Propagation of waves in thin crystal layers at different potentials of interaction of sublattices

E. L. Aero A. N. Bulygin Yu. V. Pavlov
bulygin_an@mail.ru

Abstract

Wave processes in thin membranes are considered on the basis of the nonlinear dynamic theory of crystals with the complex lattice consisting of two sublattices. System of the connected nonlinear equations for finding acoustic U and optical u modes were integrated by a method of the functionally invariant solutions construction. The nonlinear Klein-Fock-Gordon equation takes place for optical mode u. Its solution can be found as inversion of an integral which depends on the potential of interaction of sublattices $\Phi(u_R)$ (u_R is the module of a vector of optical mode). Potential $\Phi(u_R)$ is an even periodic function, representable by Fourier's series. The case was in detail considered, when $\Phi(u_R) = 1 - \cos u_R$. Partial solutions are found and character of perturbations is established: periodic, aperiodic, localized like kink, solitons, etc. It is shown that addition to the potential $\Phi(u_R)$ of the higher harmonicas gives the solution for optical mode as the inversion of the hyperelliptic integral. The genus of the hyperelliptic integral is defined in general case by the highest harmonic entering into $\Phi(u_R)$. The problem of the inversion of the hyperelliptic integrals and character of dependence of acoustic and optical modes from a type of the potential $\Phi(u_R)$ is discussed.

1 Introduction

The continual nonlinear theory of elastic and inelastic deformations of crystal mediums with the complex lattice consisting of two sublattices is developed in works [1]–[3]. As well as in the theory M. Born, K. Huang [4], shift of the center of inertia of couple of atoms (an elementary cell) describes vector U (acoustic mode), and mutual shifts of atoms in a cell — vector u (optical mode). Unlike the theory of [4] of shift of sublattices u can be any the big. The new element of translation symmetry is entered into the theory — rigid shift of one sublattice relative another for one period (or their integer) reproduces again the structure of a complex lattice. It means that its energy has to be periodic function relative to the rigid shift of sublattices of u, invariant to this translation. Certainly, the classical principle of translation symmetry resulting in invariant of energy of a lattice relative to joint translation U of both sublattices on one (or integer) the periods of a complex lattice remains also.

The equations of movement for $U(x, y, z, t)$ and $u(x, y, z, t)$ are derived from the Lagrange's variation principle:

$$\rho \ddot{U}_i = \lambda_{ikmn} U_{k,mn} - s_{ik}[\Phi(u_R)]_{,k} \,, \tag{1}$$

$$\mu_0 \ddot{u}_i = k_{ikmn} u_{k,mn} - (p - s_{ik} U_{i,k}) \frac{\partial \Phi(u_R)}{\partial u_i} \,. \tag{2}$$

Here ρ is average density of mass of atoms; μ_0 is density of the reduced mass of atoms pairs (they differ if masses of the atoms are different); the point over symbols designates a derivative with respect to time, and a comma in an index — a derivative with respect to coordinate; λ_{ikmn} is a tensor of macroelasticity, which is symmetric to permutation of pairs of indexes and indexes of a pair; k_{ikmn} is a tensor of microelasticity, which is symmetric to permutation of pairs of indexes; s_{ik} is the symmetric tensor of a nonlinear striction describing reorganization of a microstructure under the influence of external tension; p is an energy of activation of interatomic couplings. Function $\Phi(u_R)$ describes periodic energy of interaction of sublattices. In general case

$$u_R = \sqrt{u_i \alpha_{ik} u_k}, \qquad \alpha_{ik} = a_1^{-2} k_i k_k + a_2^{-2} m_i m_k + a_3^{-2} n_i n_k. \tag{3}$$

Here ($\mathbf{k}a_1$, $\mathbf{m}a_2$, $\mathbf{n}a_3$,), (a_1, a_2, a_3) are respectively vectors and sizes of Bravais lattice. For a cubic lattice one has $(a_1 = a_2 = a_3)$

$$u_R = \frac{1}{a}\sqrt{u_x^2 + u_y^2 + u_z^2}. \tag{4}$$

The function $\Phi(u_R)$ (with Dirichlet conditions) can be expanded in the Fourier series

$$\Phi(u_R) = (1 - \cos u_R) + \delta(1 - \cos 2u_R) + \ldots \tag{5}$$

Here we take into account that $\Phi(u_R)$ is an even function and the potential of interaction of sublattices is equal to zero, when $u_R = 0$. Only the first term of the series (5) was taken into account in works of [1]–[3], [5]–[10]. The main aim of the present work — the analyse of dependence of plane deformation of crystal with a cubic lattice from form of the potential $\Phi(u_R)$.

The first equation can be written in a standard form of the equations of mechanics of the continuous medium

$$\rho \ddot{U}_i = \sigma_{ik,k} \tag{6}$$

with a stress tensor

$$\sigma_{ik} = \lambda_{ikmn} U_{m,n} - s_{ik}\Phi(u_R)\,. \tag{7}$$

The stress tensor (7), unlike classical, contains the additional term. It can be considered as a source of internal macrostress which is defined unambiguously by a field of microstress.

2 Solution of the equations of movement

The equations of movement (1), (2) are the system of six connected nonlinear equations in partial derivatives. Their analytical integration in a general case is connected with overcoming of great mathematical difficulties. For this reason we made the simplifying assumptions.

2.1 One-dimensional solitary waves

The simplest are one-dimensional problems. Let's consider one-dimensional processes of distribution of perturbations with constant velocities when both fields are functions of a phase of a wave

$$U_x = U(q), \qquad u_x = u(q), \qquad q = x - vt, v \gtrless 0, \tag{8}$$

Here v is a constant phase velocity. Different signs corresponds to two waves running in opposite directions. For this case Eqs. (1), (2) take form

$$(\lambda + 2\mu)\left(1 - \frac{v^2}{v_a^2}\right) U_{,qq} = s\,\Phi(u)_{,q}\,, \tag{9}$$

$$k\left(1 - \frac{v^2}{v_k^2}\right) u_{,qq} = (p - s\,U_{,q})\frac{d\Phi(u)}{du}\,, \tag{10}$$

where λ and μ are Lamé coefficients, v_a and v_k are correspondent phase velocities of the acoustic and optic modes.

$$v_a^2 = \frac{\lambda + 2\mu}{\rho}, \qquad v_k^2 = \frac{k}{\mu_0}, \qquad k = k_1 + k_2 + k_3. \tag{11}$$

Integrating (9), we obtain

$$(\lambda + 2\mu)\left(1 - \frac{v^2}{v_a^2}\right) U_{,q} = s\,\Phi(u) + \sigma_0. \tag{12}$$

The constant σ_0 of integration is the external tension. Macrogradients are excluded from Eq. (10) with the help (12). As a result we receive the separate equation for optical mode

$$l_0^2\left(1 - \frac{v^2}{v_k^2}\right) u_{,qq} = V'(u). \tag{13}$$

Here

$$V'(u) = [p_1 + 2p_2(1 - \Phi)]\frac{d\Phi}{du}, \tag{14}$$

$$l_0^2 = \frac{k}{p}, \qquad p_1 = 1 - \frac{s(s+\sigma)}{p(\lambda + 2\mu)\left(1 - \frac{v^2}{v_a^2}\right)}, \qquad p_2 = \frac{s^2}{2p(\lambda + 2\mu)\left(1 - \frac{v^2}{v_a^2}\right)}. \tag{15}$$

From Eq. (13) we find microshifts as inversion of the integral

$$\int \frac{du}{\sqrt{E + V(u)}} = \frac{\pm 1}{L}(q + q_0), \tag{16}$$

where

$$\frac{l_0}{L} = \frac{\sqrt{2}}{\sqrt{1 - \frac{v^2}{v_k^2}}},$$

E and q_0 are constants of integration. If $\Phi(u) = 1 - \cos u$, then by corresponding substitution $x = f(u)$ the expression under radical in (16) can be transformed to a polynom of the fourth degree $P_4(x)$. In this case (16) is an elliptic integral. The inversion of elliptic integral can be expressed through the theta functions of one variable (Jacobi's elliptic functions). This variant has been considered in details in [11]. Addition to $\Phi(u)$ of the second or the following harmonicas transforms integral (16) in ultraelliptic $n = 5, 6$ or into the hyperelliptic $n > 6$. The problem of the inversion of ultra- and hyperelliptic integrals will be considered below.

2.2 Plane deformation of crystals of cubic symmetry

The important part of mechanics of the continuous mediums is researches of the plane deformations. The body is subject to the plane deformation, if

$$U_x = U_x(x,y,t), \quad U_y = U_y(x,y,t), \quad U_z = 0, \tag{17}$$

$$u_x = u_x(x,y,t), \quad u_y = u_y(x,y,t), \quad u_z = 0. \tag{18}$$

Tensors of elasticity λ_{ikmn} and microelasticity k_{ikmn} for crystals of cubic symmetry take the forms

$$\lambda_{ikmn} = \mu\left(\delta_{im}\delta_{kn} + \delta_{in}\delta_{km}\right) + \lambda\delta_{ik}\delta_{mn}, \tag{19}$$

$$k_{ikmn} = k_1\delta_{ik}\delta_{mn} + k_2\delta_{in}\delta_{mk} + k_3\delta_{im}\delta_{kn}. \tag{20}$$

Taking into account (17)–(20) we write the equations of movement of crystal mediums with a cubic lattice as

$$\rho\ddot{U}_x = \sigma_{xx,x} + \sigma_{xy,y}, \tag{21}$$

$$\rho\ddot{U}_y = \sigma_{yx,x} + \sigma_{yy,y}, \tag{22}$$

$$\mu\ddot{u}_x = k_1\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2}\right) + (k_2+k_3)\frac{\partial}{\partial x}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) - (p - s_{ik}U_{i,k})\frac{\partial \Phi}{\partial u_x}, \tag{23}$$

$$\mu\ddot{u}_y = k_1\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2}\right) + (k_2+k_3)\frac{\partial}{\partial y}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) - (p - s_{ik}U_{i,k})\frac{\partial \Phi}{\partial u_y}. \tag{24}$$

Here components of a stress tensor are expressed through gradients of macroshifts $U_{i,k}$ and microshifts $u_{i,k}$ as follows

$$\begin{aligned}
\sigma_{xx} &= \lambda\theta + 2\mu\frac{\partial U_x}{\partial x} - s_{11}\Phi(u_R),\\
\sigma_{yy} &= \lambda\theta + 2\mu\frac{\partial U_y}{\partial y} - s_{22}\Phi(u_R),\\
\sigma_{xy} &= \mu\left(\frac{\partial U_x}{\partial y} + \frac{\partial U_y}{\partial x}\right) - s_{12}\Phi(u_R).
\end{aligned} \tag{25}$$

For plane deformation

$$u_R = \sqrt{u_x^2 + u_y^2},$$

and the components of a vector of microshifts are measured in units of a constant lattice.

Gradients of macroshifts can be expressed through components of a stress tensor from Eqs. (25). Then the effective potential barrier of $p - s_{ik}U_{i,k}$ takes the form

$$p - s_{ik}U_{i,k} = p\left[p_1 + 2p_2(1 - \Phi(u_R))\right], \tag{26}$$

where

$$p_1 = 1 - 2p_2 - \frac{1}{2\mu p}\left(s_{ik}\sigma_{ik} - \frac{\lambda}{2(\lambda+\mu)}s_{ii}\sigma_{kk}\right), \tag{27}$$

$$p_2 = \frac{1}{4\mu p}\left(s_{ik}s_{ik} - \frac{\lambda}{2(\lambda+\mu)}s_{ii}s_{kk}\right). \tag{28}$$

For identical indexes summation is meant, $(i, k) = (1, 2)$.

The parameter p_2 is a constant. It is defined by only material properties of the medium. The coefficient p_1 depends on a stress of medium. Let's consider stress as smoothly changing function. In this case p_1 can consider as a constant

$$p_1 = \text{const}. \tag{29}$$

Besides, we will accept that $k_2 + k_3 = 0$. With taking into account the made assumptions the equation for microshifts take the form

$$\frac{\partial^2 u_x}{\partial \tau^2} = \frac{\partial^2 u_x}{\partial \xi^2} + \frac{\partial^2 u_x}{\partial \eta^2} - \frac{u_x}{u_R} V'(u_R), \tag{30}$$

$$\frac{\partial^2 u_y}{\partial \tau^2} = \frac{\partial^2 u_y}{\partial \xi^2} + \frac{\partial^2 u_y}{\partial \eta^2} - \frac{u_y}{u_R} V'(u_R). \tag{31}$$

Here

$$\xi = \frac{x}{l_0}, \quad \eta = \frac{y}{l_0}, \quad \tau = \frac{t}{T}, \quad T = \sqrt{\frac{\mu}{p}}, \tag{32}$$

$$V'(u_R) = \left[p_1 + 2p_2(1 - \Phi(u_R))\right] \Phi'(u_R), \tag{33}$$

and the prime denotes a derivative with respect to argument.

Microshifts we will seek in the form

$$u_x = \rho \cos\psi, \quad u_y = \rho \sin\psi, \quad \rho = \rho(\xi, \eta, \tau), \quad \psi = \psi(\xi, \eta, \tau). \tag{34}$$

Substituting (34) in (30), (31) we can see that the equations for microshifts will be fulfilled if ρ and ψ are the solution of system of the equations

$$\frac{\partial^2 \psi}{\partial \tau^2} = \frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2}, \tag{35}$$

$$\left(\frac{\partial \psi}{\partial \tau}\right)^2 = \left(\frac{\partial \psi}{\partial \xi}\right)^2 + \left(\frac{\partial \psi}{\partial \eta}\right)^2, \tag{36}$$

$$\frac{\partial \psi}{\partial \tau}\frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \xi}\frac{\partial \psi}{\partial \xi} + \frac{\partial \rho}{\partial \eta}\frac{\partial \psi}{\partial \eta}, \tag{37}$$

$$\frac{\partial^2 \rho}{\partial \tau^2} = -V'(\rho) + \frac{\partial^2 \rho}{\partial \xi^2} + \frac{\partial^2 \rho}{\partial \eta^2}, \tag{38}$$

The system (35), (36) can be solved if to use a method of construction of functionally invariant solutions [12]–[16].

The function ψ we will seek in the form of arbitrary function of an ansatz $\alpha(\xi, \eta, \tau)$

$$\psi = F_1(\alpha). \tag{39}$$

Ansatz $\alpha(\xi, \eta, \tau)$ is a root of equation

$$\xi\, l(\alpha) + \eta\, m(\alpha) - \tau p(\alpha) + g(\alpha) = 0. \tag{40}$$

The coefficients $l(\alpha), m(\alpha), p(\alpha), g(\alpha)$ are arbitrary functions of α. Function ψ will satisfy simultaneously to Eqs. (35)–(38), if

$$l^2(\alpha) + m^2(\alpha) = p^2(\alpha) \tag{41}$$

Eq. (38) is the nonlinear Klein-Fock-Gordon equation. Its solution we will seek in the form of complex function

$$\rho = \rho(W), \qquad W = W(\xi, \eta, \tau). \tag{42}$$

The type of function ρ is defined by choice the function W. Let's accept that $W(\xi, \eta, \tau)$ simultaneously satisfies the wave equation

$$\frac{\partial^2 W}{\partial \xi^2} + \frac{\partial^2 W}{\partial \eta^2} - \frac{\partial^2 W}{\partial \tau^2} = 0 \tag{43}$$

and eikonal type equation

$$\left(\frac{\partial W}{\partial \xi}\right)^2 + \left(\frac{\partial W}{\partial \eta}\right)^2 - \left(\frac{\partial W}{\partial \tau}\right)^2 = 1. \tag{44}$$

Then $\rho(W)$ has to satisfy the automodel nonlinear ordinary differential equation

$$\frac{d^2 \rho}{dW^2} = V'(\rho), \tag{45}$$

which has a first integral. Function ρ is given as inversion of integral

$$\int \frac{d\rho}{\sqrt{E + V'(\rho)}} = \pm 2(W - W_0), \tag{46}$$

Here E, W_0 are constants of integration.

The function $W(\xi, \eta, \tau)$ can be taken in the form

$$W(\xi, \eta, \tau) = a_1 \xi + a_2 \eta - \sigma \tau + F_2(\alpha), \tag{47}$$

where (a_1, a_2, σ) are constants, and $F_2(\alpha)$ is an arbitrary function of ansatz $\alpha(\xi, \eta, \tau)$. Function $W(\xi, \eta, \tau)$ will be solution of Eqs (43), (44), if the conditions (41) are fulfilled and as addition

$$a_1 l + a_2 m = \sigma p. \tag{48}$$

From Eqs. (41), (48) one obtains $l(\alpha)/p(\alpha)$ and $m(\alpha)/p(\alpha)$. If the found relations to substitute in (40) and to accept that $g(\alpha)/p(\alpha) = -\alpha$, then we will find

$$\alpha = \xi \cos(\omega \pm \delta) + \eta \sin(\omega \pm \delta) - \tau, \tag{49}$$

where

$$a_1 = \sqrt{1 + \sigma^2} \cos \omega, \qquad a_2 = \sqrt{1 + \sigma^2} \sin \omega, \qquad \operatorname{tg} \delta = \frac{1}{\sigma}. \tag{50}$$

The function $W(\xi, \eta, \tau)$ can be chosen another way. Let $W(\xi, \eta, \tau)$ simultaneously satisfies the nonlinear differential equations

$$\frac{\partial^2 W}{\partial \xi^2} + \frac{\partial^2 W}{\partial \eta^2} - \frac{\partial^2 W}{\partial \tau^2} = W, \tag{51}$$

$$\left(\frac{\partial W}{\partial \xi}\right)^2 + \left(\frac{\partial W}{\partial \eta}\right)^2 - \left(\frac{\partial W}{\partial \tau}\right)^2 = W^2. \tag{52}$$

Then function ρ has to be the solution of the nonlinear ordinary differential equation

$$W^2\rho'' + W\rho' = -V'(\rho). \tag{53}$$

Eq. (53) is reduced to the equation of a nonlinear pendulum (45) by replacement of variable

$$\zeta = \ln W. \tag{54}$$

In this case

$$W(\xi,\eta,\tau) = e^{\varphi}F_3(\alpha), \qquad \varphi = a_1\xi + a_2\eta - \sigma\tau, \tag{55}$$

where $F_3(\alpha)$ is an arbitrary function of ansatz $\alpha(\xi,\eta,\tau)$. One can convince by direct calculation that the found functions $\rho(\xi,\eta,\tau)$ and $\psi(\xi,\eta,\tau)$ satisfies Eq. (37). It means that they satisfy system Eqs. (35)–(38) and also are the solution of the equations of microfields (30) and (31).

Solutions of Eq. (45) are in detail considered in [7]–[11]. It is established that depending on parameters (p_1, p_2) the function ρ changes periodically, aperiodically, looks like the localized perturbations like kinks or solitons. The microshifts u_x, u_y and the module u_R, which are solutions of the system (30), (31) for $\Phi(u_R) = 1 - \cos(u_R)$, in the form (34), (39), (47) with $F_1(\alpha) = \operatorname{arctg}(\alpha)$, $F_2(\alpha) = 1/\operatorname{ch}(\alpha)$, $\sigma = 1$, $W_0 = 0$, $\omega = 0$, are shown on figures 1, 2. For Fig. 1 as the solution of (45) the function

$$\rho = 2\operatorname{arctg}\,\exp\big(-\sqrt{2p_2}\,W\big) \tag{56}$$

is chosen. Fig. 2 corresponds to the choosing

$$\rho = 2\operatorname{arctg}\frac{\sqrt{\frac{2p_2}{|p_1|}-1}}{\operatorname{ch}\sqrt{2p_2-|p_1|}\,W}. \tag{57}$$

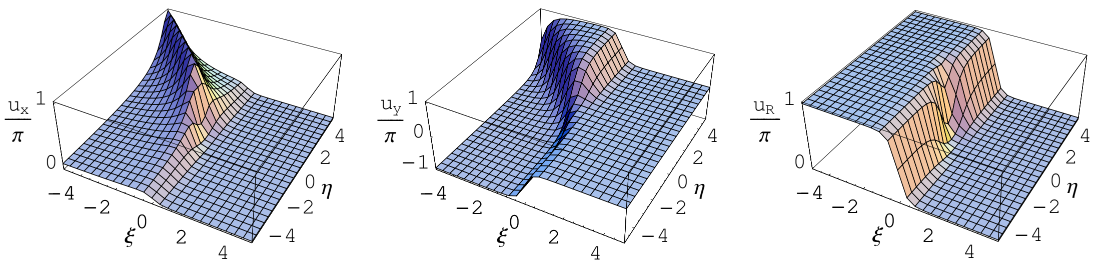

Figure 1: Microshifts u_x, u_y, and u_R for the case (56) and $p_1 = 0$, $p_2 = 4.5$.

3 Problem of inversion of hyperelliptic integrals

The large number of works is devoted to research of the integral

$$I = \int R(x,y)\,dx, \tag{58}$$

where R is a rational function of two variables, and $y = \sqrt{P_n(x)}$; $P_n(x)$ is polynom of degree n which doesn't have multiple roots. If n is the even number ($n = 2\rho + 2$), this

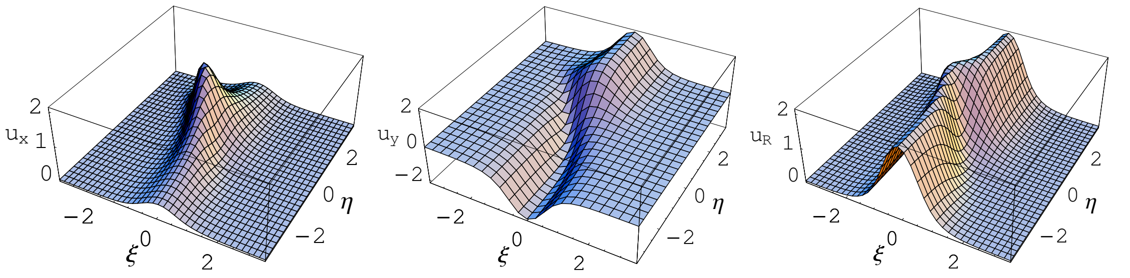

Figure 2: Microshifts u_x, u_y, and u_R for the case (57) and $p_1 = -1$, $p_2 = 2$.

integral can be transformed by an rational substitution to another integral of the same form but with a polynom of the odd degree $n = 2\rho + 1$ under the square root. Only one root of the polynom $P_n(x)$ is enough to be known for this purpose. Remarkable property of integral (58) consists that his behavior depends not so much on function $R(x, y)$, how many from degree n. If $n = 2$, then the integration (58) leads to calculation of Euler's integrals. For $n = 3, 4$, elliptic integrals take place; $n = 5, 6$ — ultraelliptic; $n > 6$ — hyperelliptic.

Depending on function $R(x, y)$ the integral (58) is reduced to integration of a rational function and calculation of integrals of the types

$$I_{12} = \int \frac{Q_m(x)}{\sqrt{P_{2n+1}(x)}}dx, \qquad I_3 = \int \frac{1}{(x-a)\sqrt{P_{2n+1}(x)}}dx. \tag{59}$$

Here a is constant, and $Q_m(x)$ is a polynom of degree m. If $m < n - 1$, then I_{12} is the integral of the first kind. It converges for all values x from $-\infty$ to $+\infty$. If $m > n$ — the integral of the second kind. He approaches to ∞, if $x \to \infty$. The integral I_3 belongs to the third kind. It has logarithmic infinity in a point $x = a$. The important characteristic of ultra- and hyperelliptic integrals is a genus p. He is determined by the polynom's degree n. For even degree $p = (n-2)/2$, for odd degree $p = (n-1)/2$.

At researches of integral (58) the main efforts were directed on a solution of the problem of the inversion. This problem was solved by outstanding mathematicians of the past and their followers. Gauss, Abel and Jacobi established the formulas of inversion of the elliptic integrals [17]. Jacobi constructed the theory of theta functions of one complex variable and found formulas for inversion of elliptic integrals with their help. The solution took place in a class of doubly-periodic functions. Jacobi's ideas were developed by Göpel and Rosenhain for ultraelliptic integrals of the first kind [18], [19]. They introduced the theta function of two complex variables into the analysis, investigated their algebraic and differential properties and showed that formulas of the inversion of ultraelliptic integrals can be expressed through the relations of the introduced theta functions. Solution took place in the class of functions with four period. Weierstrass [20] constructed the theory of theta functions of many complex variables and showed that a problem of the inversion of the hyperelliptic integral of a genus p can be solved with help of theta functions of p complex variables. Solution took place in the class of $2p$-periodic functions.

Riemann developed [21] essentially another method of the inversion of the hyperelliptic integrals. He gave geometrical interpretation of the many-valued functions in a form of multibranch surfaces (Riemann surfaces) and developed also the theory of inversion of the hyperelliptic integrals by calculations of the integrals on the special contours. Prym developed ideas of Riemann in relation to ultraelliptic integrals [22]. In further Klein developed [23] the method Jacobi, Weierstrass, and Riemann. The researches of the reduction (i.e. finding of conditions at which performance the hyperelliptic integrals of a genus of p will be transformed to the hyperelliptic integral of a smaller genus) is taken the important place in the theory of the hyperelliptic integrals. These researches are interest to applied tasks.

Coming back to studying of dependence of nature of perturbations from a type of potential of interaction of sublattices $\Phi(u_R)$, we come to a conclusion that addition of a new harmonic to $\Phi(u_R)$ transfers integral (58) of genus p in integral of a genus $p + 1$. However, if the reduction is possible, the genus can not only increase, but also to decrease. It is possible when certain ratios between amplitudes of harmonicas take place. Research of these conditions represents undoubted physical interest. Unfortunately, in literature there is no available statement of algorithm of the inversion of the hyperelliptic integrals that doesn't allow to use the modern computing technologies.

4 Conclusion

In classical mechanics of continuous mediums the plane tasks take a special place. The effective mathematical methods based on achievements of the theory of functions of a complex variable has been developed for research of the plane deformation. These methods allowed to find exact analytical solutions of many problems, which are important for practical appendices. Problems were solved about distribution of stress near lines of cut, cracks, boundaries of the section of different mediums etc. The received solutions allowed to formulate criteria of destruction of materials of the different nature and became a basis of modern mechanics of the destruction. However the classical mechanics of continuous mediums proceeds from the assumption of a smallness of shift of sublattices. This hypothesis obviously isn't fair near the cut's lines, the crack mouth etc. The new nonlinear model allows any shifts of sublattices and allows to describe more adequately the physical phenomena proceeding near lines of singularity, such as cardinal reorganization of a lattice, formation of defects of different type, structural and phase transitions, formation of a superlattice, etc. That is why the research of plane deformation on the basis of the nonlinear models are actual and perspective.

Acknowledgements

This work was supported by RFBR, grant 12-01-00521-a.

References

[1] E. L. Aero. Fiz. Tverdogo Tela **42**, (2000) 1113–1119.

[2] E. L. Aero. Uspeki Mekhaniki **1**, (2002) 130–176.

[3] E. L. Aero. Fiz. Tverdogo Tela **45**, (2003) 1482–1490.

[4] M. Born, K. Huang. Dynamical Theory of Crystal Lattices. Oxford Univ. Press, Oxford, 1954.

[5] E. L. Aero, A. N. Bulygin. Vychislitelnaya Mekh. Splosh. Sred. **1**, (2008) 14–30.

[6] A. V. Porubov, E. L. Aero, G. A. Maugin. Phys. Rev. E **79**, (2009) 046608.

[7] E. L. Aero, A. N. Bulygin. Acoustical Phys. **56**, (2010) 811–830.

[8] E. L. Aero, A. N. Bulygin. Int. J. Eng. Sci. **49**, (2011) 1494–1501.

[9] E. L. Aero, A. N. Bulygin. Continuum Mech. Thermodyn. **23**, (2011) 35–43.

[10] E. L. Aero, A. N. Bulygin, Yu. V. Pavlov. Mater. Phys. Mech. **15**, (2012) 126–134.

[11] E. L. Aero, A. N. Bulygin, Yu. V. Pavlov. Mater. Phys. Mech. **11**, (2011) 137–156.

[12] H. Bateman. The Mathematical Analysis of Electrical and Optical Wave-Motion on the Basis of Maxwell's Equations. Cambridge Univ. Press, Cambridge, 1915.

[13] S. L. Sobolev. Tr. Fiz.-Mat. Inst. Steklova, **5**, (1934) 259–264.

[14] E. L. Aero, A. N. Bulygin, Yu. V. Pavlov. Teor. Matem. Fiz. **158**, (2009) 370–377 [English transl.: Theor. Math. Phys. **158**, (2009) 313–319].

[15] E. L. Aero, A. N. Bulygin, Yu. V. Pavlov. Nelineinii Mir **7**, (2009) 513–517.

[16] E. L. Aero, A. N. Bulygin, Yu. V. Pavlov. Differentsial'nye Uravneniya **47**, (2011) 1428–1438 [English transl.: Differential Equations **47**, (2011) 1442–1452].

[17] A. I. Markushevich. Introduction to the Classical Theory of Abelian Functions. Nauka, Moscow, 1979.

[18] A. Göpel. J. Reine Angew. Math. **35**, (1847) 277–312.

[19] G. Rosenhain. Mémoires des savants étrangers **9**, (1851) 361–468.

[20] K. Weierstrass. J. Reine Angew. Math. **47** (1854) 289–306.

[21] B. Riemann. J. Reine Angew. Math. **54** (1857) 115–155.

[22] F. Prym. Neue Theorie Ultraelliptischen Functionen. Mayer & Müller, Berlin, 1885.

[23] F. Klein. Gesammelte Mathematische Abhandlungen. Springer, Berlin, 1923.

E. L. Aero, A. N. Bulygin, Yu. V. Pavlov,
Institute of Problems in Mechanical Engineering, RAS,
61 Bol'shoy, V.O., Saint Petersburg, 199178, Russia