Localization in Mechanics and Nature

Vladimir A. Babeshko    Olga V. Evdokimova    Olga M. Babeshko
Dmitry V. Grishenko
babeshko41@mail.ru, evdokimova.olga@mail.ru, babeshko49@mail.ru, infocenter@kubsu.ru

Abstract

It has been established that it is convenient to describe a number of natural anomalous processes, which consist in localizing the energy within certain spatial regions, as a result of manifestation of a natural virus. The natural virus makes it possible to reveal the conditions of occurrence of anomalies and, thus, to search for various ways of decreasing fatal natural processes. Introduction of natural virus is a result of detecting new regularities in the natural processes, which are reasonably adequately describes by mixed boundary-value problems in a great variety of fields. It is convenient to represent these regularities by reproducing the concept of a natural virus, and to interpret the occurrence of anomalies as its manifestation at different levels up to an anomalous state of the natural processes under consideration.

Thus, natural viruses are some kind of indicators of the state of natural processes, when the cause and effect relation of such a state given in terms of physical parameters of the process is mathematically described reasonably well instead of revealing only qualitative and certain quantitative estimates of the process.

It can assert that the distribution of the natural-process energy is reasonably uniform in a region when the virus does not manifest itself. As the virus manifests itself, the process energy is localized in one or several zones of the region. The degree of localization depends on the virus-manifestation level. At the limit, the localization results in anomalous behavior of the natural process in the indicated zone or outside of it depending on the natural-process type. As examples of such processes, we can mention the following: waterspouts (tornadoes), when the energy originally distributed in a sufficiently large region is localized within the zone of the narrow whirlwind cylinder – the tornado trunk; storm phenomena accompanied by intense chaotic motion of air and water masses in a certain zone and a significant calmness outside of it; the occurrence of considerably directed displacement in the contact zones on the deformed-medium surface and their smallness outside of the zone; and the localization of temperatures directly related to the localization by the thermal energy. There are other examples as well.

It succeeded in conducting the investigation with the use of the block-element method, which enabled us to establish new mathematical relations describing the anomalous natural regularities.

We found the conditions of manifestation of the virus and its properties from the example of reasonably widespread natural processes containing the normal natural virus.

1 Localization

Due to the possibility of formalizing certain natural phenomena, it proved to be possible to single out a class of objects called “natural viruses” by constructing the mixed boundary-value problems describing them; these viruses have similar mathematical descriptions, and
it is unique to them to cause an extreme behavior of natural processes. These objects are imperceptible and manifest themselves only under certain conditions for natural processes in which they occurred and for certain values of parameters, as a rule, various. In the cases of unmixed boundary-value problems, no similar properties are revealed. The manifestation of natural viruses often has an atypical character not inherent to conventional process and the possibility of occurrence of which can even give rise to doubts. The vibration-strength viruses introduced in [1] are referred to such objects, which were investigated in reasonable detail in [2, 3, 4, 5, 6, 7]. Their feature consists in wave-process localization and the initiation of resonances in semi-bounded regions in those frequency ranges in which, from the point of view of common sense, they cannot exist. The extension of the manifestation range of this virus failed; most frequently, it was found in layered media of deformable solids [8, 9, 10, 11].

However, the further development of the mathematical apparatus and the creation of the block-element method [12, 13] considerably modified the representation of viruses. The so-called “modified” natural viruses were found, which manifest themselves already in a considerably wider range of natural processes, as well as “degenerate” viruses, the range of existence of which is till wider and especially significant in seismology (local, global). As was already noted, the viruses for which the mathematical description was given by us, certainly, concern only a small fraction of natural cataclysms. It is of importance to note that the viruses singled-out and investigated are of the same type, mathematically identical; however, they are encountered in different natural processes.

A natural virus can be considered as investigated if its mathematical description is given, the mathematical conditions under which it starts to demonstrate itself are determined, and the really achievable behavior of the natural process under conditions can serve as the means for anticipation of virus manifestation, i.e., an anomalous natural phenomenon if it is undesirable, or for its manifestation if it is necessary.

Frequently the conditions of manifestation of the virus have a complex character in which certain factors can participate simultaneously introducing a negligible contribution to the virus-manifestation conditions, but sufficient as a whole for fulfilling these conditions. Therefore, the viruses are frequently imperceptible, and only knowledge of the conditions of their manifestation can explain the cause of one natural cataclysm or another as a consequence of fulfillment of the necessary conditions for the virus.

2 Case of Elasticity

We present the examples of the simplest of them and briefly recall the vibration-strength virus [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. It is formed in layered linearly deformable semi-bounded media for the mixed boundary-value problems, in particular, in the presence of inclusions, cracks, and stamps, and manifests itself in the localization of the wave process in the vicinity of inhomogeneity’s. The localization can be referred both to displacements and to stresses. In the case that the medium is without attenuation, resonances are possible under localization conditions. The virus manifests itself by fulfilling the “staticity relation” [4, 6, 8] connecting the sizes and arrangement of inhomogeneity, the medium parameters of the external action on the semi-bounded solid. For example, in the case of the stamp action on an elastic layer, the boundary-value problem results in a virus of the type [1, 2, 3, 4, 5, 6]

\[
\int_{-a}^{a} k(x - \xi)q(\xi) d\xi = 1, \quad |x| \leq a
\]
\[ k(x) = \frac{1}{2\pi} \int_{-\frac{1}{2}}^{\frac{1}{2}} K(u) e^{-iu \pi} \, du, \quad Q(u) = \int_{-a}^{a} q(x) e^{iux} \, dx \]  

The integral-equation symbol is an even meromorphic function and has a countable number of zeros and poles, which were described in detail in \[2, 3, 4, 5, 6\]. The finite zero and pole numbers are material; others are a complex having asymptotic behavior at infinity of the type.

\[ z_{n}^{\pm} \sim \pm (icn + r_{1} \ln n + O(n^{-\varepsilon})), \]
\[ \xi_{n}^{\pm} \sim \pm (ic(n + 0.5) + r_{2} \ln n + O(n^{-\varepsilon})), \]
\[ c, r_{1}, r_{2}, \varepsilon > 0, \quad n > N, \quad n \to \infty \]

Thus, we write the representation in the form of.

\[ K(u) = \Pi(u)K_{0}(u), \quad \Pi(u) = \prod_{n=1}^{N} \left( u^{2} - z_{n}^{2} \right) \left( u^{2} - \xi_{n}^{2} \right)^{-1} z_{n} > 0, \quad \xi_{n} > 0, \]
\[ n = 1, 2, \ldots, N, \quad K_{0}(u) > 0, \quad K_{0}(u) |u| \to s, \quad |u| \to \infty, \quad s < \infty, \quad Imu = 0 \]

The number of material zeros and poles depends on the stamp-vibration frequency. If the stamp-vibration frequency is lower than the critical frequency; there are no material zeros and poles \[14\]. With increasing frequency, the number of material zeros and poles increases, the material pole appearing first, then, the material zero. For simplicity, we consider the case of \( N = 1 \) at the frequency at which the energy is radiated in a layer. The function has its representation in the form of.

\[ K(u) = \Pi(u)K_{0}(u), \quad \Pi(u) = \left( u^{2} - z_{1}^{2} \right) \left( u^{2} - \xi_{1}^{2} \right)^{-1} z_{1} > 0, \quad \xi_{1} > 0 \]  

\[ n = 1, 2, \ldots, N, \quad K_{0}(u) > 0, \quad K_{0}(u) |u| \to s, \quad |u| \to \infty, \quad s < \infty, \quad Imu = 0 \]

The localization condition, which means that the vibration amplitude for the layered medium outside the region occupied with the stamp decreases exponentially when removed from the stamp, results in the fact that no energy is radiated in the layered-medium-stamp system at infinity in this case.
As is established in the works listed, such behavior of the layered medium is typical for stamps having the sizes.

\[ a_p = \pi p + \theta + O(p^{-\varepsilon}) , \quad \varepsilon > 0 , \quad \theta = \arg K_0^+(z_1) , \quad a_p \gg 1 \]

Here, \( K_0^+(u) \) is the result of factorization of the function, the representations [12, 13].

\[ K(u) = K_0^+(u)K_0^-(u) \]

It is proved in [4, 6] that it is its mass that can be found in this case for which the vibration amplitude under the action of harmonic force is unlimited, which means an unlimited resonance.

The resonant mass in the supercritical region for the chosen for which the staticity relation is fulfilled has the following value [4].

\[ m = Q_0(0)\xi_2 z^{-2} \omega^{-2} , \quad Q_0(0) = \int_{-a}^{a} q_0(x)dx \]  

(5)

The obtained result cardinally modified the accepted opinion that the resonance of a massive stamp is impossible in an elastic layer for frequencies higher than the critical ones since the energy is radiated in the layer to infinity. This resonance is possible for massive stamps with the half-width and the mass. As was told above, this virus was unsuccessfully sought in the boundary-value problems describing various natural processes.

For example, in the case of a mixed static problem for a layered medium in which the integral-equation symbol has no material zeros and poles, there is no vibration-strength virus considered above; at the same time, it is known that extreme situations in such problems and processes take place. Precisely the search for methods of their description resulted in the detection of an already considerably more widespread virus, which was called “natural”, and which contained, in a special case, a vibration-strength virus. Its feature, contrary to the one considered, consists in the ability to localize the process described by the boundary-value problem with the mixed boundary conditions to an arbitrary level set beforehand not necessarily causing resonance. This approach enabled us to single out an anomalous, i.e., ultimately permissible, behavior of the natural processes under consideration, which is called the manifestation of the natural virus because it arises when fulfilling the manifestation condition in correspondence with the mathematical formulation. The vibration-strength virus is a special case of the natural virus.

3 Case of Nature

As the simplest example, we consider the natural virus of a temperature process. It is known that the equations describing the hydrometeorological conditions in the atmosphere of a certain territory represent a complicated set of partial differential equations, the number of parameters of which has a stochastic character. They include temperature, pressure, density, mass-transfer speeds, and other special parameters, as well as the characteristics of external actions. In the general case, they are accessible to analysis with computer facilities. Certainly, this boundary-value problem being formulated with the mixed boundary conditions contains the natural virus. However, cases are known when zones of anomalously high or anomalously low temperature are established under conditions of significant atmospheric pressure, while quite normal meteorological conditions take place in neighboring zones.

We present a simple example of a natural virus, which is contained in the mixed boundary-value problem for the heat-conduction equation describing the state of the atmosphere under anticyclonic conditions, which resulted in considering this boundary-value problem.
In the assumptions described above, we investigated the problem of localization of temperature in certain zones caused by the seasonal change in the solar action at the surface. In recent years, such phenomena have become repeating, steady, and extended. There is no explanation of the causes of such a phenomenon. We have only general assumptions about the effect of various factors, mainly of the exogenous nature of the origin. In this study, this phenomenon may be a manifestation of a natural virus.

Thus, we investigated the boundary-value problem for the heat-conduction equation in an atmospheric layer $\Omega$ under the assumption of unlimited length of its relief boundaries $\partial\Omega$ and, for the sake of simplicity, in the absence of directed motion of the atmosphere. It is assumed that there are the zones on the day side of the Earth’s surface, $\partial\Omega_p$, $p = 1, 2, \ldots, P$ which are restricted and able to be heated up intensely by solar beams, which the presence of the temperatures gradient indicates. We designate as the zones having this ability to a smaller degree and assume as $\partial\Omega^+$, $n = 1, 2, \ldots, N$ they have the zero gradient. Among the zones covering the entire lower layer boundary, there are also unlimited regions. Thus, we consider that $\cup \partial\Omega^n$, $n = 1, 2, \ldots, N$ is an addition $\cup \partial\Omega_p$, $p = 1, 2, \ldots, P$ for to the entire lower layer boundary. In the zones subjected to heating, the temperature values on $T(x, t)$, $x = (x_1, x_2, x_3)$ the Earth’s surface are known. For the upper layer boundary designated as $\partial\Omega^+$, we accept the condition of a certain stabilized temperature equal to.

The described mixed initial-boundary-value problem is set by the relation.

$$\Delta \psi = D \frac{\partial \psi}{\partial t}, \quad \psi = \sigma (x, t), \quad x \in \partial\Omega^+, \quad \frac{\partial \psi}{\partial n} = 0, \quad x \in \partial\Omega^n$$

$$\psi = T_p(x, t), \quad x \in \partial\Omega_p, \quad \psi = \psi_0(x, 0), \quad x \in \Omega, \quad x = (x_1, x_2, x_3)$$  \hspace{1cm} (6)

Here $\psi(x, t)$ is the desired temperature in the region $\Omega$, $D^{-1} > 0$ is the temperature-conductivity coefficient of the medium, which is considered as constant; $n$ is the normal $t \geq 0$ external to the boundary; $t$ is the time; $\psi_0(x, 0)$ is the initial value of the temperature in the region $\Omega$. The initial-boundary-value problem can be investigated by the block-element method in four-dimensional space [14]; however, it is unnecessary for solving the above problem. For this purpose, we take into account that the investigation is carried out over a reasonably long time interval of the temperature change removed from the initial state. Therefore, we consider that the temperature was stabilized during the conditional fall-winter (FW) season $T(x, t)$ and accepted at the boundary of the cooling mode, which is established in time and described by the relation.

$$T_p(x, t) = f_{1p}(x) e^{-\omega t}, \quad \omega > 0$$  \hspace{1cm} (7)

The mode at the upper boundary $\partial\Omega^+$ is considered as that established in time $\sigma(x, t) = \sigma_0 e^{-\omega t}, \omega > 0$ in the same form, where $\sigma_0$ can be considered as constant.

Similarly, during the conditionally called spring-summer (SS) season, this function is described by the relation of heat increase of the territory.

$$T_p(x, t) = f_{2p}(x) e^{\varepsilon t}, \quad \varepsilon > 0$$  \hspace{1cm} (8)

Here, $\omega$, $\varepsilon$ are certain constants. For new unknown functions $\varphi(x)$ introduced for both cases by the relations

$$\psi(x, t) = \varphi(x) e^{-\omega t}, \quad \psi(x, t) = \varphi(x) e^{\varepsilon t}$$  \hspace{1cm} (9)

we obtain the boundary-value problems for the FW in the form of,

$$\Delta \varphi + k_1^2 \varphi = 0, \quad \frac{\partial \varphi}{\partial n} = 0, \quad x \in \partial\Omega^n$$

$$\varphi = f_{1p}(x), \quad x \in \partial\Omega_p, \quad k_1^2 = D \omega;$$

$$\varphi = \sigma_0(x), \quad x \in \partial\Omega^+$$  \hspace{1cm} (10)
and that for the SS in the form of

$$
\begin{align*}
\Delta \varphi - k_0^2 \varphi &= 0, & &\varphi = f_{\alpha \lambda}(x), & &x \in \partial \Omega, \\
\varphi &= f_{\alpha \lambda}(x), & &x \in \partial \Omega_p, & &k_0^2 = D \varepsilon \\
\varphi &= \sigma_0(x), & &x \in \partial \Omega^+.
\end{align*}
$$

(11)

Considering for simplicity the boundary-value problem in the rectangular Cartesian system of coordinates in the layer $-\infty < x_1, x_2 < \infty$, $0 \leq x_3 \leq h$ with plane boundaries, we come to the integral equations of the form.

$$
\begin{align*}
K_{\lambda \cdot q_{\lambda}} &= \sum_p \int_{\partial \Omega_p} k_{\lambda} (x_1 - \xi_1, x_2 - \xi_2) q_{\lambda \cdot p} (\xi_1, \xi_2) d\xi_1 d\xi_2 = f_{\lambda s}^0 (x_1, x_2), \\
x_1, x_2 &\in \partial \Omega_s
\end{align*}
$$

(12)

$$
\begin{align*}
K_{\lambda} (x_1, x_2) &= \int_{-\infty}^{\infty} K_{\lambda} (\alpha_1, \alpha_2) e^{-i(\alpha x)} d\alpha_1 d\alpha_2, & &\langle \alpha x \rangle = \alpha_1 x_1 + \alpha_2 x_2, \\
K_{\lambda} (\alpha_1, \alpha_2) &= \frac{sh_{\alpha \lambda \lambda \lambda} h}{\alpha \lambda \lambda \lambda h}, & &\alpha_{13} = i \sqrt{\alpha_1^2 + \alpha_2^2 - k_1^2}, & &\alpha_{23} = i \sqrt{\alpha_1^2 + \alpha_2^2 + k_2^2}, \\
q_{\lambda s} (x_1, x_2) &= P_{\Omega_s} (x_1, x_2) q_{\lambda}, & &\lambda = 1, 2 \\
f_{\lambda s}^0 (x_1, x_2) &= -f_{\lambda s} (x_1, x_2) + \sigma_0 \int_{-\infty}^{\infty} \frac{e^{-i(\alpha x)}}{\alpha \lambda \lambda \lambda h} d\alpha_1 d\alpha_2
\end{align*}
$$

(13)

Here, in the case of problems (10), (11), we have for the FW $\lambda = 1$ and for the SS $\lambda = 2$, respectively $P_{\Omega_s}$, is the projector on the region $\partial \Omega_s$. The distributions of zeros $z_m$ and poles $\xi_m$ of the meromorphic functions $K_{\lambda} (\alpha_1, \alpha_2)$ at $\lambda = 1$ for the parameter $u = \sqrt{\alpha_1^2 + \alpha_2^2}$ are given respectively by the relations for, where $\pm z_m$, $\pm \xi_m$, where

$$
\begin{align*}
&z_m = \sqrt{k_1^2 - \pi^2 m^2 h^{-2}}, & &\xi_m = \sqrt{k_1^2 - \pi^2 (m - 0.5)^2 h^{-2}}, & &k_1 \geq \pi m h^{-1}; \\
z_m = i \sqrt{\pi^2 m^2 h^{-2} - k_1^2}, & &\xi_m = \sqrt{k_1^2 - \pi^2 (m - 0.5)^2 h^{-2}}, \\
\pi m h^{-1} \geq k_1 \geq \pi (m - 0.5) h^{-1} \\
z_m = i \sqrt{\pi^2 m^2 h^{-2} - k_1^2}, & &\xi_m = \sqrt{\pi^2 (m - 0.5)^2 h^{-2} - k_1^2}, \\
\pi (m - 0.5) h^{-1} \geq k_1, & &m = 1, 2, 3, \ldots
\end{align*}
$$

For $\lambda = 2$, the zeros $\pm z_m$ and poles $\pm \xi_m$ have the form of

$$
\begin{align*}
&z_m = i \sqrt{\pi^2 m^2 h^{-2} + k_2^2}, & &\xi_m = i \sqrt{\pi^2 (m - 0.5)^2 h^{-2} + k_2^2}, & &m = 1, 2, 3, \ldots
\end{align*}
$$

Let $R$ be the radius of the region completely containing all inhomogeneities $\partial \Omega_p$, $p = 1, 2, \ldots, P$ and while $r, \rho$ are the radius vectors of fixed points in their regions. We consider that the function $\Pi_M (u)$ contains all zeros and poles up to $m = m_0$ the number and $K_{M \lambda} = K_{\lambda} \Pi_M^{-1}$ at $\lambda = 1$ for a certain $k_1^2$.  

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(i) Let virus-manifestation conditions (9) from [4] be fulfilled for set (12) of integral equations with the kernel $K_{\lambda}^{M}(\alpha_1, \alpha_2)$ with the special right-hand sides at $\lambda = 1$ subsequently for all $\pi(m_0 - 0.5)h^{-1} \geq k_1m \leq m_0$. Then, the following relation takes place in the region $r > R$.

$$
\frac{P_{r>R}(\varphi(x))}{P_{\Omega_{s}}(\varphi(\rho))} \rightarrow O\left(e^{-|\text{Im}\xi_{m}|(r-\rho)}\right), \quad m = m_0 + 1, \quad s = 1, 2, \ldots, P
$$

Similarly, we consider that the function $\Pi_{M}(u)$ contains all zeros and poles up to the number $m = m_0$ and $K_{\lambda}^{M} = K_{\lambda}^{1-1} = 2$ for a certain $k_2^2$.

(ii) Let, for set (12) of integral equations with the kernel $K_{\lambda}^{M}(\alpha_1, \alpha_2)$ with the special right-hand sides, virus-manifestation conditions (9) from [4] be fulfilled at $\lambda = 2$ subsequently for all $m \leq m_0$. Then, the following property exists for an arbitrary fixed $r$ and $\rho$ in their regions

$$
\frac{P_{\Omega_{s}}(x_1, x_2)(\varphi(\rho))}{P_{r>R}(x_1, x_2)(\varphi(r))} \rightarrow O\left(e^{|\text{Im}\xi_{m}|(r-\rho)}\right), \quad m = m_0 + 1, \quad s = 1, 2, \ldots, P
$$

Conclusion

Thus, the viruses can either decrease the temperature outside the zone of inhomogeneities or increase it in there.

More general natural viruses are encountered in the mixed boundary-value problems of ecology, seismology, and durability theory [15, 16, 17, 18].

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Vladimir A. Babeshko, Kuban State University, Russia
Olga V. Evdokimova, Kuban State University, Russia
Olga M. Babeshko, Kuban State University, Russia
Dmitry V. Grishenko, Kuban State University, Russia