{8}$$

The boundary conditions are as follows:

$$\Theta|_{X=0} = 0, \tilde u|_{X=0} = 0, \Theta|_{X=1} = 0, \tilde u|_{X=1} = 0 \tag{9}$$

3 Parameters of calculation and numerical scheme

The main positions of the numerical method will be described here. In obtaining a numerical solution, we choose a number of locations (n grid points) and seek the solution there. The differential equations (4)-(7) with (8), (9) are solved by converting them into discretization equations (algebraic equations). The discretization equations are obtained by the control volume approach and the second-order treatment of the numerical scheme [3]. The system (4)-(7) is written as:

$$a_\Phi \Phi_i = b_\Phi \Phi_{i+1} + c_\Phi \Phi_{i-1} + d_\Phi \tag{10}$$

$$\tilde{\rho}_i = \tilde{\rho}_i(\Phi_i) \tag{11}$$

$$a_\Theta \Theta_i = b_\Theta \Theta_{i+1} + c_\Theta \Theta_{i-1} + d_\Theta \tag{12}$$

$$\tilde{P}_i = \tilde{P}_i(\Theta_i) \tag{13}$$

where

$$\Phi_i = \tilde{\rho}_i \tilde{u}_i \tag{14}$$

is the mass flux;$a_\Phi, b_\Phi, c_\Phi, d_\Phi, a_\Theta, b_\Theta, c_\Theta, d_\Theta$ are the known numeric functions, $i = 2..n-1$, (11) is the equation of continuity, (13) is the equation of state for gas. We use the staggered grid for the mass flux Φ and for the velocity u. In the staggered grid, the velocity components are calculated in points that lie on the faces of the control volume [4].

This is the sequence of operations:

1. Start with a guessed fields of velocity, pressure and density;

2. Calculate the coefficients for the equations (10), and solve this system to obtain Φ by the TriDiagonal-Matrix Algorithm [3];

3. Calculate the density by use of (11);

4.Calculate the coefficients for the equations (12), and solve this system to obtain Θ by the TriDiagonal- Matrix Algorithm [3];

5. Calculate the pressure by use of (13);

6. Calculate the velocity by use of (14);

7. Return to step 2 and repeat until convergence.

The computation grid was uniform and consisted of 1000 nods. The time step was taken to be $\Delta\tau = 0.1\Delta X$(i.e., on condition that the wave will pass a distance of ΔX or less during the time $\Delta\tau$ where ΔX is the size of control volume).

The dimensionless parameters were taken following:$\Omega = 0.144, 0.288, 0.432, 0.576, 0.720, 0.864, 1.008, 1.152, \tilde{A} = 2, Re = 100000, Pe = 80000, \gamma = 1.4$. We will use the dimensionless parameters for the analysis of the results.

4 Analysis of the results

We will first describe the behavior of the gas at the beginning of the vibration exposure. At the initial instant of time the cavity was situated at the rightmost position. Then the cavity begin to move to the left. Consequently, there are increasing of mass of the gas and a pressure jump near the right boundary. The calculations showed that the pressure to the right boundary at this time will be the maximum for all time of vibration. At the left boundary at the same time, however, there are great gas rarefaction and minimum pressure. Fig.1 shows the temperature at the nearest to the boundary points depending on time. Two frequencies are showed here. They are the natural frequency of the system

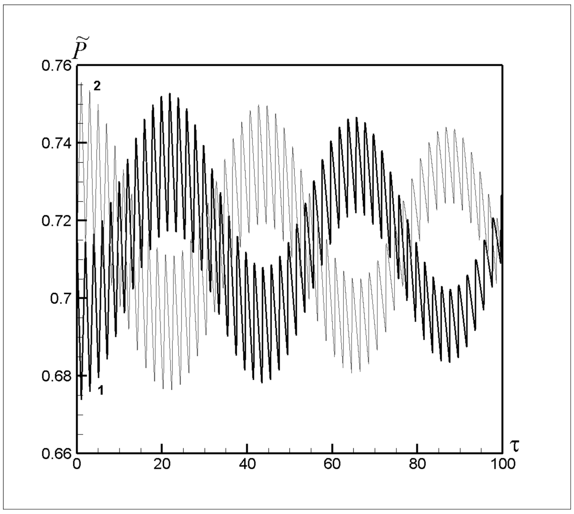

Figure 1: The time dependence of pressure: 1 - at the point $X = 0.0005$, 2 - at the point $X = 0.9995$

and the oscillation frequency caused by the boundary conditions ($\Omega = 0.144$)[2]. However, all further collision of waves with boundaries will cause less pressure jumps as compared with pressure jump at the right boundary which was a result of impact to the resting gas.With increasing frequency of vibration the maximum pressure in the boundary points is increased.

The numerical experiments showed that the maximum temperature in the nearest to borders grid points is not achieved at the same time with the maximum pressure. One reason for this is the influence of given constant wall temperature, which constrains the temperature jumps at the boundary. Let us consider the most intense vibration frequencies of the cavity since $\Omega = 0.720$. With such frequencies even the first acoustic wave is a shock wave. Therefore, it causes an intense temperature jump, and several less intense pressure jump. Following collision of shock waves cause a smaller increase in temperature, because the intensity of the shock wave decreases with time. Fig.2 and Fig.3 give the distributions

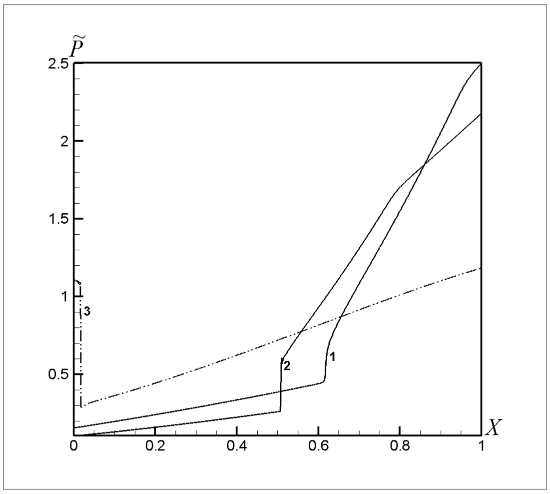

Figure 2: The pressure as a function of coordinate: 1 - $\tau = 0.74$, 2 - $\tau = 0.89$, 3 - $\tau = 1.38$

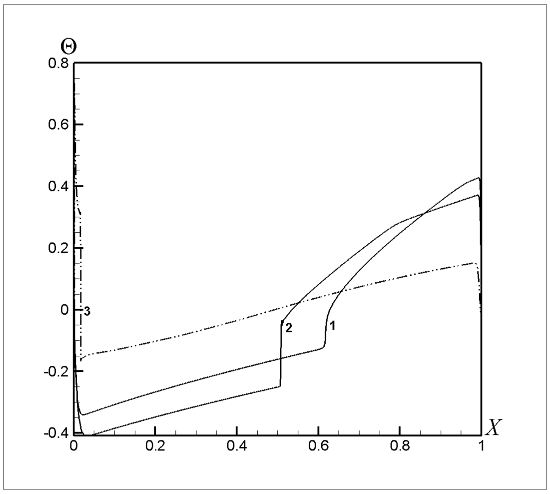

Figure 3: The temperature as a function of coordinate: 1 - $\tau = 0.74$, 2 - $\tau = 0.89$, 3 - $\tau = 1.38$

of pressure and temperature depending on coordinate at instants of time corresponding to maximum pressure ($\tau = 0.74$), maximum temperature ($\tau = 1.38$) and intermediate time when where is a forming of the shock wave ($\tau = 0.89$).

With less intense vibration frequencies of the cavity the shock waves reach maximum intensity later than the first passage of the field. Therefore, the strongest collision of shock wave with the wall are later than with frequencies which more intensify than was considered earlier. The results are presented in Tab.1. Here for each of the considered frequencies are showed the period of cavity vibration T, maximum pressure $\tilde{P}_{max}$ near boundaries of the cavity, maximum pressure $\tilde{\Theta}_{max}$ near boundaries of the cavity and times when this values are showed ($\tau(\tilde{P}_{max})$ and $\tau(\Theta_{max})$).

Table 7:

Ω	T	$\tilde{P}_{max}$	$\tau(\tilde{P}_{max})$	Θ_{max}	$\tau(\Theta_{max})$
0.144	43.617	0.756	0.99	0.006	55.57
0.288	21.809	0.884	0.96	0.045	11.52
0.432	14.539	1.111	0.91	0.114	5.50
0.576	10.904	1.456	0.85	0.190	3.49
0.720	8.723	1.940	0.79	0.305	1.41
0.864	7.270	2.593	0.74	0.897	1.38
1.008	6.231	3.445	0.68	1.413	1.30
1.152	5.4521	4.511	0.63	2.337	1.20

5 Conclusions

There are two frequencies: the natural frequency of the system and the oscillation frequency caused by the boundary conditions. The maximum values of the pressure and the temperature at the boundaries of the frequencies with investigated range are obtained. The time of observation of these values is found. The results provide an opportunity to determine the maximum frequency of vibration for given other vibration parameters, taking into account the specific properties of the material forming the wall of the cavity.

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