

# Thermal brittleness and creep fracture of metallic materials

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## Abstract

The problem of damage and high-temperature creep fracture of metallic materials is demanded in such areas of modern engineering as the thermal and nuclear power plants, aircraft, spacecraft and others. In this regard, intensive studies on this problem are carried out. It has been found that under the prolonged action of relatively small stresses and high temperatures metallic materials embrittled due to development of damage (cracks, pores etc.). These effects have been studied in details by the methods of physics and materials science. For engineering applications, it became necessary to develop mechanical models of creep damage and fracture. The first such models have been proposed by G. Hoff, L.M. Kachanov, Yu.N. Rabotnov. In these models some controversy assumptions are made, for example the incompressibility condition, which can be overcome, if we formulate the creep fracture criteria using the mass conservation law. Taking into account these propositions and remaining within the concept of damage mechanics, interrelated kinetic equations of creep, damage, and creep fracture criterion are formulated. The proposed approach does not contain the mentioned contradictions and can be considered as the basis for a more accurate description of the damage and fracture of metallic materials and structural elements under the high temperature creep condition.

Financial support of the Russian Foundation for Basic Research (Grant N 15-01-03159) is gratefully acknowledged.

Under the long action of high temperatures and relative small stresses many metallic alloys and pure metals lose plasticity and collapse as brittle (the phenomenon of thermal brittleness). Because these effects are observed in elements of many important engineering objects, in particular, in power and nuclear, the problem of brittle fractures became a subject of numerous theoretical and experimental researches. For the description of brittle fractures the concept of continuity (Kachanov [1]) and damage (Rabotnov [2, 3]) was developed. To materialize the damage parameter various definitions were offered: the relative size of pores or irreversible change of volume (loosening on Novozhilov's terminology [4]) or density (Arutyunyan [5, 6]). In the paper the parameter of continuity is determined by the ratio  $\psi = \rho/\rho_0$  ( $\rho_0$  is initial,  $\rho$  is current density) and it is an integral measure of the accumulation of structural micro defects during long-term of high-temperature loading [7-16]. In the initial condition  $t = 0$ ,  $\rho = \rho_0$ ,  $\psi = 1$ , at the time of fracture  $t = t_f$ ,  $\rho = 0$ ,  $\psi = 0$ .

In the general statement the continuity (damage) parameter  $\psi$  and the kinetic equation for this parameter was considered by Haward [17]. According to Haward brittle fracture proceeds with a speed depending on stress  $\sigma(t)$

$$\frac{d\psi}{dt} = -f[\sigma(t)], \quad (1)$$

or, according to representations of statistical physics, from stress and the damage parameter

$$\frac{d\psi}{dt} = -f[\sigma(t), \psi]. \quad (2)$$

Basic provisions of the concept of Kachanov-Rabotnov brittle fracture are based on the equations (1), (2) which right part is taken in the form of power relation. In the brittle model of Kachanov the continuity parameter  $\psi$  ( $1 \geq \psi \geq 0$ ) is introduced randomly without giving of a certain physical meaning to it. It is supposed that creep deformation doesn't influence fracture processes, and the kinetic equation of the continuity parameter is taken as a power function from effective stress [1]

$$\frac{d\psi}{dt} = -A \left( \frac{\sigma_{\max}}{\psi} \right)^n, \quad (3)$$

where  $A > 0$ ,  $n \geq 0$  are empirical constants, not depending on stress,  $\sigma_{\max}/\psi$  is effective stress.

The tension problem of specimen stretched under the action of constant load  $P$  is solved. It is considered that brittle fracture happens at small deformations therefore it is possible to neglect change of specimen cross section, i.e. the conditions  $F = F_0$ ,  $\sigma_{\max} = \sigma = P/F = P/F_0 = \sigma_0 = \text{const}$ , ( $\sigma$  is true stress,  $\sigma_0$  is nominal stress,  $F_0$ ,  $F$  are the initial and current area of cross section of a specimen) are accepted. At these assumptions the equation (3) can be expressed in the form

$$\frac{d\psi}{dt} = -A \left( \frac{\sigma_0}{\psi} \right)^n. \quad (4)$$

In the Rabotnov's brittle fracture model [3] the damage parameter  $\omega$  ( $0 \leq \omega \leq 1$ ) is introduced and it is defined by the following kinetic equation

$$\frac{d\omega}{dt} = A\sigma^n. \quad (5)$$

The damage parameter is introduced as  $\omega = F_T/F_0$  ( $F_T$  is the total area of pores) and it characterizes extent of reduction of a specimen area of cross section. Then from condition  $F = F_0 - F_T$ , we have  $F = F_0(1 - \omega)$ ,  $\sigma = P/F = \sigma_0 F_0/F = \sigma_0/(1 - \omega)$ . Taking into account these relations the kinetic equation (5) can be written as

$$\frac{d\omega}{dt} = A \left( \frac{\sigma_0}{1 - \omega} \right)^n. \quad (6)$$

The equations (4) and (6) are identical at  $\omega = 1 - \psi$ ,  $d\psi = -d\omega$ . From the solution of these equations under the initial condition  $t = 0$ ,  $\psi = 1$ ,  $\omega = 0$ , we have

$$\psi = 1 - \omega = [1 - (n + 1)A\sigma_0^{nt}]^{\frac{1}{n+1}}. \quad (7)$$

Accepting a fracture condition,  $t = t_f^b$ ,  $\psi = 0$ ,  $\omega = 1$ , from (7) follows the criterion of purely brittle fracture

$$t_f^b = \frac{1}{(n + 1) \cdot A\sigma_0^n}. \quad (8)$$

Such an approach provides a physical interpretation of the Kachanov's parameter. However, the condition  $F = F_0$ , used in the Kachanov's concept, corresponds to zero value of the damage parameter what disagrees with the concept of damage accumulation. Thus, similar interpretation of Kachanov's continuity parameter isn't represented fully correct.

To define the creep deformation Rabotnov [3] introduced a system of two interconnected equations for the rate of creep and damage parameter

$$\frac{d\varepsilon}{dt} = b\sigma^m(1 - \omega)^{-q}, \quad (9)$$

$$\frac{d\omega}{dt} = c\sigma^n(1 - \omega)^{-r}, \quad (10)$$

where  $b$ ,  $c$ ,  $m$ ,  $n$ ,  $q$ ,  $r$  are constants,  $\varepsilon = \ln(l/l_0)$  is creep deformation,  $l_0$ ,  $l$  are the initial and current length of specimen.

In the case of a purely brittle fracture and small deformations when  $F = F_0$ ,  $\sigma = \sigma_0 = \text{const}$  solving the system of equations (9)-(10) we will obtain the relation of the creep deformation

$$\varepsilon = \frac{k}{m} \frac{t_f^b}{t_f^v} \left[ 1 - \left( 1 - \frac{t}{t_f^b} \right)^{1/k} \right], \quad (11)$$

$$\text{where } k = \frac{r+1}{r+1-q}, \quad t_f^b = \frac{1}{c(1+r)\sigma_0^n}, \quad t_f^v = \frac{1}{bm\sigma_0^m}.$$

Relation (11) is considered as a major result in the Rabotnov's theory, because by using this formula it is possible to describe the third phase of the creep curve, which, in the case of brittle fracture, is completely determined by the damage of material. At the same time, the output of this formula is based on the condition  $F = F_0$  and  $\omega = 0$ , which, as it was noted, is contrary to the very concept of damage. Further, in determining the criteria of ductile-brittle fracture using equations (9)-(10) the condition of incompressibility is introduced, which is also contrary to the damage concept.

To overcome these contradictions in [18] a system of equations for the rate of creep and damage, based on the continuity parameter  $\psi = \rho/\rho_0$ , have proposed. This paper presents a modified version of these equations, which can describe the main experimental results on creep and creep rupture of metallic materials. Let's consider the following system of equations

$$\frac{d\varepsilon}{dt} = B\sigma^m,$$

$$\psi^\alpha \frac{d\psi}{dt} = -A\sigma^n,$$

where  $B$ ,  $\alpha$  are constants.

The last equation of this system corresponds to the equation (2). Taking into account the mass conservation law  $\rho_0 l_0 F_0 = \rho l F$  from which follows the relation  $\sigma = \sigma_0 \psi e^\varepsilon$  these equations can be written in the form

$$\frac{d\varepsilon}{dt} = B\sigma_0^n \psi^m e^{m\varepsilon}, \tag{12}$$

$$\frac{d\psi}{dt} = -A\sigma_0^n \psi^{n-\alpha} e^{n\varepsilon}. \tag{13}$$

If we consider the case of brittle fracture and small deformations, we can assume  $e^{m\varepsilon} \approx 1$ ,  $e^{n\varepsilon} \approx 1$ , then the solution of equation (13) with the initial condition  $t = 0$ ,  $\psi = 1$  has the form

$$\psi = [1 - (\alpha - n + 1)A\sigma_0^n t]^{\frac{1}{\alpha - n + 1}}. \tag{14}$$

Fig. 1 presents the curves, corresponding to equation (14) for various values of the constants:  $\alpha = 6$  (curve 1),  $\alpha = 4$  (curve 2),  $\alpha = 2$  (curve 3) and  $\alpha = 1, 1$  (curve 4). The curves agree with the experimental curves [7-15]. In the calculations the following values of coefficients were used:  $n = 2$ ,  $A = 10^{-9} [\text{MPa}]^{-2}$ ,  $\sigma_0 = 100 \text{ MPa}$ .

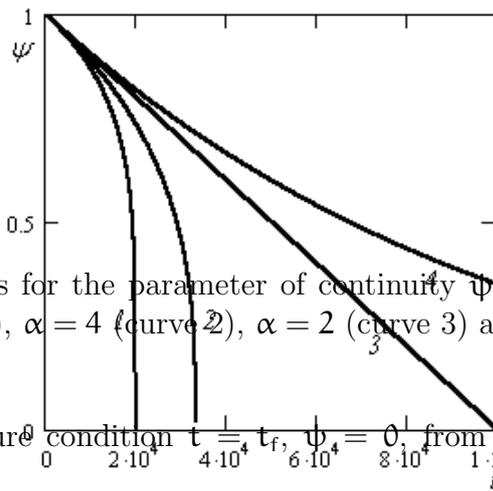


Figure 1: The curves for the parameter of continuity  $\psi$  according to the formula (11):  $\alpha = 6$  (curve 1),  $\alpha = 4$  (curve 2),  $\alpha = 2$  (curve 3) and  $\alpha = 1, 1$  (curve 4).

Taking the fracture condition  $\psi = 0$  from (14) we obtain the creep fracture criterion

$$t_f^b = \frac{1}{(\alpha - n + 1) \cdot A\sigma_0^n}. \tag{15}$$

Figure 2: Curves of long-term strength under criterion (15):  $\alpha = 6$  (curve 1),  $\alpha = 4$  (curve 2),  $\alpha = 2$  (curve 3) and  $\alpha = 1, 1$  (curve 4).

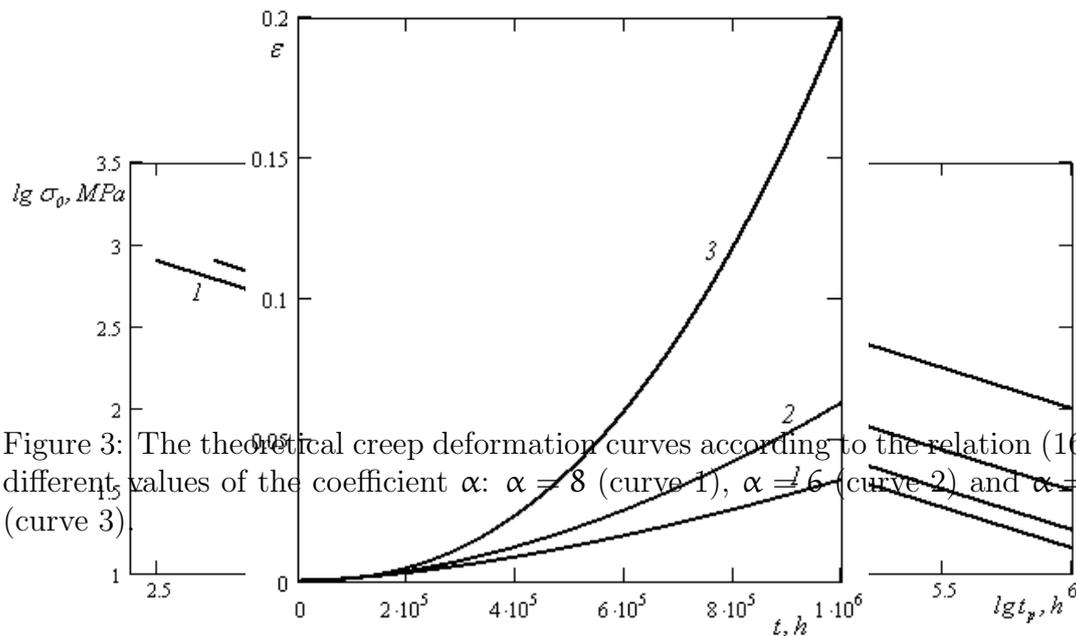


Figure 3: The theoretical creep deformation curves according to the relation (16) for different values of the coefficient  $\alpha$ :  $\alpha = 8$  (curve 1),  $\alpha = 6$  (curve 2) and  $\alpha = 1, 1$  (curve 3).

When  $\alpha = 2n$  the criterion (15) coincides with the Kachanov-Rabotnov criterion (8). On Fig. 2 in the double logarithmic coordinates are shown the creep fracture curves according to the formula (15) for different values of the coefficients:  $\alpha = 6$  (curve 1),  $\alpha = 4$  (curve 2),  $\alpha = 2$  (curve 3) and  $\alpha = 1, 1$  (curve 4). In the calculations the following values of coefficients were used:  $n = 2$ ,  $A = 10^{-9} [\text{MPa}]^{-2}$ .

Taking into account (14) and the initial condition  $t = 0$ ,  $\varepsilon = 0$ , from equation

(12) follows the relation of the creep deformation

$$\varepsilon = \frac{B\sigma_0^{m-n}}{A(\alpha-n+1)(m-n+1)} \left\{ 1 - \left[ 1 - (\alpha-n+1)A\sigma_0^{nt} \right]^{\frac{m-n+1}{\alpha-n+1}} \right\}. \quad (16)$$

On Fig. 3 are shown the theoretical creep deformation curves according to the relation (16) for different values of the coefficient  $\alpha$ :  $\alpha = 8$  (curve 1),  $\alpha = 6$  (curve 2) and  $\alpha = 1, 1$  (curve 3). As can be seen from this figure, the system of equations able to describe the third phase of creep curves, which is determined by the processes of damage accumulation.

In the calculations the following values of coefficients were used:  $n = 2$ ,  $m = 4$ ,  $A = 10^{-9} [\text{MPa}]^{-2}$ ,  $B = 5 \cdot 10^{-17} [\text{MPa}]^{-4}$ ,  $\sigma_0 = 100 \text{ MPa}$ .

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