Contact elastic bodies with taking into account their adhesion

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Abstract

This paper considers the potential force field. The presence of a potential force field is taken into account in the study of contact interaction of elastic bodies, taking into account their adhesion. To carry out the calculation of the stress and strain of the material is necessary to know the potential distribution of the force field and the surface energy of body. The method of determining the type and parameters of the potential force field near the free surface of a solid was offered. It is based on a continuous version of the model of an elastic medium. At the heart of this version is the supposition multiparticle potential nonlocal interaction infinitely small particles, which forms the medium.

Keywords: contact of elastic bodies, adhesion, surface energy, nonlocal surface forces, the potential force field.

1   Introduction

Problem of adhesive interaction of two elastic bodies viewed in works [1-5]. Bodies $B_{(1)}$ and $B_{(2)}$ have convex boundary surface $A_{(1)}$ and $A_{(2)}$. Force fields are near each of them. Potential force field decreases rapidly to zero with increasing distance to $A_{(1)}$ and $A_{(2)}$. Dependence of the potential on the distance assumed to be known. The total potential energy field, which per unit of surface area, is equal to the surface energy.

When approaching bodies $B_{(1)}$ and $B_{(2)}$ each of them falls into the field of another body. Therefore, the bodies $B_{(1)}$ and $B_{(2)}$ are gravitated to each other. The attractive forces are the adhesive forces. They are determined by the field gradient between the bodies $B_{(1)}$ and $B_{(2)}$. In the final state the bodies are pressed towards each other and are in equilibrium. Attractive forces are balanced by the elastic forces of repulsion. Strains and stresses developed in the bodies $B_{(1)}$ and $B_{(2)}$. They contacted along a contact surface $A_{(12)}$. This surface has a certain form and size.

To calculate the stresses and strains, shape and size of the contact surface we need to know the dependence potential of the force field on the distance and the surface energy of bodies $B_{(1)}$ and $B_{(2)}$. For some materials the surface energy is known and the dependence potential of the force field on the distance is unknown. Therefore, assumptions are made about it in the works [1-5]. In these works proposed a method that helps determine this relationship and to calculate the surface energy. The method is based on the proposed in works [6-8] model of gradient elastic medium. The model is based accounting nonlocal potential interaction of the particles of an elastic material. The interaction potential is determined by the known mechanical properties.

2   The theoretical principles

2.1   The essence of the model gradient medium

Arbitrary body $B_{(k)}$ $(k = 1, 2)$ is the union of infinitely small particles $dB_{(k)}$.

In the reference state, they have the radius-vector $\mathbf{r}_{(k)}$. In the current state, they have the radius-vector $\mathbf{R}_{(k)} = \mathbf{r}_{(k)} + \mathbf{u}_{(k)}$. Where $\mathbf{u}_{(k)}$ is the displacement vector. On the particles $dB_{(k)}$ in the reference and current states are volumetrically distributed external and internal forces. Density of the external forces is equal to $\mathbf{f}_{(kp)}$ $(k \neq p)$. Density of internal forces is equal to $\mathbf{f}_{(kk)}$.

Internal forces are due to two-factor, triple and etc. interactions between the particles of the body B [6]. Next (for simplicity), we consider only two-factor interactions. External forces also arise because two-factor interactions of particles $dB_{(k)}$ body $B_{(k)}$ with the particles $dB_{(p)}$ of the another body $B_{(p)}$.

Potential $d^2\Psi_{(kp)}$ $(k, \ p = 1, 2)$ of a particle interacting $dB_{(k)}$ with a particle $dB_{(p)1}$ is proportional to their volume $dV_{(k)}$ and $dV_{(p)1}$: $d^2\Psi_{(kp)} = \Phi_{(kp)} dV_{(k)} dV_{(p)}$. The coefficient of proportionality $\Phi_{(kp)}$ (hereinafter potential) is a function of the distance $L_{(kp)}$ between the particles.

$$L_{(kp)} = |\mathbf{L}_{(k,p)}|, |\mathbf{L}_{(k,p)} = \mathbf{R}_{(p)1} - \mathbf{R}_{(k)} = (\mathbf{r}_{(p)1} - \mathbf{r}_{(k)}) - (\mathbf{u}_{(p)1} - \mathbf{u}_{(k)}) = \mathbf{l}_{(kp)} + \Delta\mathbf{u}_{(kp)} \quad (1)$$

Here $\mathbf{l}_{(kk)}$ – the radius-vector of the particle $dB_{(p)1}$ relative to the particle $dB_{(k)}$, and $l_{(kp)} = |\mathbf{l}_{(kp)}|$ – the distance between particles in the reference state; $\Delta\mathbf{u}_{kp} = \mathbf{u}_{kp} - \mathbf{u}_{(p)1}$ – relative displacement of particles.

Vector $\Delta\mathbf{u}_{kp}$ is decomposed into a series of external powers of the vector $\Delta\mathbf{l}_{kp}$.

$$\Delta\mathbf{u}_{kp} = \sum_{n=1}^{\infty} \frac{1}{n!}(\nabla_l^n \mathbf{u}) \cdot \overbrace{\cdots}^{n \ times} \cdot \mathbf{l}_{(kp)}^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}(\nabla^n \mathbf{u}) \cdot \overbrace{\cdots}^{n \ times} \cdot (\mathbf{l}_{(kp)}^T)^n \quad (2)$$

In the expression (2) upper index $(\ldots^T)$ means operation of transposition; $\nabla = d\ldots/d\mathbf{r}$ – Hamiltonian differential operator on the vector $\mathbf{r}$ and $\nabla_l = d\ldots/d\mathbf{l}$ – on the vector $\mathbf{l}$. While the position of particle $dB_{(k)}$ is changed, another particle $dB_{(p)1}$ is fixed. Therefore, when the vector $\mathbf{r}_{(k)}$ receives an increment $d\mathbf{r}_{(k)}$, vector $\mathbf{l}_{(kp)}$ receives an increment $d\mathbf{l} = -d\mathbf{r}$. So, $\nabla_l^n = (-1)^n \nabla^n$.

The Equation (1) is evidence of the dependence of potential Φ_{kp} on the vector $\Delta\mathbf{u}_{(kp)}$. The authors assumed that its unit is a small quantity. Then we can assume that the power series converges:

$$\Phi_{(kp)}(L_{(kp)}) = \Phi_{(kp)}(l_{(kp)}) + \sum_{m=1}^{\infty} \frac{1}{m!} \left[\nabla_L^m \Phi_{(kp)}(L_{(kp)}) \right]_{\mathbf{L}_{(kp)} = \mathbf{l}_{(kp)}} \cdot \ldots \cdot \left[\left(\mathbf{L}_{(kp)} - \mathbf{l}_{(kp)} \right)^T \right]^m =$$

$$= \Phi_{(kp)}(l_{(kp)}) + \sum_{m=1}^{\infty} \frac{1}{m!} \left[\nabla_l^m \Phi_{(kp)}(l_{(kp)}) \right] \cdot \ldots \cdot \left[\left(\Delta\mathbf{u}_{(kp)} \right)^T \right]^m \quad (3)$$

The potential interaction of the particle $dB_{(k)}$ with its surroundings in the bodies $B_{(k)}$ and $B_{(p)1}$ $(k \neq p)$ is defined by:

$$\Phi_{(k)} dV_{(k)} = \left[\iint_V \Phi_{(kp)}(L_{(kp)}) dV_{(p)1} \right] dV_{(k)} \begin{cases} V_{(k)} \ V_{(p)} \ k \neq p \\ V_{(k)} \ V_{(k)} = V_{(k)} \ k = p \end{cases} \quad (4)$$

In the expression (3), the authors take into account the terms with numbers $m=1,2$. If in this case the expression (2) to take into account only the first term, we can obtain the potential of the particle, which corresponds to the classical theory of elasticity with initial stress state. If it is remembered that the expression (2) to take into account only the first item, we can obtain the potential of the particle $dB_{(k)}$, which corresponds to the classical theory of elasticity with initial stress state. If we consider two terms, we obtain the potential of the particle $dB_{(k)}$, which corresponds to the gradient theory of second-order elasticity with initial stress state. This potential is represented by the equation:

$$\Phi_{(k)} = \Phi_{(k)}(l_{(kp)}) + \sum_{m=1}^{2} P_{(k)}^{0(m)} \cdot \cdot (\nabla^m u)^T + \sum_{m=1}^{2} C_{(k)}^{(mn)} \cdot \ldots \cdot \left[(\nabla^n \mathbf{u}_{(k)})^T (\nabla^m \mathbf{u}_{(k)})^T \right]^m \quad (5)$$

The first item on the right side of this equation is cubic density of the potential energy of the particle $dB_{(k)}$ until deformations. The amount is an addition to this energy, which arises due to the deformations. Tensors $P_{(k)}^{0(m)}$ ($m=1,2$) describe the initial state of stress, which arises in the environment because of the mutual attraction of its particles. Tensors $C_{(k)}^{(mn)}$ ($m,n=1,2$) describe its elastic properties. If $m,n=1$, we have the classical linear theory of elasticity. In this theory the tensor $P_{(k)}^{0(1)}$ is not considered [9].

If the potential interaction of the particles $\Phi_{(kk)}(l_{(kk)})$ is known, the tensors $P_{(k)}^{0(m)}$ and $C_{(k)}^{(mn)}$, which characterize the mechanical properties of the material of the body $B_{(k)}$, may be expressed through it.

$$P_{(k)}^{0(m)} = \int\limits_{V} \frac{1}{m!} \big(\nabla_l \Phi_{(kk)}(l_{(kk)}) \big) \mathbf{l}_{(kk)}^m \, dV_{(k)1} \quad (6)$$

$$C_{(k)}^{(mn)} = \int\limits_{V} \frac{(-1)^{m+n}}{m!n!} \big(\nabla_l^2 \Phi_{(kl)}(l_{(kk)}) \big) \big(\mathbf{l}_{(kk)}^m \mathbf{l}_{(kk)}^n \big) \, dV_{(k)1} \quad (7)$$

If the shape and parameters of the potential $\Phi_{(kk)}(l_{(kk)})$ are not known, they can be defined by specifying a particular form (eg, Morse potential) [7, 8] by using information about the experimentally determined characteristics of the elastic state of the material.

2.2 Surface potential field and its impact on the balance

The presence of a stationary force field $A_{(k)}$ near the boundary surface and determine its characteristics is carried out by using of a test particle $dB_{(p)1}$. Its potential interactions with the body $B_{(k)}$ is defined by

$$d\Psi_{(p)1} = \Phi_{(p)} dV_{(p)} \left[\int\limits_{V(k)} \Phi_{(kp)} \big(L_{(kp)} \big) dV_{(k)} \right] dV_{(p)} \quad (8)$$

The force, which acts on the body $dB_{(p)1}$, defined by:

$$d\mathbf{F}_{(p)} = \mathbf{f}_{(p)} dV_{(p)1} = \left[\int\limits_{V(k)} \big(-\nabla_{L(pk)} \Phi_{(kp)} \big(L_{(pk)} \big) dV_{(k)} \right] dV_{(p)1} =$$

$$= \left[\int\limits_{V(k)} \mathbf{f}_{(pk)}\big(L_{(pk)}\big)\, dV_{(k)} \right] dV_{(p)1} \tag{9}$$

In this expression: $\nabla_{L(pk)} = \frac{d}{d\mathbf{L}_{(pk)}}$, since
$\mathbf{L}_{(kp)} = \mathbf{R}_{(p)1} - \mathbf{R}_{(k)} = -\big(\mathbf{R}_{(k)} - \mathbf{R}_{(p)1}\big) = -\mathbf{L}_{(pk)}$

Equations (8) and (9) are evidence that the test particle $dB_{(p)1} \notin B_{(k)}$ from the outside body $B_{(k)}$ would be affected, which is characterized by the potential force $d\mathbf{F}_{(p)} = \mathbf{f}_{(p)}\, dV_{(p)1}$. It defined by the sum its interactions with all particles of the body, each of which is characterized by a potential $\Phi_{(kp)}$. Potentials depend not only on the distance between the particles, but also on their material.

Particle $d_{(p)1}$ will not have a noticeable effect on the positions of the particles $dB_{(k)}$ of the body $B_{(k)}$. Therefore, the symbol L in the expressions (8) and (9) can be replaced by the symbol l. That is, we can consider $B_{(k)}$ strainless body. When we looked interaction of finite bodies $B_{(k)}$ and $B_{(p)}$, we suggest that deformation of these bodies do not have a noticeable effect on the capacity and strength of the interaction of particles of one of them with other particles.

Thus, if the potential $\Phi_{(kp)}$ is known, then for points $\mathbf{r}_{(2)} \notin V_{(1)}$ potential is

$$\Phi_{(2)}(\mathbf{r}_{(2)}) = \int\limits_{V(1)} \Phi_{(12)}\big(|\mathbf{r}_{(2)} - \mathbf{r}_{(1)}|\big)\, dV_{(1)}\ \mathbf{r}_{(1)} \in V_{(1)} \tag{10}$$

In the works [1в Ђ"5] is assumed that the potential field near a solid surface is created only its surface (the force field acts on boundary surface of plots). Therefore, the possibility of transition in the expression (10) from the volume integration to integration over the surface, is of interest.

It is assumed that the dependence of $\Phi_{(12)}$ on $\mathbf{r}_{(1)}$ is such, that there exists a vector field $\xi_{(12)}(\mathbf{r}_{(1)})$ with the property:

$$\Phi_{(12)} = \nabla \cdot \xi_{(12)}\ \nabla = \frac{d}{d\mathbf{r}_{(1)}} \tag{11}$$

In this case,

$$\Phi_{(12)}(\mathbf{r}_{(2)}) = \int\limits_{A(1)} \mathbf{r} \cdot \xi_{(12)}\big(|\mathbf{r}_{(2)} - \mathbf{r}_{(1)}|\big)\, dA_{(1)} \tag{12}$$

Role of the density of the surface potential played a scalar quantity $\varphi_{(12)} = \mathbf{n}_{(1)}\xi_{(12)}$. Vector field $\xi_{(12)}(\mathbf{r}_{(1)})$ can be determined if further prevent its potential: $\xi_{(12)}(\mathbf{r}_{(1)}) = \nabla\zeta(\mathbf{r}_{(1)})$ on conditions that at the surface $A_{(1)}$: $\zeta(\mathbf{r}_{(1)}) = const$. In this case $\xi_{(12)}(\mathbf{r}_{(1)}) = \mathbf{n}_{(1)}\xi_{(12)}(\mathbf{r}_{(1)})$. Density $\varphi_{(12)}(\mathbf{r}_{(2)}, \mathbf{r}_{(1)} \in A_{(1)})$ surface (along $A_{(1)}$) distributed non-local forces, which act on test particles $dB_{(2)}$, is defined by

$$\varphi_{(12)}(\mathbf{r}_{(2)}, \mathbf{r}_{(2)} \in A_{(1)}) = -\nabla_{(2)}\varphi_{(12)}(\mathbf{r}_{(2)}, \mathbf{r}_{(2)} \in A_{(1)})$$

Thus, the hypothesis of surface potential distribution is justified.

There is an easier simplified version. We consider the situation, where the radius of curvature of the boundary surfaces exceeds the thickness of the boundary layer on a lot. In the context, the nonlocal effects of the body to another is detected. (Estimated of [7] it is of the order of several interatomic distances). Then, as the magnitude of $\varphi_{(12)}(\mathbf{r}_{(2)}, \mathbf{r}_{(2)} \in A_{(1)})$ is proposed to use the integral over the thickness of $\mathbf{f}_{(12)}$.

Next we consider the balance of the body $B_{(1)}$, which interacts with the body $B_{(2)}$.

On the basis of equation (4) can be argued, that the equation of motion of a particle $dB_{(1)} \subset B_{(1)}$ is given by:

$$\rho_{(1)}\frac{\partial^2 \mathbf{u}}{\partial t^2} = \mathbf{f}_{(1)} = \mathbf{f}_{(11)} + \mathbf{f}_{(12)} \tag{13}$$

In accordance with (4) vector $\mathbf{f}_{(1)}$ is defined by the expression:

$$\mathbf{f}_{(1)} = -\nabla\Phi_{(1)} \quad \Phi_{(1)} = \int\limits_{V(1)} \Phi_{(11)}\big(l_{(11)1}\big)\,dV_{(1)1} + \int\limits_{V(2)} \Phi_{(12)}\big(l_{(12)1}\big)\,dV_{(2)1} \tag{14}$$

It is the sum of two terms $\mathbf{f}_{(11)}$ and $\mathbf{f}_{(12)}$. The first term is conditioned to the interaction of particle $dB_{(1)} \subset B_{(1)}$ with $dB_{(1)1}$ particle of the same body $B_{(1)}$. It is represented as divergence of double tensor: $\mathbf{f}_{(11)} = \nabla \cdot P_{(1)}$. In particular, if the description of the material properties of the body is used classical theory of linear elasticity, then $P_{(1)} = P_{(1)}^{0(1)} + C_{(1)}^{(11)}\cdot\cdot\big(\nabla\mathbf{u}_{(1)}\big)$. Here it can be assumed (when removing the boundaries of order of the interatomic distance [7]) that $P_{(1)}^{0(1)} = const$, and used as the tensor $P_{(1)}$ a supplement to $P_{(1)}$, which is caused by strains. In view of the spherical symmetry of dependence $\Phi_{(11)}\big(l_{(11)}\big)$ on the basis of (6) and (7) we can prove that the tensors $P_{(1)}^{0(1)}$ and $C_{(1)}^{(11)}$ also have this property.

The second term is conditioned to the interaction of particle $dB_{(1)} \subset B_{(1)}$ with particle $dB_{(2)1}$ of another body $B_{(2)}$. In the works [8, 9] for the second term is offered to use an idea that is similar to (5). Then the body $B_{(2)}$ effect $B_{(1)}$ on the body affects its mechanical properties in the presence of the adhesion between the bodies. In this paper we offer not to do this and keep in equation (13) definition for the vector B:

$$\mathbf{f}_{(12)} = \int\limits_{V(2)} \big(-\nabla_{l(12)}\Phi_{(12)}\big(l_{(12)}\big)\big)\,dV_{(2)1} = \int\limits_{V(2)} \mathbf{f}_{(12)1}\,dV_{(2)1} \tag{15}$$

Taking into consideration the realized reasoning, which consider nonlocality of the interaction of the particles of the bodies $B_{(1)}$ and $B_{(2)}$, equation (13) takes the form:

$$\rho_{(1)}\frac{\partial^2 \mathbf{u}_{(1)}}{\partial t^2} = \nabla \cdot P_{(1)} + \int\limits_{V(2)} \big(-\nabla_{l(12)}\Phi_{(12)}\big(l_{(12)}\big)\big)\,dV_{(2)} = \nabla \cdot P_{(1)} + \mathbf{f}_{(12)} \tag{16}$$

If the potential $\Phi_{(12)}\big(l_{(12)1}\big)$ is known, we can calculate the interaction of bodies $B_{(1)}$ and $B_{(2)}$ taking into account their adhesion.

At the lesser curvature of the boundary surfaces of the contacting bodies and large scale volumetric bodies on each other may be replaced a surface effect. In this case, instead of equation (16) is necessary to use an equation of the form:

$$\rho_{(1)} \frac{\partial^2 \mathbf{u}_{(1)}}{\partial t^2} = \nabla \cdot P_{(1)} \qquad (17)$$

In the boundary conditions to the existing external surface effects need to add the value $\psi_{(21)}$ of the action of the body $B_{(2)}$ per unit of area of the surface $A_{(1)}$ of body $B_{(1)}$.

$$\psi_{(21)} = \int\limits_0^\infty dh_{(1)} \int\limits_{A_{(2)}} \varphi_{(12)}\big(\mathbf{r}_{(2)}, \mathbf{r}_{(1)} \in A_{(1)}\big) dA_{(2)} \qquad (18)$$

Here $h_{(1)}$ – distance from the surface $A_{(1)}$, in the depths of the body $B_{(1)}$. It is believed that the size $B_{(1)}$ in the direction of reference for many more than the distance at which the potential $\Phi_{(12)}$ decreases from maximum to zero.

2.3   The example calculation of the field between solids

The purpose of this work – the method for determining the type and parameters of the force potential near the free surface of a solid. In other words – the method of determining the forces $\mathbf{f}_{(12)}$ in equation (16) or force $\psi_{(12)}$ to the boundary condition of the classical equation of equilibrium.

Therefore, there are no solutions of the equilibrium equations in the example. There is provided a calculation result value $\psi_{(12)} = |\psi_{(12)}|$ depending on the distance between the planar surfaces $A_{(1)}$ and $A_{(2)}$ of semiinfinite solid $B_{(1)}$ and $B_{(2)}$, which are made of copper (Cu) and aluminum (Al).

The potential two-factor interaction of particles of one material selected in the form of Morse [7,11].

$$\Phi_{(k,k)}^{(2)} = \Phi_{0(k,k)}^{(2)}\big(e^{-2\beta_{(k,k)}L_{1(k,k)}} - 2e^{-\beta_{(k,k)}L_{1(k,k)}}\big)$$

If particles of different materials ($k \neq p$), the form of the potentials saved. The parameters are defined by expressions $\Phi_{0(k,p)}^{(a)} = \sqrt{\Phi_{0(k,k)}^{(a)}\Phi_{0(p,p)}^{(a)}}$, $\beta_{0(k,p)} = \sqrt{\beta_{(k,k)}\beta_{(p,p)}}$ (similar to [12]).

For determining the parameters $\beta_{(k,k)}$, $\Phi_{0(k,k)}^{(2)}$ and $\Phi_{0(k,k)}^{(3)}$ (in accordance with the [7]) used data on parameter Lame $\big(\lambda_{(k)}$ and $\mu_{(k)}\big)$ and average interatomic distance $l_{at(k)}$.

Matrix components of the tensor of traditional characteristics of mechanical properties of isotropicu linear elastic material in an ortho-normal basis ($\mathbf{e}_n, n = 1,2,3$) has the form:

$$\big(C_{(k)}^{(1,1)C,l}\big)_{ijkm} = \lambda\delta_{ij}\delta_{km} + \mu(\delta_{ik}\delta_{jm} + \delta_{im}\delta_{jk}) \qquad (19)$$

Here δ_{ij} – the Kronecker symbol; λ and μ – first and second parameters Lame.

Taking into account only two-factor interaction, difference $\lambda - \mu$ must be neglected. If this is impossible, it is necessary to take into account three-particle interaction [8]. Taking into account only pair two-factor interaction, comparing (19) and tensor matrix (7) (when $m=1$, $n=1$) you can get the first value for the parameters $\beta_{(k,k)}$, $\Phi_{(k,k)}^{(2)}$. The second relation obtained on the basis the analysis of nonlinear dispersion relation for plane longitudinal acoustic waves [7,9]. This can be done only when the material properties of the contacting
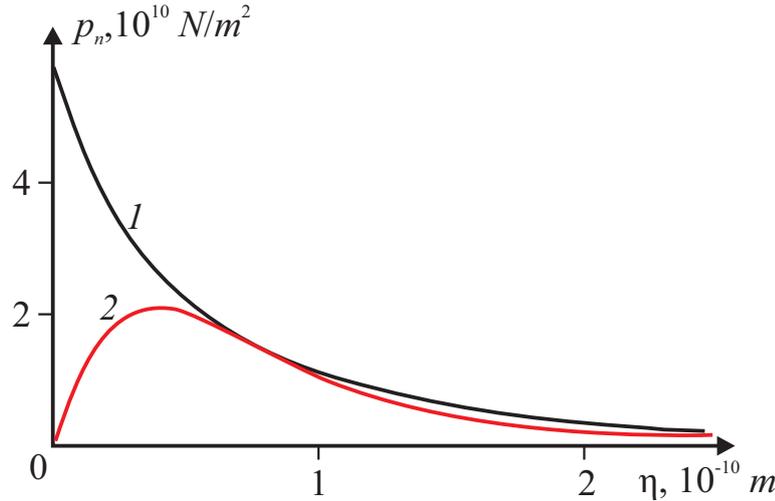
Figure 1: The forces of the adhesive attraction

bodies described gradient elasticity theory of second-order materials. The simplest form of the dispersion law has the form [9]:

$$\omega^2 = \left(\frac{2C_1}{M}l_{at}^2\right)\sum_{q=1}^{\infty}\frac{(-1)^{q-1}}{(2q)!}l_{at}^{2(q-1)}K^{2q} \tag{20}$$

Where ω -вЂ" circular frequency; K -вЂ" wave number; M -вЂ" the mass of an atom; $C_1 = \left(d^2U/dx^2\right)_{x=l_{at}}$, $U=U(x)$ – dependence of the atoms potential from the distance x.

Taking into account only first two displacement gradients in this model environment [7] the same dependence has the form:

$$\omega^2 = \left(C_{(k)}^{(11)}\right)_{1111}\left[K^2 - \left(\frac{(C_{(k)}^{(22)})_{111111}}{(C_{(k)}^{(11)})_{1111}}\right)K^4\right] \tag{21}$$

Calculation of the magnitude l_{at} conducted on the basis of formulas [13]: $l_{at} = \frac{1}{\sqrt[3]{n}}$. Here $n = \frac{N_A}{A}\rho$, $N_A = 6{,}02 \cdot 10^{23}$ $1/mol$ – Avogadro's number, A – atomic mass, ρ – the density of materials.

The calculation was obtained the graph of the dependence the forces p_n of the adhesive attraction of semi-infinite bodies of copper and aluminum from the distance η between them (figure 1, line 2). The line 1 in the figure 1 represents the interaction of copper and aluminum semi-infinite body, which was calculated by the method of physics [14].

The best match of calculated data obtained by the method, which was proposed in this paper, it was possible to get at $\beta = 2{,}1 \cdot 10^{10}$ $1/m$. The calculated value of this parameter is close to fitted value and amounts the magnitude $\beta = 2{,}3 \cdot 10^{10}$ $1/m$. The parameter value is $\Phi_{0(12)}^{(2)} = 5{,}7 \cdot 10^{40}$ J/m^6. It is close to the square root of the product of the total interaction energies of all pairs of atoms of aluminum and copper, which contained in a unit volume: $\epsilon n^2 = 7{,}52$ J/m^6 – aluminum, and $\epsilon n^2 = 4{,}02$ J/m^6 – copper.

Using the idea of the distribution of surface adhesion influence leads to the fact: the body cease to act on each other in the direction normal to the surface of contact with full contact (dashed line). It is natural, because in this case the attractive force of particles

of one body to the particles of another body are directed along the surface and have no component perpendicular to it. This conclusion does not depend on the choice of the potential shape.

3   Conclusion

In the paper we proposed the method of determining the type and parameters of the potential force field near the free surface of a solid. Results of calculation of the attractive force of semi-infinite body carried out in accordance with the proposed method. At the distances greater than one angstrom proposed method corresponds satisfactorily to the calculation results, which were conducted using Solid-state physics. At shorter distances, there is a significant discrepancy.

The authors consider, that it is due to the accepted hypothesis: the forces, which created the potential field between bodies, are distributed on their surfaces rather than volumes.

Acknowledgements

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