Behavior of neutral buoyancy solid in cavity with liquid under rotational vibration

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Abstract

The dynamics of a cylindrical body in a horizontal annulus with a longitudinal partition filled with a viscous fluid and subject to high-frequency rotational oscillations is experimentally investigated. The experiments are performed with “light” and “heavy” bodies, the density of which is only slightly different from the density of the liquid. With increase of the vibration intensity the repulsion from the boundary and suspension of light body near the upper boundary of the layer (the heavy one – near the bottom) is found. The frequency and the amplitude of vibration vary in the experiments. It is shown that vibrational suspension of the body in the gravity field is not associated with inertial oscillations of the body relative to the fluid but is determined by shear oscillations of the liquid itself and viscous interaction of solid with the cavity boundary.

1 Introduction

Occurrence of the averaged vibration lift force acting on a rigid body in the cavity subjected to rotational oscillations was studied experimentally and theoretically in [1]. Experiments were carried out with a heavy cylinder in the simply connected coaxial gap filled with a viscous fluid, and performing rotational oscillations around a horizontal axis of symmetry. In experiments with a spherical body [2] it was found that under the influence of vibrations the body may be in quasi-equilibrium state near both the outer and the inner layer boundaries. In [3] while studying the averaged dynamics of a heavy cylinder in the cavity with liquid the attention was paid to the elucidation of the nature of the azimuthal displacement of the body as well as the role of viscous hydrodynamic interaction of the body with the cavity boundaries. At the theoretical treatment of the listed problems the oscillations of fluid and body are assumed to be high-frequency, while the boundary layers negligibly thin, and the fluid is regarded as a non-viscous one.

The present paper is devoted to the experimental study of the cylindrical body dynamics in the liquid-filled annulus with a partition and subjected to high-frequency rotational vibrations. Density difference between the body $\rho_S$ and the fluid $\rho_L$ is small and varies: experiments are performed with both the “light” ($\rho_L > \rho_S$) and “heavy” ($\rho_L < \rho_S$) cylinders.

2 Experimental technique and procedure

Cavity 1 (Fig.1) is filled with liquid and mounted on the plate of mechanical vibrator; the cylinder 2 is placed in the cavity. The mechanical vibrator with crank and connecting rod mechanism is used [4], it imparts to the cavity the periodic rotational oscillations according the law $\varphi = \varphi_0 \cos(\Omega t)$. The angular amplitude varies in the range $\varphi_0 = (0.01 – 0.8)$ rad
and is measured by an optical cathetometer with accuracy 0.001 rad. The frequency of vibration varies in the range \( f \equiv \Omega/2\pi = (1 - 40) \) Hz and is measured by digital tachometer with accuracy 0.05 Hz.

The cavity is formed by two cylindrical surfaces. The radii of the internal and external boundaries of the layer are \( R_1 = 2.7 \) cm and \( R_2 = 4.5 \) cm, the layer thickness cm, depth -2.5 cm. The front wall of the cavity is sealed with a transparent Plexiglas lid that allows the visual observations and photo- and video registration. Inside the layer there is a rigid impermeable partition \( 3 \) forcing the liquid to oscillate together with the cavity. Air bubbles and other inclusions (except the cylindrical solid) are absent inside the cavity.

The kinematic viscosity of the water-glycerol solutions, used as the working fluid, varies in the range \( \nu = 6.0 - 18 \) cSt by changing the glycerol concentration; it leads to the variation of liquid density: \( \rho_L = 1.14 - 1.24 \) g/cm\(^3\). The density of water-glycerol solution could be also increased by addition of salt.

A body is an ebonite cylinder of circular cross-section of diameter \( d = (4.50 \pm 0.02) \) mm or \( (2.95 \pm 0.01) \) mm. The length of the bodies is the same \( l = (23.29 \pm 0.03) \) mm, the density is \( \rho_S = 1.18 \) g/cm\(^3\). The relative density of the solid \( \rho \equiv \rho_S/\rho_L \) could be greater or smaller than unit due to the change of liquid density. Main experiments were carried out with the “heavy” body of density \( \rho = 1.02 \) and the “light” one, \( \rho = 0.95 \).

In the absence of vibrations the “light” cylinder is situated near the ceiling of the cavity (Fig.1a), the “heavy” one – near the bottom (Fig.1b); with increase of the vibrational frequency there is a gap between the body and the boundary of the layer.

Visual observations are made under continuous and stroboscopic illumination. The high resolution digital camera Canon EOS600D is used for photo and video-registration. Vibrational dynamics of the body relative to the cell is studied by the method of high-speed video registration. Measurement and processing of experimental data are performed using the specialized software and computer.

3 Experimental results

“Heavy” body

Let us dwell on the dynamics of a cylindrical body the density of which is slightly higher than the liquid density, \( \rho = 1.02 \). In the absence of vibrations the cylinder is situated near the lower (external) cavity boundary (Fig.2a). With a smooth increase of the vibrational frequency \( f \) (amplitude \( \varphi_0 \) is fixed) a gap \( h \) appears between the “heavy” body and the
bottom of the cavity \((b)\). Further rise of the vibrational frequency results in the increase of the gap width and the threshold transition of the body to the internal boundary of the layer \((c)\). Reverse transition of the body occurs with decrease of \(h\).

![Fig.2](image1)

![Fig.3](image2)

The variation of the gap width \(h\) between the body and the cavity boundary with the change of vibration frequency is shown in Fig.3. The value of \(h\) is measured using the photographs obtained at each step of frequency change.

Here and further dark points on the graph correspond to the results obtained at the frequency rising (direct course), light points - with decreasing \(f\) (reverse course). Arrows indicate the threshold transitions of the body to the inner boundary layer and return to the lower (outer) boundary. The gap between the body and the lower layer boundary increases monotonically with increasing the frequency of vibrations (at a certain value of \(\varphi_0\)). Just before transition of the body to the upper border the gap size is comparable to the body size. After the transition of the cylinder to the ceiling the gap between the body and the cavity boundary persists, and with a further increase in \(f\) it decreases, but does not disappear. Threshold frequency of the body return to the bottom of the cavity is much smaller than the frequency at which the cylinder moves to the inner boundary. This means that for retaining the body in quasi-equilibrium state near the top of the cavity vibration of lower intensity are required.

The increase of the amplitude of angular oscillations \(\varphi_0\) leads to the shift of the curves to the lower frequencies (Fig.3). So the curve \(\varphi_0 = 0.12\) rad corresponds to larger values of the frequency \(f\), than the curve obtained in experiments with the same liquid \((\nu = 18\) cSt) at \(\varphi_0 = 0.16\) rad.

In experiments with liquids of different viscosity for a given value of the amplitude of angular oscillations \(\varphi_0 = 0.16\) rad, characteristic distance over which the body departs from the layer wall decreases with decreasing viscosity. At this the critical value of the frequency of vibration corresponding to the threshold transitions of the body for the fluids of lower viscosity are higher.

"Light" body

The experiments are carried out with the same bodies \((d = 2.95\) mm and \(4.50\) mm, \(l = 23.29\) mm, \(\rho_S = 1.18\) g/cm\(^3\)). However, the addition of salt in the water-glycerol solution allowed to increase the liquid density and decrease the relative density of the
body up to the value $\rho = 0.95$.

The vibrational dynamics of the “light” body is similar to the “heavy” body dynamics. Typical positions of the cylinder with change of vibration frequency are shown in Fig.4a,b. The difference from the “heavy” body dynamics is that at any vibration frequency the “light” body does not move to the internal boundary of the layer.

The change of the distance between the cylinder and the cavity ceiling with vibration frequency at $\nu = 18$ cSt and different amplitudes $\varphi_0$ is presented in Fig.5. With increase of vibration amplitude the curves shift to lower values of $f$. At the same time, the gap width $h$ is growing with amplitude. In all experiments there is a maximum value of $h$ which remains constant with further increase of vibration frequency.

4 Discussion

The dimensionless parameter $\text{Wr} = (\varphi_0 \Omega)^2 R_0 / g$ which characterizes the dimensionless lift force (ratio of the vibrational lift force to gravity) was introduced in [1], where $R_0 = (R_1 + R_2)/2$ is mean curvature radius, $g$ is gravity acceleration.

The experiments are carried out over a wide range of dimensionless frequency $\omega = \Omega d^2 / \nu = 10 - 340$. The gap width between the oscillating body and the cavity boundary characterizes the radius of action of the repulsion force, which is determined by viscous interaction of the body with the boundary and manifests itself at a distance comparable to the thickness of Stokes layer $\delta = \sqrt{2 \nu / \Omega}$.

Near the cavity bottom the lift force acts on the cylinder, expulsing the body outside the Stokes layer. Regardless of the frequency $\omega$, the body goes away from the cavity bottom at the same maximal distance $h/\delta \approx 4$ (Fig.6), above which it goes to the internal boundary of the layer. The curve corresponding to the amplitude $\varphi_0 = 0.16$ rad is shifted to lower values of $\text{Wr}$ than the curves corresponding to $\varphi_0 = 0.12$ rad.

After the body transition to the upper cavity boundary, between the latter and the body there is a gap comparable to Stokes layer thickness that is caused by the repulsion force which does not allow the cylinder to come close to the wall. The gap width is practically constant $h/\delta \approx 2.5$ for different vibration amplitudes.

With $f$ decrease the gap width increases, then the cylinder falls down and comes back to the bottom. The distance from the ceiling at which the body falls down is about $h/\delta \approx 4$.

In experiments with the “light” body ($\rho = 0.95$) the characteristic distance over which the cylinder moves from the cavity ceiling does not exceed the value $h/\delta \approx 3$ (Fig.7). The transition from one wall to another was not found in the experiments. Results obtained at different $\varphi_0$ repeat each other and practically coincide on the plane, $\text{Wr}, h/\delta$ (the liquid viscosity and the size of the body did not change in the experiments).

In Fig.8 the trajectories of oscillating movement of “heavy” body in the laboratory frame are presented. The curves are obtained from time-lapse video processing and analysis of
vibrating cavity using high-speed video camera Basler A402k. The vibrations of the cylinder relative to the cavity are studied at a given amplitude $\varphi_0 = 0.16$ rad and different frequencies of vibration $f$. 

At low intensity of vibration ($f = 2.2$ Hz) the body oscillates continuously touching the lower boundary of the cavity. After the separation from the surface ($f = 13.4$ Hz) body oscillates being at a constant distance from the wall during the entire period ($h/\delta \approx 3$). The unit of distance between the body and the wall the thickness of Stokes layer $\delta$ is selected, as a unit of length $\tau$ tangential displacement of the body - its diameter; tangential displacement is calculated as follows $\tau \equiv \beta(R_2 - d/2)$, where $\beta$ - angular coordinate of the body axis. After the transition of the cylinder to the ceiling ($f = 19.9$ Hz) the gap between the body and the boundary of the cavity persists ($h/\delta \approx 3$) and remains un-changed during the oscillations of the body.

Tangential displacement of body relative to the cavity is defined by three components: 1 – tangential displacement of the cavity itself; 2 – displacement of fluid relative to the cavity [1]; 3 – inertial offset of body relative to the fluid, which in this case is negligible due to the small difference in densities of the fluid and the body. Thus, the dimensionless amplitude of the horizontal oscillations of “heavy” body $\tau/d$ relative to the cavity (Fig.8) varies depending on the distance between the cylinder and the cavity wall.
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Trajectory of the “light” cylinder oscillations before its separation from the cavity ceiling \( (f = 2.2 \text{ Hz}) \) and after its repulsion from the upper layer boundary \( (f = 13.4 \text{ Hz}) \) are shown at Fig.9. Behavior patterns of “light” and “heavy” cylinders are similar. At this the characteristic value of the dimensionless gap between the cavity ceiling and the body, after its separation from the border, is less \( (h/\delta \approx 1.5) \) than in the case of “heavy” body (Fig.8). With increasing distance from the top of the cylinder the amplitude of horizontal oscillation of the body gradually reduces.

In [1] the dimensionless amplitude of the body oscillations with respect to the liquid is represented by the expression
\[
B = \left(\frac{(\rho - 1)}{(\rho + 1)R_0 \varphi_0}\right)/d,
\]
where the amplitude of displacement is a theoretical value, which is valid for a potential flow around the cylinder located in the middle of the layer. Fig.10 presents the threshold curves of the “heavy” body transitions in dependence on \( B \). The dark points correspond to the direct transitions of the body: the detachment from the bottom \( (1) \), the transition to the internal boundary of the layer \( (2) \); the corresponding light points indicate the reverse transitions. The increase of the dimensionless amplitude of vibration leads to the rise of the threshold values of parameter \( W_r \). A hysteresis is observed between the body transitions from one boundary to another; the depth of hysteresis is not constant in the studied range of \( B \).

![Fig.10](image)

The condition of quasi-equilibrium of the cylinder at an arbitrary distance from the axis of vibration can be written as \( (1) \): \( W_r = 2(\rho + 1)(R/R_0)/3[2 - (R/R_0)]^2 \).

It is evident from \( (1) \) that the quasi-equilibrium state of the cylinder is determined by the distance from the cylinder to the axis of vibration: the larger value of \( R \) requires the larger \( W_r \) in order to maintain the body in suspended state. This proves that the transitions of the body to the internal boundary and back should occur abruptly and with hysteresis, that is consistent with the experimental results. Threshold values of vibrational parameter \( W_r \) calculated according \( (1) \) and corresponding to the transitions of the “heavy” cylinder of relative density \( \rho = 1.02 \) are shown in graph by points 3. Taking into account that in the extreme positions of the body its center of inertia is at distance \( d/2 \) from the walls we get the value \( W_{r_1}^* = 2.2 \) for lifting the body from the bottom, and the value \( W_{r_2}^* = 0.9 \) for the reverse transition. The theoretical and experimental values of both transitions are in satisfactory agreement in the limit of small amplitudes \( B \rightarrow 0 \).

Parameter \( W_r \) is valid in the limiting case of high frequencies and characterizes the interaction between the body and the oscillating flow of liquid when the cylinder is far from the boundaries of the layer. Since the transitions occur near the external and internal cavity walls, the latter may be expected to influence even in the high-frequency limit. In viscous liquids, with growth of Stokes layer thickness, the influence of the walls on the
body dynamics increases significantly because of increase of hydrodynamic interaction.

Behavior of the body near the wall is characterized by the dimensionless vibrational parameter $W = (bΩ)^2/(gd)$ [1]. In case of rotational vibration the parameter $W = (ϕ_0 RΩ)^2/(gd)$ is defined as, where $R = (R_2 - d/2)$ – the distance from the axis of rotation to the center of the body near the external or internal layer boundary.

Thresholds of retention of “light” and “heavy” body near the cavity border (Fig.11) are practically independent on the dimensionless amplitude of the of the body oscillations $B$.

Hysteresis in the transitions was not found: thresholds of separation and return of the body coincide within the confidence interval determined by a discrete frequency step. The characteristic values of the parameter $W$, necessary to keep the “light” body near the cavity ceiling ($W \approx 3.5$), is larger than the threshold value of retention of “heavy” body near the bottom ($W \approx 2.1$).

Experimental study of vibrational suspension of a body near the cavity wall versus the relative density in case of translational vibrations [1] revealed a non-monotonic dependence of the vibrational lift force on $ρ$ (Fig.12). The dashed curve on the graph is a theoretical one and corresponds to the limiting case $ω \to \infty$. At $ρ \sim 1$ the critical value of $W$ rapidly increases and tends to infinity, meanwhile the vibrational lift force vanishes. Under the translational vibrations it is impossible to repulse from the cavity boundary the body whose density is close to the liquid one. This is because the lift force is generated due to the body oscillations relative to the cavity, the amplitude of which depends on the difference of liquid and the body densities.

The results of studies of the dynamics of cylindrical bodies with a density significantly different from the density of the fluid ($ρ = 2.22$ and $0.47$), with rotational vibration of the cavity are presented in [3], [6]. In these studies, the threshold of return of the body to the boundary of the cylindrical layer (Fig.12, points 2) agrees satisfactorily with the same threshold at the translational vibrations.

The experiments with the bodies of almost neutral buoyancy ($ρ = 0.95$ and $1.02$) carried out in a frame of this work revealed different results. Threshold of the suspending of such bodies (Fig.12, points 3,4) near the cavity walls was found in the region of small $W$ values. This suggests the occurrence of additional vibration repulsive force of a different nature. This force is not associated with inertial oscillations of the body relative to the liquid (as in case of translational vibrations), but is due to the oscillations of the liquid itself relative to the boundary caused by rotational cavity vibrations.

**Conclusion**

The vibrational dynamics of the cylindrical body of practically neutral buoyancy in the annulus with the impermeable longitudinal partition filled with a viscous fluid and subjected to high-frequency rotational oscillations is experimentally studied. The investigations were performed with the “heavy” and “light” solid bodies.
With increase of the frequency of vibration the threshold repulsion of the “heavy” body from the cavity bottom (the “light” one – from the ceiling) is found at a critical intensity of cavity oscillations. Further intensification of vibrations results in the transition of the “heavy” cylinder to the internal boundary of the layer. In case of “light” body the transition to the internal boundary is not observed. Repulsion and return of body to the border with increasing vibration and weakening occur in a critical way.

Threshold transitions of the body, obtained with different angular amplitude of cavity oscillations are in satisfactory agreement on the plane of the control dimensionless parameters. Meanwhile, the thresholds of the vibrational parameter $W$ at rotational vibrations are substantially smaller than in the experiments with the bodies of higher density [3]. The importance of fluid shear oscillations associated with the nontranslational character of the cavity oscillations in generating of the averaged lift force near the wall is concluded.

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References


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