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# Robust and Adaptive Passification Based Consensus Control of Dynamical Networks ⋆

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**Abstract:** An output synchronization problem for networks of dynamical agents is examined based on passification method and recent results in graph theory. Static delayed output feedback controllers of two types are investigated and an adaptive control is proposed. Sufficient conditions for synchronization under incomplete measurements and incomplete control are established. Example of the adaptive synchronization in a network of seven double integrators is presented.

## 1. INTRODUCTION

Controlled synchronization of networks has a broad area of important applications: cooperative control of mobile robots, control of power, biochemical, ecological networks, etc. Olfati-Saber and Murray [2004], Boccaletti et al. [2006], Bullo et al. [2009], Scardovi and Sepulchre [2009], Li et al. [2010], Proskurnikov [2012]. However most existing papers deal with control of networks of dynamical systems (agents) with full state measurements and full control, i. e. vectors of agent input, output and state have equal dimensions. In the case of synchronization by output feedback additional dynamical systems (observers) are incorporated into network controllers.

In this paper the synchronization problem for networks of agents with arbitrary numbers of outputs and states by static delayed output neighbor-based feedback (consensus protocol) is solved based on passification method Fradkov et al. [1999], Fradkov [2003] and recent results in graph theory. For the case of non delayed output control an adaptive controller is proposed and investigated.

The proposed solution for output feedback synchronization unlike those of Scardovi and Sepulchre [2009], Li et al. [2010] does not use observers. Compared to static output feedback result of Scardovi and Sepulchre [2009] (Theorem 4) the proposed synchronization conditions relax passivity condition for agents to their passifiability that allows for unstable agents. The task of delayed consensus control of *semi-passive systems* was considered in Steur and Nijmeijer [2011]. *Passifiability condition* imposed in our work in some cases is less restrictive than semi-passivity condition and easy verifiable.

The presented results extend our previous results. In Fradkov and Junussov [2011], Dzhunusov and Fradkov [2011] a static output consensus control was analyzed for networks of hyper-minimum-phase systems. In Fradkov et al. [2011] adaptive decentralized controller for interconnected systems with disturbances and time delays was proposed and analyzed.

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## 2. PROBLEM STATEMENT

Consider a network of $N$ agents dynamics of which are governed by the following equations:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + \varphi(t, x_i(t)), \\ y_i(t) &= Cx_i(t), \quad i = 1, \dots, N, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$ is a state vector of the $i$-th node, $u_i \in \mathbb{R}$ is a controlling input (control), $y_i \in \mathbb{R}^l$ is a vector of measurements (output), $A$, $B$ and $C$ are matrices of appropriate dimensions and $\varphi(t, x_i(t))$ is globally Lipschitz continuous with Lipschitz constant $L_\varphi$: $\forall t \in [0, +\infty)$ $\forall x', x'' \in \mathbb{R}^n$

$$\|\varphi(t, x') - \varphi(t, x'')\| \leqslant L_\varphi \|x' - x''\|.$$

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a digraph with a set of vertices $\mathcal{V}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ such that for $i = 1, \dots, N$ the vertex $v_i$ is associated with the $i$-th agent and the edge $(v_i, v_j)$ indicates that the output of the $i$-th node is available for the controller of the $j$-th node. An important result was obtained by R.P. Agaev and P.Yu. Chebotarev in 2000 (see Agaev and Chebotarev [2011], Chebotarev and Agaev [2009]).

*Theorem 1.* (Agaev-Chebotarev). The rank of the Laplace matrix of the graph $\mathcal{G}$ is equal to $N - \nu$, where $\nu$ is the forest dimension of the graph by converging trees. In particular, $\operatorname{rank} L = N - 1$, i. e. the zero eigenvalue of the matrix $L$ has the unit multiplicity if and only if the digraph $\mathcal{G}$ has the converging spanning tree.

We consider the task of synchronization, that is a control goal is:

$$\lim_{t \to \infty} (x_i(t) - x_j(t)) = 0, \quad i, j = 1, \dots, N. \tag{2}$$

## 3. PASSIFICATION LEMMA

*Definition 1.* A linear system $\dot{x}(t) = Ax(t)+Bu(t)$, $y(t) = g^T Cx(t)$ with the transfer matrix $W(\lambda) = g^T C(\lambda I - A)^{-1}B$, where $u \in \mathbb{R}$, $g$, $y(t) \in \mathbb{R}^l$ and $\lambda \in \mathbb{C}$ is called *hyper-minimum-phase* if the polynomial $\varphi(\lambda) = W(\lambda)\det(\lambda I - A)$ is Hurwitz and $g^T CB = lim_{\lambda\to\infty}\lambda W(\lambda)$ is a positive number.

Main results are based on the passification lemma that can be formulated in the following form Fradkov [2003].

*Lemma 2.* (Passification lemma). Let the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$, $g \in \mathbb{R}^{l\times m}$ be given and the full-rank condition $rank(B) = m$ holds. Then for existence of a positive-definite $n \times n$-matrix $P = P^T > 0$ and $l \times m$-matrix $\theta_*$ such that

$$PA_* + A_*^T P < 0, \quad PB = C^T g, \quad A_* = A - B\theta_*^T C$$

it is necessary and sufficient, that the system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = g^T Cx(t) \tag{3}$$

is hyper-minimum-phase.

In our particular case $m = 1$. It can be proved (see Fradkov [2003]) that $\theta_*$ can always be chosen as $\theta_* = \varkappa g$ where $\varkappa > 0$ is sufficiently large. Therefore the following corollary can be formulated.

*Corollary 3.* If (3) is hyper-minimum-phase then there exist $P > 0$, $\varkappa > 0$, $\varepsilon > 0$ such that

$$PA_* + A_*^T P < -\varepsilon I,\ PB = C^T g,\ A_* = A - \varkappa Bg^T C. \tag{4}$$

## 4. ROBUST STATIC CONTROL WITH DELAYS

In this section we investigate robustness of the consensus control with respect to time-varying delay. Consider two types of delayed control laws.

Type-I:

$$u_i(t) = K \sum_{j\in\mathcal{N}_i} (y_i(t - \tau(t)) - y_j(t - \tau(t))), \tag{5}$$

Type-II:

$$u_i(t) = K \sum_{j\in\mathcal{N}_i} (y_i(t) - y_j(t - \tau(t))), \tag{6}$$

where $0 \le \tau(t) \le h$ is a bounded communication delay, $K \in \mathbb{R}^{1\times l}$ and $\mathcal{N}_i = \{k = 1, \dots, N | (v_i, v_k) \in \mathcal{E}\}$ is the set of neighbor vertices to $v_i$.

The problem is to find $K$ from (5) or (6) such that the goal (2) holds.

The problem is analyzed under the following assumptions:

(A1) *The interconnection digraph $\mathcal{G}$ has a converging spanning tree.*

(A2) *There exists a vector $g \in \mathbb{R}^l$ such that the function $g^T W(s)$ is hyper-minimum-phase, where $W(s) = C^T(sI - A)^{-1}B$.*

### 4.1 Consensus conditions for Type-I controller

The main result is as follows.

*Theorem 4.* Let assumptions A1 and A2 hold with some $g \in \mathbb{R}^l$ and $2L_\varphi < \varepsilon\lambda_{\max}^{-1}(P)$ where $\varepsilon$, $P$ are from (4). If $k > 0$ is sufficiently large and $kh$ is sufficiently small then the control law (5) with the feedback gain $K = -kg^T$ ensures the goal (2) for the closed loop system (1), (5).

*Proof.* Let $L$ be a Laplacian matrix of the interconnection digraph $\mathcal{G}$. Closed-loop system (1), (5) can be written as

$$\dot{x}(t) = (I_N \otimes A)x(t) + (L\otimes BKC)x(t-\tau(t)) + \bar{\varphi}(t, x), \tag{7}$$

where $x = \mathrm{col}(x_1, \dots, x_N)$, $u = \mathrm{col}(u_1, \dots, u_N)$, $\bar{\varphi}(t, x) = \mathrm{col}(\varphi(t, x_1), \dots, \varphi(t, x_N))$, and $A\otimes B$ stands for Kronecker product of matrices $A$ and $B$.

Consider $(N \times N)$-matrix of the form

$$M = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & -1 \end{pmatrix}. \tag{8}$$

As far as $L$ is a Laplacian matrix,

$$MLM = \begin{pmatrix} 0 & * \\ \mathbf{0} & \Lambda \end{pmatrix},$$

where $\Lambda \in \mathbb{R}^{(N-1)\times(N-1)}$ and $\mathbf{0} = (0, \dots, 0)^T$.

Since (A1) is true one can apply Agaev-Chebotarev theorem (Theorem 1) and find that

$$\Lambda + \Lambda^T > 0.$$

Consider the following change of variable:

$$\bar{z}(t) = (M \otimes I_n)x(t).$$

As far as $M^{-1} = M$, (7) can be rewritten in the form

$$\dot{\bar{z}}(t) = (I_N \otimes A)\bar{z}(t) + (MLM \otimes BKC)\bar{z}(t - \tau(t)) + \\ +(M \otimes I_n)\bar{\varphi}(t, x).$$

Since $\bar{z}_i = x_1 - x_i$ $(i = 2, \dots, N)$, it is sufficient to investigate the stability of $\bar{z}_i \equiv 0 \quad \forall i = 2, \dots, N$. Denote

$$z(t) = \begin{pmatrix} \bar{z}_2(t) \\ \vdots \\ \bar{z}_N(t) \end{pmatrix}, \Phi(t, \bar{z}_1, z) = \begin{pmatrix} \varphi(t, \bar{z}_1) - \varphi(t, \bar{z}_1 - z_2) \\ \vdots \\ \varphi(t, \bar{z}_1) - \varphi(t, \bar{z}_1 - z_N) \end{pmatrix}.$$

Then

$$\dot{z} = (I_{N-1}\otimes A)z + (\Lambda\otimes BKC)z(t-\tau(t)) + \Phi(t, \bar{z}_1, z). \tag{9}$$

Assumption (A2) implies existence of $P$ and $\varkappa$ such that (4) are true. Consider the following function:

$$V(z) = z(t)^T(I_{N-1} \otimes P)z(t). \tag{10}$$

Differentiation of (10) along (9) yields

$$\dot{V} = z(t)^T(I_{N-1} \otimes \{A^T P + PA\})z(t) + 2z(t)^T \times \\ (\Lambda \otimes PBKC)z(t - \tau(t)) + 2z(t)^T(I_{N-1} \otimes P)\Phi(t, \bar{z}_1, z).$$

Using $z(t - \tau(t)) = z(t) - \int_{t-\tau(t)}^t \dot{z}(s)\, ds$ we find that:

$$\dot{V} = z(t)^T(I_{N-1} \otimes \{A_*^T P + PA_*\})z(t) + \\ z(t)^T(\{2\varkappa I_{N-1} - k[\Lambda + \Lambda^T]\} \otimes \{C^T gg^T C\})z(t) + \\ 2kz(t)^T(\Lambda \otimes C^T gg^T C)\int_{t-\tau(t)}^t \dot{z}(s)\, ds + \\ 2z(t)^T(I_{N-1} \otimes P)\Phi(t, \bar{z}_1, z).$$

where $K = -kg^T$, $A_* = A - \varkappa Bg^T C$. The first term is less than $-\varepsilon\|z(t)\|^2$ since (4) is true. As far as $\Lambda + \Lambda^T > 0$, the second term can be made less than zero by choosing $k \geq 2\varkappa/\lambda_{min}[\Lambda + \Lambda^T]$. The forth term satisfies

$$2|z(t)^T(I_{N-1} \otimes P)\Phi(t, \bar{z}_1, z)| \leqslant 2\lambda_{\max}(P)L_\varphi \|z\|^2. \quad (11)$$

If $V(z(t+\theta)) < pV(z(t))$ $\forall\theta \in [-2h, 0]$ with $p > 1$, then for any $\theta \in [-2h, 0]$, $\|z(t+\theta)\| < q\|z(t)\|$ with $q = \sqrt{p\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}$. Therefore,

$$\left\|\int_{t-\tau(t)}^{t} \dot{z}(s)\,ds\right\| = \left\|\int_{t-\tau(t)}^{t} (I_{N-1} \otimes A)z(s) + (\Lambda \otimes BKC)z(s-\tau(s)) + \Phi(s, \bar{z}_1, z)\,ds\right\| \leq$$

$$\int_{t-\tau(t)}^{t} \lambda_A\|z(s)\| + k\lambda_1\lambda_B\lambda_C\|z(s-\tau(s))\| + L_\varphi\|z(s)\|\,ds,$$

where $\lambda_A = \sqrt{\lambda_{max}(A^TA)}$, $\lambda_1 = \sqrt{\lambda_{max}(\Lambda^T\Lambda)}$, $\lambda_B = \sqrt{B^TB}$, $\lambda_C = \sqrt{\lambda_{max}(C^Tgg^TC)}$. Since $\forall\theta \in [-2h, 0]$, $\|z(t+\theta)\| < q\|z(t)\|$,

$$\left\|\int_{t-\tau(t)}^{t} \dot{z}(s)\,ds\right\| \leq qh\|z(t)\|(\lambda_A + k\lambda_1\lambda_B\lambda_C + L_\varphi).$$

Thereby,

$$\begin{aligned}\dot{V}(t) &\leqslant (2\lambda_{\max}(P)L_\varphi - \varepsilon)\|z(t)\|^2 + \\ &z^T(t)(\{2\varkappa I - k[\Lambda + \Lambda^T]\} \otimes \{C^Tgg^TC\})z(t) + \\ &2k\lambda_1\lambda_C^2 qh(\lambda_A + k\lambda_1\lambda_B\lambda_C + L_\varphi)\|z(t)\|^2.\end{aligned}$$

Conditions of the theorem imply that $2\lambda_{\max}(P)L_\varphi - \varepsilon < 0$. As one can see if

$$k \geqslant \frac{2\varkappa}{\lambda_{min}[\Lambda + \Lambda^T]} \quad (12)$$

and

$$0 \leqslant kh < \frac{\varepsilon - 2\lambda_{\max}(P)L_\varphi}{2\lambda_1\lambda_C^2 q(\lambda_A + k\lambda_1\lambda_B\lambda_C + L_\varphi)}, \quad (13)$$

then $\dot{V} < 0$. It follows from Lyapunov-Razumikhin theorem (Razumikhin [1956]) that $z(t) \equiv 0$ is globally asymptotically stable, that is synchronous solution of (1), (5) is globally asymptotically stable and the control goal (2) is achieved. ■

*Remark 1.* The proof of Theorem 4 gives a lower bound for a control gain (12) and an upper bound for time-delay magnitude (13).

The above result was extended to the case of non-identical delays, i. e. controller of Type-Ia was considered:

$$u_i(t) = K \sum_{j\in\mathcal{N}_i} \left[y_i(t - \tau_{ji}(t)) - y_j(t - \tau_{ij}(t))\right]. \quad (14)$$

In order to ensure the existence of the synchronous solution for the closed loop system an additional assumption should be imposed:

(A3) $\tau_{ij}(t) = \tau_{ji}(t) \quad \forall t \quad \forall i, j = 1, \ldots, N.$

That is the delay matrix $T = \{\tau_{ij}(t)\}$ should be symmetric.

### *4.2 Consensus conditions for Type-II controller*

Now consider the closed-loop system (1), (6). To ensure the existence of a synchronous solution for this system we assume the following.

(A4) $\text{card}(\mathcal{N}_i) = c \quad \forall i = 1, \ldots, N,$

where card stands for cardinality and $c$ is arbitrary constant.

The main result is as follows.

*Theorem 5.* Let assumptions A1, A2 and A4 hold with some $g \in \mathbb{R}^l$ and $2L_\varphi < \varepsilon\lambda_{\max}^{-1}(P)$ where $\varepsilon$, $P$ are from (4). If $k > 0$ is sufficiently large and $kh$ is sufficiently small then the control law (6) with the feedback gain $K = -kg^T$ ensures the goal (2) for the closed loop system (1), (6).

The proof is similar to the proof of Theorem 4 and, therefore, is omitted here.

The results presented in 4.1 and 4.2 were extended to the case of changing topology.

## 5. ADAPTIVE CONTROL

Now consider a network dynamics of which is described by (1) with $\varphi \equiv 0$. Let the $i$-th agent be able to adjust its control gain, i. e. each local controller is adaptive. Denote $\overline{y}_i = \sum_{j\in\mathcal{N}_i}(y_i - y_j)$ and consider the following adaptive controller:

$$\begin{aligned}u_i(t) &= -k_i(t)g^T\overline{y}_i(t), \\ \dot{k}_i(t) &= \overline{y}_i(t)^Tgg^T\overline{y}_i(t),\end{aligned} \quad (15)$$

where $k_i \in \mathbb{R}$ and $g \in \mathbb{R}^l$.

Further we will consider the case of undirected graphs, that is the Laplacian matrix of an interconnected graph is symmetric. Since now we deal with undirected graphs we need to reformulate assumption (A1):

(A1′) *The interconnection graph $\mathcal{G}$ has a spanning tree.*

Adaptive synchronization conditions are formulated as follows.

*Theorem 6.* Let assumptions A1′ and A2 hold with some $g \in \mathbb{R}^l$. Then adaptive controller (15) ensures achievement of the goal (2) for the closed loop system (1), (15) with $\varphi \equiv 0$.

*Proof.* Let $L$ be a Laplacian matrix of the interconnection graph $\mathcal{G}$. Closed-loop system (1), (15) can be written as

$$\begin{aligned}\dot{x}(t) &= (I_N \otimes A)x(t) - (K(t)L \otimes Bg^TC)x(t), \\ \dot{K}(t) &= x^T(t)(L^2 \otimes C^Tgg^TC)x(t),\end{aligned} \quad (16)$$

where $x = \text{col}(x_1, \ldots, x_N)$, $u = \text{col}(u_1, \ldots, u_N)$, $K = \text{diag}(k_1, \ldots, k_N)$, and $A \otimes B$ stands for Kronecker product of matrices $A$ and $B$. It follows from A2 that there exist $P > 0$, $\varkappa > 0$, $\varepsilon > 0$ such that (4) is true. Consider the following Lyapunov function candidate:

$$V(x, k) = x^T(t)(L \otimes P)x(t) + \sum_{i=1}^{N}(k_i(t) - k_*)^2, \quad (17)$$

where $k_*$ will be chosen later. Taking derivative of (17) along (16) we find that

$$\dot{V}(x, k) = x^T(t)(L \otimes \{PA + A^TP\})x(t) - 2x^T(t)(LK(t)L \otimes PBg^TC)x(t) + 2\sum_{i=1}^{N}(k_i(t) - k_*)\dot{k}_i(t).$$

Note that $PB = C^Tg$ and consider the third term:

$$2\sum_{i=1}^{N}(k_i(t)-k_*)\dot{k}_i(t) = 2x^T(t)(LK(t)L\otimes PBg^TC)x(t)-$$
$$2k_*x^T(t)(L^2\otimes C^Tgg^TC)x(t).$$

Laplacian matrix $L$ is real and symmetric, therefore there exists real orthogonal matrix $Q$ such that $QLQ^T$ is diagonal. Then there exists $\eta > 0$, $\eta \in \mathbb{R}$ such that $\eta L^2 > L$. Let us choose $k_* \geq \eta\varkappa$, where $\varkappa > 0$ is taken from (4). Now we conclude that

$$\dot{V}(x,k) \leq x^T(t)(L\otimes\{PA_* + A_*^TP\})x(t),$$

where $A_* = A - \varkappa Bg^TC$. Consider the change of variable

$$\bar{z}(t) = (M\otimes I_n)x(t)$$

with $M$ given by (8). We recall that $M^{-1} = M$. The derivative of $V(x,k)$ transforms to

$$\dot{V}(\bar{z},k) \leq \bar{z}^T(t)(M^TLM\otimes\{PA_* + A_*^TP\})\bar{z}(t).$$

Since $PA_* + A_*^TP < 0$ and

$$M^TLM = \begin{pmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & F \end{pmatrix}$$

with $F > 0$, the following inequality holds

$$\dot{V}(\bar{z},k) \leq -\mu\|z(t)\|^2,$$

where $\mu > 0$ and $z(t) = (\bar{z}_2(t),\ldots,\bar{z}_N(t))^T$. The function $V(\bar{z}(t),k(t))$ is not negative, $V(\bar{z}(0),k(0))$ is finite and

$$V(\bar{z}(t),k(t)) = V(\bar{z}(0),k(0)) + \int_0^t \dot{V}(\bar{z}(s),k(s))\,ds \leq$$
$$V(\bar{z}(0),k(0)) - \mu\int_0^t \|z(s)\|^2\,ds,$$

therefore, there exists a finite $\lim_{t\to\infty} V(\bar{z}(t),k(t))$. Furthermore,

$$\mu\int_0^t \|z(s)\|^2\,ds \leq V(\bar{z}(0),k(0)) - V(\bar{z}(t),k(t)),$$

thus $\int_0^\infty \|z(s)\|^2\,ds < \infty$. It follows from Barbalat's lemma (Khalil [2002]) that $z(s)\to 0$ when $t\to\infty$, i. e. the control goal (2) is achieved. ■

## 6. EXAMPLE. NETWORK OF DOUBLE INTEGRATORS

### 6.1 System description

Consider network $S$, consisting of seven agents $S_i, i = 1,\ldots,7$. Each agent $S_i, i = 1,\ldots,7$ is modeled as follows:

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = C^Tx_i,$$

where $x_i \in \mathbb{R}^2$ is a state vector, $u_i \in \mathbb{R}$ is a control, $y_i \in \mathbb{R}$ is a vector of measurements and

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad C = (0.5\ 0.5).$$

Let the interconnection graph $\mathcal{G}$ be undirected and such as illustrated in Fig. 1.

Let us apply Theorem 6. Transfer function

$$W(\lambda) = g^TC(\lambda I - A)^{-1}B = g^T\frac{\lambda+1}{\lambda^2},$$

is hyper minimum phase when $g = 1$. Thus, according to Theorem 6 the adaptive controller (15) ensures achievement of the control goal (2).

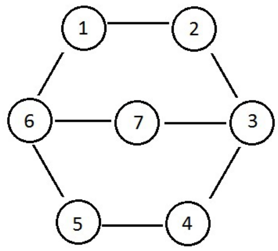


Fig. 1. Interconnection graph $\mathcal{G}$.

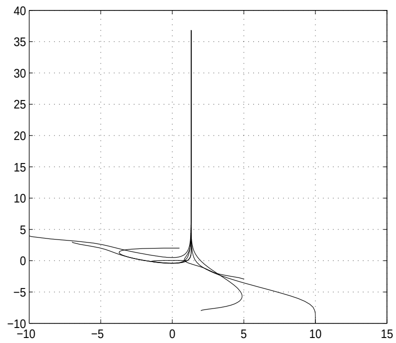


Fig. 2. Phase plane.

First component of a single agent's state vector can be interpreted as a velocity, second component can be interpreted as a position on a straight line. Achievement of the control goal means convergence of seven points on straight line and their motion with constant non-zero velocity.

### 6.2 Simulation results

Let the agents have following initial conditions

$$x_1(0) = \mathrm{col}(0.5,2), \quad x_2(0) = \mathrm{col}(-7,3),$$
$$x_3(0) = \mathrm{col}(1,0), \quad x_4(0) = \mathrm{col}(10,-10),$$
$$x_5(0) = \mathrm{col}(5,-3), \quad x_6(0) = \mathrm{col}(-10,4),$$
$$x_7(0) = \mathrm{col}(2,-8).$$

Achievement of the control goal is illustrated by results of 30 second modeling. Trajectories of agents on the same phase plane are shown in Fig. 2. Sum of error norms $\Delta = \sum_1^6 \|x_i - x_{i+1}\|$ is shown in Fig. 3.

## 7. CONCLUSION

The control algorithm for synchronization of networks based on static delayed output feedback to each agent from the neighbor agents is proposed. Since the number of inputs and outputs of the agents are less than the number of agent state variables, synchronization of agents is achieved under incomplete measurements and incomplete control. Synchronization conditions include passifiability (hyper-minimum-phase property) for each agent and some connectivity conditions for interconnection graph: existence of the converging spanning tree. Similar conditions are

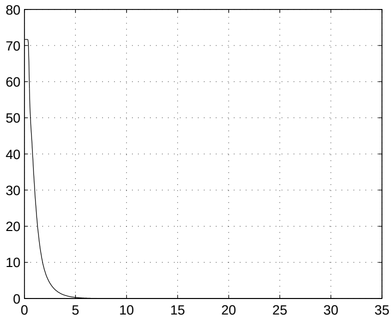


Fig. 3. Sum of error norms $\Delta$.

obtained for adaptive passification-based control of network. Example of adaptive synchronization in a network consisting of seven double integrators is presented.

## REFERENCES

R.P. Agaev, P.Yu. Chebotarev: The matrix of maximal outgoing forests of a digraph and its applications, *Automation and Remote Control*, 61(9): 1424-1450, 2000.

S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D. Hwang. Complex networks: Structure and dynamics. *Physics Reports*, 424(4-5):175–308, 2006.

F. Bullo, J. Cortes, and S. Martinez. *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms*, Princeton University Press, 2009.

P.Yu. Chebotarev, R.P. Agaev: Coordination in multiagent systems and Laplacian spectra of digraphs, *Automation and Remote Control*, 70(3): 128-142, 2009.

I.A. Dzhunusov, A.L. Fradkov: Synchronization in Networks of Linear Agents with Output Feedbacks. *Automation and Remote Control*, 72(8): 1615-1626, 2011.

A.L. Fradkov. Passification of Non-square Linear Systems and Feedback Yakubovich-Kalman-Popov Lemma, *European journal of control*, (6):573–582, 2003.

A.L. Fradkov, G.K. Grigoriev, A.A. Selivanov: Decentralized Adaptive Controller for Synchronization of Dynamical Networks with Delays and Bounded Disturbances. In: *Proc. 50th IEEE Conf. Dec. Contr.*, pp. 1110-1115, 2011.

A.L. Fradkov, I.A. Junussov: Output feedback synchronization for networks of linear agents. In: *7th European Nonlinear Dynamics Conference*, 2011.

A.L. Fradkov, I.V. Miroshnik, and V.O. Nikiforov. *Nonlinear and Adaptive Control of Complex Systems*. Systems. Dordrecht: Kluwer, 1999.

H.K. Khalil. *Nonlinear Systems*, Prentice Hall, 2002.

Z. Li, Z. Duan, G. Chen, and L. Huang. Consensus of Multiagent Systems and Synchronization of Complex Networks: A Unified Viewpoint. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 57(1):213–224, 2010.

R. Olfati-Saber and R.M. Murray. Consensus Problems in Networks of Agents with Switching Topology and Time-Delays. *IEEE Transactions on automatic control*, 49(9): 1520–1533, 2004.

A. Proskurnikov: Average consensus in symmetric nonlinear multi-agent networks with non-homogeneous delays. *Cybernetics and Physics*, 1(2): 128-133, 2012.

B. Razumikhin: On the stability of system with a delay. *Prikl. Math. Mech.*, 20: 500-512, 1956. [In Russian]

L. Scardovi and R. Sepulchre. Synchronization in networks of identical linear systems. *Automatica*, 45(11):2557–2562, 2009.

E. Steur and H. Nijmeijer. Synchronization in Networks of Diffusively Time-Delay Coupled (Semi-)Passive Systems. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 58(6):1358–1371, 2011.