ISSN 0005-1179, Automation and Remote Control, 2007, Vol. 68, No. 7, pp. 1186–1200. © Pleiades Publishing, Ltd., 2007.
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Adaptive Observer-based Synchronization of the Nonlinear Nonpassifiable Systems[1]

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Received December 4, 2006

Abstract—The general approach to synchronization of the dynamic systems on the bassis on the adaptive observer and the passification method was extended to the nonpassifiable nonlinear systems—in particular, to those whose model has the relative order higher than one. Two schemes of synchronization relying on the extended-error adaptive observers and the high-order tuning algorithms were proposed. Solution of the problem of synchronization relies on a new canonical form of the adaptive observer. The conditions for convergence of the parameter estimates to the true values in the case of no system noise were established, and also robustness of the adaptive synchronization to the bounded measurement error was proved. The feasibility of information transmission by modulation of the chaotic signal with the use of the proposed method was demonstrated by the example of the controllable Lorentz system.

PACS number: 02.30.Yy

DOI: 10.1134/S0005117907070077

1. INTRODUCTION

Synchronization which is one of the most important properties of the interacting oscillatory systems founds numerous applications in mechanics and physics [1–3], vibration technologies [1, 2], radio and communication engineering [4, 5], and other areas. In the recent years, the interest in the problems of *control of synchronization* introducing additional feedbacks with the purpose of supporting synchronous course of the system processes is growing. In particular, the interest in the so-called *chaotic synchronization* where each of the synchronized subsystems continues to perform complex chaotic motions after stabilization of the synchronous mode [6–8] has grown appreciably since the early 1990's. A number of practices using the effect of chaotic synchronization to enhance security and reliability of information transmission was proposed (see, for example, the reviews and special issues of journals [9–16]). A number of results concerning application of the adaptive methods was obtained in this field [17–20]. The problem of *adaptive observer*-based synchronization was considered in [8, 16, 21–24] along the same lines as that of adaptive synchronization control. The arising problems differ from the traditional problems of observation in that the observed plant model is nonlinear and, possibly, unstable.

A new method of adaptive synchronization on the basis of the Lyapunov functions and passification was proposed in [17, 20] and extended in [21–23, 25, 26] to the problems of synchronization of communication systems on the basis of adaptive observers. However, the approach of [17, 20] was constrained by the condition for plant (signal source or "master system") *passifiability* which, in particular, implies that the plant relative degree should not be greater than one. The requirement

[1] This work was supported by the Russian Foundation for Basic Research, projects nos. 05-01-00869, 06-08-01386, and the Research Program no. 22 "Control Processes" of the Presidium of the Russian Academy of Sciences, project no. 1.8.

of passifiability prevents improvement of information security because the stepwise change of the useful signal gives rise to an easily detectable jump in the derivative of the transmitted signal [27].

In what follows, the adaptive observer-based method of synchronization is extended to the *nonpassifiable* systems including those with the relative degree of the linear part greater than one. The proposed solution relies on the recent results obtained by the theory of adaptive control of the nonlinear plants [22, 28–30] including the structure of adaptive observers and new classes of the adaptation algorithms proposed in [31, 32].

We note that some methods of adaptive synchronization of the systems with the relative degree greater than one were proposed earlier in [27, 33], but the synchronization algorithm of [33] makes use of the state feedback, rather than the output feedback and, in addition, assumes that all parameters of the signal source (master system) are known. The approach of [27] proceeds from the canonical forms of the linear state observers (see, for example [34]). The design of the *nonlinear* observers on the basis of [27] is applicable only to particular forms of systems and requires special techniques for each kind of nonlinearity. In particular, it is not clear how the method of [27] can be used to synchronize the Lorentz systems. Finally, the results of [27] do not allow one to take into consideration the errors in the transmitted signal.

In distinction to [27, 33], the present paper suggests a general method of adaptive synchronization enabling also synchronization in the presence of bounded noise in the communication channel by robustification ("roughening") of the adaptation algorithms. The results of simulation presented in what follows demonstrate higher speed and robustness of the adaptive observer ("slave system") as compared with the method of [25]. The preliminary results were presented in [35–37].

The problem is formulated in mathematical terms in Section 2. Sections 3 and 4 describe and substantiate the general method of designing the adaptive observers for systems with a relative degree higher than one. For presentational convenience, Section 3 considers a simpler case of no system noise for which the basic structures of the adaptive observers are presented and the convergence conditions are derived. The basic algorithms enabling system robustness under external disturbances (measurement noise) are modified in Section 4. Application of the proposed method to synchronization of the controlled Lorentz systems is demonstrated in Section 5. For this example, presented are the results of simulation that are in good agreement with the theoretical propositions and provide quantitative information about the system processes. Section 5 also demonstrates application of the proposed method of adaptive synchronization to the design of the algorithms for information transmission by modulation of the chaotic signals.

2. FORMULATION OF THE PROBLEM

We follow [21] in assuming that the dynamic plant (*master system* obeys the state equations

$$\dot{x} = Ax + \varphi_0(y) + b\varphi(y)^{\mathrm{T}}\theta, \quad y = c^{\mathrm{T}}x, \tag{2.1}$$

$$y_r = y + \xi, \tag{2.2}$$

where $x \in \mathbb{R}^n$ is the state vector whose values are not transmitted to the slave system,[2] y is the plant output, $y_r = y + \xi(t)$ is the measured output entering the input of the slave system, $\xi(t)$ is the measurement error (additive noise in the communication channel between the master and slave systems), and $\theta \in \mathbb{R}^m$ is the unknown vector of the plant parameters (in the telecommunication systems this vector carries the transmitted information). The nonlinear dependences $\varphi_0(y)$ and $\varphi(y)$ as well as the matrix A and vectors b and c are assumed to be known and can be used by the synchronization algorithm realized in the slave system. We make the following assumptions about the plant (2.1).

[2] In what follows, we draw on the analogies with the telecommunication systems and say that the slave system receives the signal from the master system through a *communication channel*.

Assumption 1. *The state vector $x(t)$ is a bounded time function for any bounded initial conditions $x(0)$ and any admissible value of the vector θ.*

Assumption 2. *The functions $\varphi_0(y)$ and $\varphi(y)$ are bounded for any bounded y.*

The problem of designing the adaptive observer which is the following dynamic system

$$\dot{z} = F(z, y_r), \quad \widehat{\theta} = h(z, y_r) \tag{2.3}$$

satisfying the objective condition $\overline{\lim_{t\to\infty}}|\theta - \widehat{\theta}| \leq \delta$ is considered under these assumptions.

3. DESIGN OF THE ADAPTIVE OBSERVERS UNDER NO PERTURBATIONS

In this section, the measurement error is disregarded, that is, it is assumed in (2.3) that $\xi(t) \equiv 0$. In what follows, we confine ourselves to the following special form of the adaptive observer (2.3) (*slave system*):

$$\dot{\widehat{x}} = A\widehat{x} + \varphi_0(y) + b\varphi(y)^{\mathrm{T}}\widehat{\theta} + k(y - \widehat{y}), \quad \widehat{y} = c^{\mathrm{T}}\widehat{x}, \tag{3.1}$$

$$\dot{\widehat{\theta}} = F_\theta(\widehat{\theta}, \widehat{x}, y), \tag{3.2}$$

where $\widehat{x}$ and $\widehat{y}$ are the estimates of the variables x and y, respectively, the vector $z = \mathrm{col}\,(x, \widehat{\theta})$, and $\widehat{\theta}$ is the vector of adjustable parameters which serves as the estimate of the vector of the master system parameters θ.

The structure of the adaptive observer (3.1), (3.2) was proposed in [21, 23] where the convergence conditions were established for the case of passifiable linear part of the plant (master system). The paper [25] considered the action of the bounded perturbations for this plant.

The passifiability condition impose tight constraints on the plant properties: the relative degree r of the plant model must be unity, which limits the domain of applicability of the method of adaptive observers and stimulates one to seek solutions to the problem of constructing the adaptive observers for systems with $r > 1$. Two methods of solving the formulated problem are proposed below with regard for the above assumptions. The first method is based on the approach of *extended error* considered in [22, 38, 39], the second method relies on the *algorithm of high-order tuning* [22, 28, 31, 32, 40].

The extended error is used by means of the adaptive observer like (3.1), (3.2) where the vector k is chosen using the condition for Hurwitz stability of the matrix $F = A - kc^{\mathrm{T}}$. To derive the adaptive algorithm, we first determine the so-called *error model* for which purpose we differentiate the estimation error $\varepsilon = x - \widehat{x}$ with regard for the expressions (2.1) and (3.1), and obtain

$$\dot{\varepsilon} = F\varepsilon + b\varphi(t)^{\mathrm{T}}\widetilde{\theta}, \quad e = c^{\mathrm{T}}\varepsilon, \tag{3.3}$$

where the *regressor* $\varphi(t) = \varphi(y(t))$, $\widetilde{\theta} = \theta - \widehat{\theta}$ is the error of parameter estimation and $e = y - \widehat{y}$ is the mismatch between the outputs of the plant and observer. The error Eq. (3.3) is rearranged in

$$e = H(p)\,[\varphi(t)^{\mathrm{T}}\widetilde{\theta}\,], \tag{3.4}$$

where $p = \mathrm{d}\,/\mathrm{d}\,t$ is the time differentiation operator and the transfer function $H(s)$ of system (3.4), $H(s) = c^{\mathrm{T}}(s\mathrm{I} - F)^{-1}b$, is asymptotically stable.

We follow [22, 38, 39] in using the adaptive algorithm

$$\dot{\widetilde{\theta}} = \gamma\omega(t)^{\mathrm{T}}\widehat{e}, \tag{3.5}$$

where $\gamma > 0$ is the gain of the algorithm, $\omega(t) = H(p)[\varphi(t)]$ is the regressor passed through a filter with the transfer function $H(s)$, and $\widehat{e}$ is the *extended error signal* obeying the following expression:

$$\widehat{e} = e + H(p)[\varphi(t)^{\mathrm{T}}\widehat{\theta}\,] - \omega(t)^{\mathrm{T}}\widehat{\theta}. \tag{3.6}$$

Now, we make use of the following definition.

Definition 1 ([38, 41]). The vector function $f : [0,\infty) \to \mathbb{R}^m$ is referred to as permanently exciting (PE) on $[0,\infty)$ if it is measurable and bounded on $[0,\infty)$ and there are $\alpha > 0$, $T > 0$, such that

$$\int\limits_t^{t+T} f(s)f(s)^T ds \geq \alpha I \tag{3.7}$$

for all $t \geq 0$.

The sense of the PE condition lies in that the vector $f(t)$ does not approach any hyperplane in the space $\mathbb{R}^m$ with t, that is, the components of the vector $f(t)$ are linearly independent in the strengthened sense. We formulate a theorem establishing the conditions for synchronization on the basis of the adaptive observer in the absence of measurement errors.

Theorem 1. *The closed-loop system consisting of the master system* (2.1)*, tuned observer* (3.1)*, and the adaptive algorithm* (3.5), (3.6)*, has the following characteristics:*

(i) *for any initial conditions and any $\gamma > 0$, all variables involved in the equations of the closed-loop system are bounded and*

$$\lim_{t\to\infty}(y(t) - \widehat{y}(t)) = 0 \tag{3.8}$$

is satisfied;

(ii) *if the vector function $\varphi(t)$ satisfies the PE condition and the transfer function $H(s)$ is minimum phase, then* (*except for* (i))

$$\lim_{t\to\infty}|\theta - \widehat{\theta}(t)| = 0 \tag{3.9}$$

also is satisfied.

Theorem 1 is proved in the Appendix.

We note that it is required to check for linear independence the components of $\varphi(t)$ in order to check the PE conditions for the function $\varphi(t) = \varphi(y(t))$. If $y(t)$ is a scalar, then the condition for linear dependence $\sum\limits_{i=1} m\varphi_i(y)C_i$ becomes a nonlinear equation. If this equation has only a finite number of roots y_j, $j = 1,\ldots,k$, (for example, the nonlinearities are representable as polynomials), then it suffices to prove that the hyperplanes $y = y_j$ $(C^{\mathrm{T}}x = y_j)$ are not attracting sets of system (2.1), which in turn is knowingly satisfied if the matrix A is unstable. Therefore, the PE condition often admits an effective check.

Now we consider another solution of the formulated problem where the high-order tuners are used. With this aim in view, we make use of the tunable observer

$$\dot{\widehat{x}} = A\widehat{x} + \varphi_0(y) + b\nu(y,\widehat{\theta}) + k(y - \widehat{y}), \quad \widehat{y} = c^{\mathrm{T}}\widehat{x}, \tag{3.10}$$

where $\nu(y,\widehat{\theta})$ is the tuned feedback whose equation is presented in what follows.

For the system under consideration, the error equation takes the form

$$\dot{\varepsilon} = F\varepsilon + b\left(\varphi(t)^{\mathrm{T}}\theta - \nu\right), \quad e = c^{\mathrm{T}}\varepsilon,$$

that is, can be rearranged in the "input–output" equations

$$e = H(p)\left[\varphi(t)^{\mathrm{T}}\theta - \nu\right]. \tag{3.11}$$

We choose the transfer function $W(s)$ as follows:

$$W(s) = (s+\lambda)H(p),$$

where λ is an arbitrary positive constant. Then, Eq. (3.11) is representable as

$$e = \frac{1}{p+\lambda}\Big[\overline{\omega}(t)^{\mathrm{T}}\theta - W(p)[\nu]\Big], \tag{3.12}$$

where $\overline{\omega}(t) = W(p)[\varphi(t)]$.

Reasoning from the form of the error Eq. (3.12), we take the tuned feedback as

$$\nu = W(p)^{-1}\Big[\overline{\omega}(t)^{\mathrm{T}}\widehat{\theta}\Big], \tag{3.13}$$

where $\widehat{\theta}$ is the tuned parameter vector. To realize feedback (3.13), one needs to know not only the tuned parameters $\widehat{\theta}$, but also their derivatives up to the order $(r-1)$ inclusively, where r is the relative degree of the transfer function $H(s)$. To overcome this difficulty caused by the differentiation of $\widehat{\theta}$, we use the following adaptive algorithm:

$$\dot{\widehat{\psi}}_i = \overline{\omega}_i e, \tag{3.14}$$

$$\dot{\eta}_i = \left(1 + \mu\overline{\omega}^{\mathrm{T}}\overline{\omega}\right)\left(\Gamma\eta_i + h\widehat{\psi}_i\right), \tag{3.15}$$

$$\widehat{\theta}_i = l^{\mathrm{T}}\eta_i, \tag{3.16}$$

where $i = 1, \ldots, m$, $\mu > 0$ is the algorithm's parameter, (l, Γ, h) is the minimum realization of the unit with the transfer function $\alpha(0)/\alpha(s)$ whose denominator $\alpha(s)$ is the Hurwitz polynomial of the order $(r-2)$, that is, $\alpha(0)/\alpha(s) = l^{\mathrm{T}}(s\mathrm{I} - \Gamma)^{-1}h$ is satisfied.

Note. There is no need to use the additional filters (3.15) and (3.16) for $r \leq 2$. For this special case, the adaptive algorithm becomes $\dot{\widehat{\theta}} = \overline{\omega}e$.

The following synchronization theorem is valid.

Theorem 2. *The closed-loop system consisting of the master system* (2.1)*, observer* (3.10)*, and the tuned feedback* (3.13)–(3.16)*, has the following properties:*

(i) *for any initial conditions and any μ such that the inequality*

$$\mu > \frac{3}{4\lambda}\left(|l| + |OF^{-1}h|\right)^2$$

is satisfied where the positive definite matrix P satisfies the equation $F^{\mathrm{T}}P + PF = -2\mathrm{I}$, all variables of the closed-loop system are bounded and (3.8) *tends to zero;*

(ii) *if the vector function $\varphi(t)$ satisfies the PE condition and the transfer function $H(s)$ is minimum phase, then the asymptotic convergence of* (3.9) *takes place in addition to i.*

The proof is based on standard reasoning; see, for example, [22, 31].

Let us consider now a more general case where the master system obeys the equations

$$\dot{x} = A(y)x + \varphi_0(y) + b\varphi(y)^{\mathrm{T}}\theta, \quad y = c^{\mathrm{T}}x \tag{3.17}$$

and make the following additional assumptions for it.

Assumption 3. *There are a vector function $k(y) \in \mathbb{R}^n$ and scalar function $V(x)$ such that*

$$c_1|x|^2 \le V(x) \le c_2|x|^2,$$
$$\frac{\partial V}{\partial x}(x)\left(A(c^{\mathrm{T}}x) - k(c^{\mathrm{T}}x)c^{\mathrm{T}}\right)x \le -c_3|x|^2,$$
$$\left|\frac{\partial V}{\partial x}\right| \le c_4|x|,$$

where c_i are some positive constants ($i = \overline{1,4}$).

Assumption 4. *For any bounded y, all elements of the matrix $A(y)$ are bounded as well.*

Stated differently, Assumption 3 implies exponential stability of the autonomous system

$$\dot{x} = G(c^{\mathrm{T}}x)x,$$

where $G(c^{\mathrm{T}}x) = G(y) = A(y) - k(y)c^{\mathrm{T}}$.

It deserves noting that the aforementioned methods of design are inapplicable to systems like (3.17). Therefore, we propose another method which is a variety of the method with the extended error signal. We introduce for that purpose a tunable observer of the form

$$\dot{\widehat{x}} = A(y)\widehat{x} + \varphi_0(y) + b\varphi(y)^{\mathrm{T}}\widehat{\theta} + k(y)(y - \widehat{y}), \quad \widehat{y} = c^{\mathrm{T}}\widehat{x}, \tag{3.18}$$

where the time-varying vector function $k(y(t))$ is chosen so as to satisfy Assumption 3. In this case, the error model takes the form

$$\dot{\varepsilon} = G(t)\varepsilon + b\varphi(t)^{\mathrm{T}}\widetilde{\theta}, \quad e = c^{\mathrm{T}}\varepsilon, \tag{3.19}$$

where $G(t) = G(c^{\mathrm{T}}x(t))$.

Now we define the extended error signal as

$$\widehat{e} = e + c^{\mathrm{T}}\eta, \tag{3.20}$$

where the additional vector η is generated by the following filters

$$\dot{\eta} = G(t)\eta - \Omega\dot{\widehat{\theta}}, \quad \eta \in \mathbb{R}^n, \tag{3.21}$$
$$\dot{\Omega} = G(t)\Omega + b\varphi(t)^{\mathrm{T}}, \quad \Omega \in \mathbb{R}^{n\times n}. \tag{3.22}$$

Then, according to [39] one may use the adaptation algorithm

$$\dot{\widehat{\theta}} = \gamma\omega^{\mathrm{T}}\widehat{e}, \tag{3.23}$$

where $\omega = c^{\mathrm{T}}\Omega$. We formulate the following theorem.

Theorem 3. *The closed-loop system consisting of the master system* (3.17)*, tuned observer* (3.18)*, filters of the extended error signal* (3.20)–(3.22)*, and the adaptive algorithm* (3.23) *has the following characteristics:*

(i) *for any initial conditions and any $\gamma > 0$, all variables in the closed-loop system are bounded and* (3.8) *converges to zero;*

(ii) *if the vector function $\varphi(t)$ also satisfies the PE condition, then* (3.9) *converges asymptotically.*

Theorem 3 is proved in the Appendix.

4. ROBUST ADAPTIVE OBSERVER FOR THE SYSTEM WITH MEASUREMENT NOISE

It is common knowledge that the adaptation algorithms based on "pure" integration can loose stability under the action of external perturbations or measurement noise [22, 41, 42]. To retain stability of the adaptive systems under these conditions, various methods of robustification ("roughening") of the adaptation algorithms are developed. Two modifications of the algorithms proposed in the last section are described below.

Let us consider the case of additive perturbations in the communication channel, that is, assume that the slave system receives the noisy signal (2.2) instead of the true signal from the output of the master system y. To make the process of adaptation stable under these conditions, proposed is an adaptive observer with the extended error signal which follows the equations

—*tuned observer*

$$\dot{\widehat{x}} = A\widehat{x} + \varphi_0(y_r) + b\varphi(y_r)^{\mathrm{T}}\widehat{\theta} + k(y_r - \widehat{y}), \quad \widehat{y} = c^{\mathrm{T}}\widehat{x}, \tag{4.1}$$

—*generator of the extended error signal*

$$\overline{e} = y_r - \widehat{y} + H(p)[\overline{\varphi}(t)^{\mathrm{T}}\widehat{\theta}\ ] - \overline{\omega}(t)^{\mathrm{T}}\widehat{\theta}, \tag{4.2}$$

where $\overline{\varphi}(t) = \varphi(y_r(t)), \overline{\omega}(t) = H(p)[\overline{\varphi}(t)]$,

—*robustified adaptation algorithm*

$$\dot{\widehat{\theta}} = \gamma\overline{\omega}(t)\overline{e} - \alpha(\widehat{\theta})\widehat{\theta}, \tag{4.3}$$

where the function $\alpha(\widehat{\theta})$ satisfies the relations

$$\alpha(\widehat{\theta}) = \begin{cases} 0, & |\widehat{\theta}| < \theta^* \\ \left(\dfrac{|\widehat{\theta}|}{\theta^*} - 1\right), & \theta^* \le |\widehat{\theta}| \le 2\theta^* \\ 1, & |\widehat{\theta}| > 2\theta^* \end{cases} \tag{4.4}$$

with some positive constant θ^*. We formulate the results concerning the properties of the robustified closed-loop system as the following theorem.

Theorem 4. *The closed-loop system consisting of the master system* (2.1), (2.2), *tuned observer* (4.1), *generator of the extended error signal* (4.2), *and the adaptation algorithm* (4.3), (4.4), *has the following properties:*

(i) *for any initial conditions and any* $\gamma > 0$, $\theta^* > 0$, *all variables in the closed-loop system are bounded and the estimate vector* $\widetilde{\theta}$ *hits the limiting set*

$$D = \left\{\widetilde{\theta} : |\widetilde{\theta}|^2 \le \max\left[(|\theta| + 2\theta^*)^2, \gamma\|\xi + \xi_e\|_\infty^2 + |\theta|^2\right]\right\}, \tag{4.5}$$

where the bounded variable ξ_e *satisfies the equations*

$$\dot{\Xi}_e = F\Xi_e + \varphi_0(y) - \varphi_0(y_r) + b\big(\varphi(y) - \varphi(y_r)\big)^{\mathrm{T}}\theta - k\xi, \quad \xi_e = c^{\mathrm{T}}\Xi_e; \tag{4.6}$$

(ii) *if* $\xi(t) \equiv 0$ *and* $\theta^* > |\theta|$, *then* (3.8) *tends to zero, except for* (i);

(iii) *if, in addition, the vector function* $\varphi(t)$ *satisfies the PE condition and the transfer function* $H(s)$ *is minimum phase, then* (3.9) *converges.*

Theorem 4 is proved in the Appendix.

To extend the result obtained to the case of measurement noise in the output of the master system (3.17), (2.2), we make use of the adaptive observer with extended error signal which obeys the equations

—*tuned observer*

$$\dot{\widehat{x}} = A(y_r)\widehat{x} + \varphi_0(y_r) + b\varphi^{\mathrm{T}}(y_r)\widehat{\theta} + k(y_r)(y_r - \widehat{y}), \quad \widehat{y} = c^{\mathrm{T}}\widehat{x}, \tag{4.7}$$

—*generator of the extended error signal*

$$\overline{e} = y_r - \widehat{y} + c^{\mathrm{T}}\overline{\eta}, \tag{4.8}$$

$$\dot{\overline{\eta}} = \overline{G}(t)\overline{\eta} - \overline{\Omega}\,\dot{\widehat{\theta}}, \quad \overline{\eta} \in \mathbb{R}^n, \tag{4.9}$$

$$\dot{\overline{\Omega}} = \overline{G}(t)\overline{\Omega} + b\overline{\varphi}(t)^{\mathrm{T}}, \quad \overline{\Omega} \in \mathbb{R}^{n\times n}, \tag{4.10}$$

where $\overline{G}(t) = A(y_r(t)) - k(y_r(t))c^{\mathrm{T}}$,

—*robustified adaptation algorithm*

$$\dot{\widehat{\theta}} = \gamma\overline{\varpi}(t)\overline{e} - \alpha(\widehat{\theta})\widehat{\theta}, \tag{4.11}$$

where $\overline{\varpi}(t) = c^{\mathrm{T}}\,\overline{\Omega}(t)$ and the function $\alpha(\widehat{\theta})$ satisfied condition (4.4). We formulate the theorem of robustified adaptive observer in noise.

Theorem 5. *The closed-loop system consisting of the master system* (3.17), (2.2)*, tuned observer* (4.7)*, generator of the extended error signal* (4.8)–(4.10)*, and the adaptation algorithm* (4.11), (4.4) *has the following properties:*

(i) *under any initial conditions and any $\gamma > 0$ and $\theta^* > 0$, all variables in the closed-loop system are bounded and the parametric error $\widetilde{\theta}$ hits the limiting set*

$$D = \left\{\widetilde{\theta} : |\widetilde{\theta}|^2 \le \max\left((|\theta| + 2\theta^*)^2, \gamma\|\xi + \xi_y\|_\infty^2 + |\theta|^2\right)\right\}, \tag{4.12}$$

where the bounded variable ξ_y satisfies the equations

$$\dot{\Xi}_y = \overline{G}(t)\Xi_y + \varphi_0(y) - \varphi_0(y_r) + b\big(\varphi(y) - \varphi(y_r)\big)^{\mathrm{T}}\theta - k(y_r)\xi + \big(A(y) - A(y_r)\big)x,$$
$$\xi_y = c^{\mathrm{T}}\Xi_y;$$

(ii) *if $\xi(t) \equiv 0$ and $\theta^* > |\theta|$, then, in addition to* (i), (3.8) *tends to zero as well;*

(iii) *if in addition the vector function $\varphi(t)$ satisfies the PE condition and the transfer function $H(s)$ is minimum phase, then* (3.9) *converges.*

Theorem 4 is proved in the Appendix.

5. EXAMPLE. INFORMATION TRANSMISSION ON THE BASIS OF ADAPTIVE SYNCHRONIZATION OF THE CONTROLLED CHAOTIC LORENTZ SYSTEMS

5.1. Design of the Adaptive Observer with an Extended Error Signal for the Lorentz System

Let us consider by way of example application of the proposed method of adaptive synchronization to the chaotic Lorentz system. Let the *Lorentz system* obeying the equations [8, 13, 23, 43]

$$\begin{cases} \dot{x}_1 = \sigma x_2 - \sigma x_1 \\ \dot{x}_2 = -x_2 - x_1 x_3 + \theta x_1 \\ \dot{x}_3 = -\beta x_3 + x_1 x_2 \end{cases} \tag{5.1}$$

be used as the master system. The constant parameters β and σ are regarded as unknown, that is, their values can be used in the design of the slave system, and the parameter θ is variable, its values varying depending on the transmitted useful information (transmitted message) and must be restored by the observer. It is assumed also that the component x_1 of the state vector is the output of the master system $y \equiv x_1$.

Obviously, system (5.1) is a special case of (3.17) where

$$A(y) = \begin{bmatrix} -\sigma & \sigma & 0 \\ 0 & -1 & -y \\ 0 & y & -\beta \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \tag{5.2}$$
$$\varphi_0(y) = [0, 0, 0], \quad \varphi(y) = y, \quad c^{\mathrm{T}} = [1, 0, 0].$$

One can readily see that Assumption 4 is satisfied for the given system. As follows from Assumption 3, in order to apply Theorem 3 one needs to determine the vector function $k(y) \in \mathbb{R}^3$ such that the system $\dot{x} = \left(A(y) - k(y)c^{\mathrm{T}}\right)x$, $y = c^{\mathrm{T}}x$ is asymptotically stable. For that we take $k(y) \equiv k = [0, -\sigma, 0]^{\mathrm{T}}$. Then, the matrix function $G(y) = A(y) - k(y)c^{\mathrm{T}}$ is the sum of the diagonal and skew-symmetric matrices:

$$G(y) = \begin{bmatrix} -\sigma & \sigma & 0 \\ -\sigma & -1 & -y \\ 0 & y & -\beta \end{bmatrix}. \tag{5.3}$$

One may easily verify that with this choice Assumption 3 is satisfied. Indeed, let us consider system $\dot{x} = G(c^{\mathrm{T}}x)x$ where the matrix $G(y)$ obeys (5.3) and introduce the Lyapunov function $V(x) = \frac{1}{2}x^{\mathrm{T}}x$. By differentiating the function $V(x(t))$ with respect to t, we obtain

$$\dot{V}(x) = \frac{1}{2}x^{\mathrm{T}}\left(G(c^{\mathrm{T}}x)^{\mathrm{T}} + G(c^{\mathrm{T}}x)\right)x$$
$$= x^{\mathrm{T}}\begin{bmatrix} -\sigma & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}x = -\sigma x_1^2 - x_2^2 - \beta x_3^2 \le c_3|x|^2$$

for $c_3 = \min\{\sigma, 1, \beta\}$, whence the *exponential stability* of the system $\dot{x} = G(c^{\mathrm{T}}x)x$ follows immediately. Therefore, Assumptions 3 and 4 are satisfied, which enables use of Theorem 3. The tuned observer for the master system in the form of the Lorentz system (5.1) obeys Eq. (3.18) with matrices of the form (5.2). To estimate the unknown parameter θ and, consequently, to restore the message, the tuning algorithm (3.20)–(3.23) with $n = 3$ is realized in the observer. The robustified adaptation algorithm (4.3), (4.4) is used for an appreciable level of noise in the communication channel. Let us consider some numerical examples of using the proposed method for message transmission by modulating the Lorentz generator.

5.2. Numerical Example: Restoration of the "Rectangular Wave" Signal

To restore the values of the parameter θ of the master system (5.1), we employ the algorithm (3.18), (3.20)–(3.23). It was assumed above that θ is an unknown constant. In the information transmission systems, this parameter varies in compliance with the transmitted data, $\theta = \theta(t)$. Therefore, applicability of the proposed method depends on the rate of tuning the observer parameter $\widehat{\theta}(t)$ which is determined by means of numerical simulation.

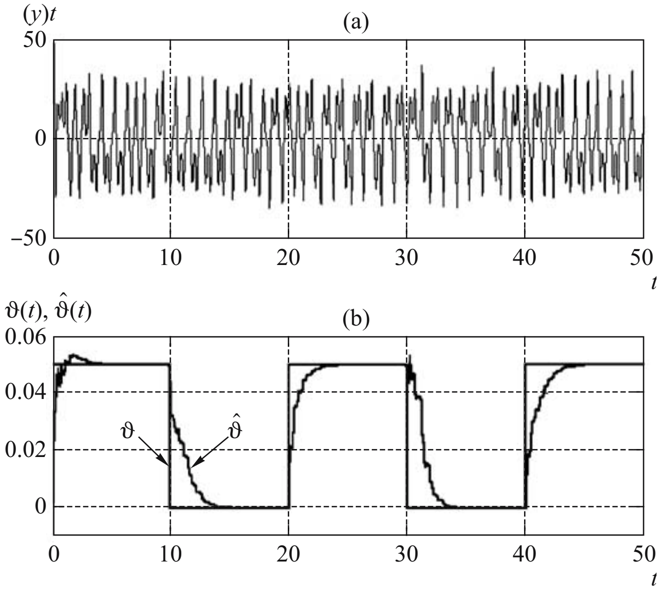

Fig. 1. (a) Chaotic input of the slave system $y_r(t)$; (b) information signal in the master system $\vartheta(t)$ and its estimate $\widehat{\vartheta}(t)$ by the slave system. Algorithm (5.5)–(5.8).

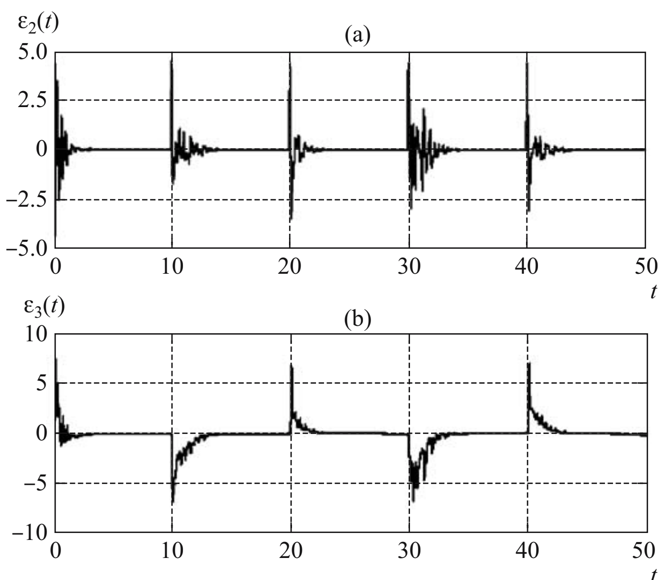

Fig. 2. Errors of state estimates $\varepsilon_2(t)$ and $\varepsilon_3(t)$ by the algorithm (5.5)–(5.8).

In the example under consideration, there exists a system defined by the following equations:
—*master system:*

$$\begin{cases} \dot{x}_1 = \sigma x_2 - \sigma x_1 \\ \dot{x}_2 = -x_2 - x_1 x_3 + r\big(1 + \vartheta(t)\big)x_1 \\ \dot{x}_3 = -\beta x_3 + x_1 x_2, \end{cases} \tag{5.4}$$
$$y(t) = x_1(t),$$

where r is a certain constant and $\vartheta(t)$ is a variable parameter. In the communication systems using of the chaotic generator, $\vartheta(t)$ plays the part of the *information signal*. In the example at hand, $\theta(t) = r(1 + \vartheta(t))$;

—*tuned observer:*

$$\begin{cases} \dot{\widehat{x}}_1 = \sigma\widehat{x}_2 - \sigma\widehat{x}_1 \\ \dot{\widehat{x}}_2 = -\widehat{x}_2 - y_r(t)\widehat{x}_3 + \sigma e(t) + r(1 + \widehat{\vartheta}(t))y_r(t) \\ \dot{\widehat{x}}_3 = -\beta x_3 + y_r(t)\widehat{x}_2, \end{cases} \quad (5.5)$$
$$e(t) = y_r(t) - \widehat{x}_1(t),$$

where $e(t)$ is the *observation error*, $y_r(t)$ is the measured input of the slave system (for the communication systems, y_r is the *received signal*). In the absence of perturbations, $y_r(t) \equiv y(t)$ is satisfied;

—*generator of the extended error signal:*

$$\begin{cases} \dot{\Omega}_1 = \sigma\Omega_2 - \sigma\Omega_1 \\ \dot{\Omega}_2 = -\sigma\Omega_1 - \Omega_2 + y_r(t)\Omega_3 \\ \dot{\Omega}_3 = -\beta\Omega_3 + y_r(t)\Omega_2, \end{cases} \quad (5.6)$$
$$\omega(t) = \Omega_1(t),$$

$$\begin{cases} \dot{\eta}_1 = \sigma\eta_2 - \sigma\eta_1 - \Omega_1(t)\dot{\widehat{\vartheta}}(t) \\ \dot{\eta}_2 = -\sigma\eta_1 - \eta_2 + y_r(t)\eta_3 - \Omega_2(t)\dot{\widehat{\vartheta}}(t) \\ \dot{\eta}_3 = -\beta\eta_3 + y_r(t)\eta_2 - \Omega_3(t)\dot{\widehat{\vartheta}}(t); \end{cases} \quad (5.7)$$

—*adaptation algorithm:*

$$\widehat{e}(t) = e(t) + c^{\mathrm{T}}\eta(t),$$
$$\dot{\widehat{\vartheta}} = \gamma\omega\widehat{e}, \quad \widehat{\vartheta}(0) = \widehat{\vartheta}_0, \quad (5.8)$$

where the positive parameter $\gamma > 0$ plays the role of the algorithm *gain*.

The following values of the parameters of system (5.5)–(5.8) were used in modeling: $\sigma = 10$, $\beta = 8/3$, $r = 97$, $\gamma = 0.45$. The "rectangular wave" was used as the information signal $\vartheta(t)$. The results are depicted in Figs. 1 and 2.

The graph of the input to the slave system $y_r(t)$ is shown in Fig. 1a. Figure 1b shows the graphs of the initial information signal $\vartheta(t)$ and that restored by the adaptive observer (5.5)–(5.8). Figure 2 represents the components of the vector of state estimate errors $\varepsilon_i(t) = x_i(t) - \widehat{x}_i(t)$, where $i = 1, 2$. The results of simulation demonstrate that the process of adaptation by the proposed algorithm proceeds with high rate and the time of identification of the unknown parameter by this algorithm is close to that of synchronization of the processes of the master and slave systems.

6. CONCLUSIONS

A general method of the nonlinear adaptive synchronization of the chaotic systems based on the approach of [17, 20, 21, 25] and the adaptation algorithms of [22, 31, 32] was obtained. It enables one to make use of the *nonpassifiable* nonlinear systems—in particular, systems with the relative order greater than one—as the sources of chaotic signals for transmission of information on the basis of adaptive synchronization. This method will extend the class of chaotic signal sources and enhance information security.

The paper proposed two kinds of algorithms of adaptive synchronization: the algorithms using the adaptive observers with extended error signal and the algorithms of high-order tuning. Conditions for asymptotic convergence of identification (Theorems 1–3) were established for the no noise (no distortions) case. A robust modification of the algorithms of adaptive synchronization for the systems with noisy communication channels (Theorems 4 and 5) was proposed as well.

The theoretical findings are illustrated by an example of adaptive synchronization of the controllable chaotic Lorentz systems. Computer simulation provided realizations of the system processes demonstrating high speed of the algorithms of adaptive synchronization and parameter identification, which shows that the proposed algorithms hold promise for information transmission by modulation of the chaotic signals.

APPENDIX

Proof of Theorem 1. The extended error signal $\widehat{e}$ (3.6) is known [22, 38] to be representable by the equivalent model

$$\widehat{e} = \omega(t)\widetilde{\theta} \tag{A.1}$$

to within the exponentially fading addends depending on the initial conditions. At that, the derivatives of the exponentially fading terms remain such also at differentiation, which justifies correctness of differentiation of equality (A.1). By calculating the derivative of the Lyapunov function $V(\widetilde{\theta}) = \dfrac{1}{2\gamma}\widetilde{\theta}^{\mathrm{T}}\widetilde{\theta}$ along the trajectories of system (3.5), with regard for (A.1) we obtain that $\dot{V}(\widetilde{\theta}) = -\widehat{e}^2$ from which it follows that the variable $\widehat{\theta}$ is bounded. Since the right-hand sides of (2.1), (3.2), (3.5) are locally Lipschitzian in x and $\widehat{x}$ and $\widehat{\theta}$ is uniform in t [29], we get that $\widehat{e}(t)$ tends to zero. By virtue of boundedness of $\omega(t)$ in (3.5), $\dot{\widehat{\theta}} \to 0$ for $t \to \infty$. Therefore, we obtain from (3.6) that $e - \widehat{e} \to 0$ and, consequently, $e \to 0$, which proves statement (i). Validity of (ii) is proved in a standard manner [22, 38].

Proof of Theorem 3. By differentiating the following expression of the auxiliary error $\delta = \varepsilon + \eta - \Omega\widetilde{\theta}$ and taking into consideration (3.19), (3.21), we obtain that

$$\dot{\delta} = G\varepsilon + b\varphi^{\mathrm{T}}\widetilde{\theta} + G\eta - \Omega\dot{\widehat{\theta}} - G\Omega\widehat{\theta} - b\varphi^{\mathrm{T}}\widetilde{\theta} + \Omega\dot{\widehat{\theta}} = G(\varepsilon + \eta - \Omega\widetilde{\theta}) = G\delta.$$

Then, the extended error signal obeying (3.20) is representable as $\widehat{e} = \omega^{\mathrm{T}}\widetilde{\theta} + \delta_e$ where $\delta_e = c^{\mathrm{T}}\delta$ decreases exponentially. Then, by following the lines of the proof of Theorem 1 one can demonstrate that all signals in the feedback loop are bounded, condition (3.8) is satisfied, and (3.9) is valid, provided that the PE condition is satisfied.

Proof of Theorem 4. By time differentiation of the estimation error $\varepsilon = x - \widehat{x}$ with regard for Eqs. (2.1), (2.2), and (4.1) we obtain after some simple transformations that

$$\dot{\varepsilon} = F\varepsilon + b\overline{\varphi}(t)^{\mathrm{T}}\widetilde{\theta} + \Delta(t), \tag{A.2}$$

where $\Delta(t) = \varphi_0(y) - \varphi_0(y_r) + b\big(\varphi(y) - \varphi(y_r)\big)^{\mathrm{T}}\theta - k\xi$.

Therefore, the extended error signal $\overline{e}$ following Eq. (4.2) takes the form

$$\overline{e} = \overline{\omega}(t)^{\mathrm{T}}\widetilde{\theta} + \xi_e + \xi, \tag{A.3}$$

where the bounded variable ξ_e satisfies Eqs. (4.6).

We make use of a Lyapunov function like $V(\widetilde{\theta}) = \dfrac{1}{2}\widetilde{\theta}^{\mathrm{T}}\widetilde{\theta}$ whose time derivative along the trajectories of system (4.3), (A.3) is as follows:

$$\begin{aligned}\dot{V}(\widetilde{\theta}) &= \widetilde{\theta}^{\mathrm{T}}\left(-\gamma\overline{\varpi e} - \alpha\widetilde{\theta} + \alpha\theta\right) = \widetilde{\theta}^{\mathrm{T}}\left(-\gamma\overline{\varpi}\,\overline{\varpi}^{\mathrm{T}}\widetilde{\theta} - \gamma\overline{\varpi}(\xi_e + \xi) - \alpha\widetilde{\theta} + \alpha\theta\right)\\ &\le -\gamma|\overline{\varpi}^{\mathrm{T}}\widetilde{\theta}|^2 + -\sigma|\widetilde{\theta}|^2 + \gamma|\overline{\varpi}^{\mathrm{T}}\widetilde{\theta}|\|\xi_e + \xi\|_\infty + \alpha|\widetilde{\theta}|\,|\theta|\\ &\le -\frac{1}{2}\gamma|\overline{\varpi}^{\mathrm{T}}\widetilde{\theta}|^2 - \frac{1}{2}\alpha|\widetilde{\theta}|^2 + \frac{1}{2}\gamma\,\|\xi_e + \xi\|_\infty^2 + \frac{1}{2}\alpha|\theta|^2\\ &\le -\frac{1}{2}\alpha|\widetilde{\theta}|^2 + \frac{1}{2}\gamma\|\xi_e + \xi\|_\infty + \frac{1}{2}\alpha|\theta|^2.\end{aligned}$$

Boundedness of all variables in the closed-loop system and satisfaction of the inequality in the right-hand side of (4.5) follow from the last inequality.

If $\xi(t) \le 0$ and $\theta^* > |\theta|$, then $\dot{V}(\widetilde{\theta}) = -\gamma|\overline{\varpi}^{\mathrm{T}}\widetilde{\theta}|^2 + \gamma\sigma(\widehat{\theta})\widetilde{\theta}^{\mathrm{T}}\widehat{\theta} \le -\gamma|\overline{\varpi}^{\mathrm{T}}\widetilde{\theta}|^2$ is satisfied for the time derivative of the Lyapunov function $V(\widetilde{\theta}) = \dfrac{1}{2}\widetilde{\theta}^{\mathrm{T}}\widetilde{\theta}$. Validity of (3.8) follows from this inequality.

Proof of Theorem 5. With regard for (3.17), (2.2), and (4.7), the time differentiation of the estimation error $e = x - \widehat{x}$ after simple transformations provides

$$\dot{\varepsilon} = \overline{G}(t)\varepsilon + b\overline{\varphi}(t)^{\mathrm{T}}\widetilde{\theta} + \Delta(t), \tag{A.4}$$

where $\Delta(t) = (A(y) - A(y_r))x + \varphi_0(y) - \varphi_0(y_r) + b(\varphi(y) - \varphi(y_r))^{\mathrm{T}}\theta - k(y_r)\xi$. By differentiating now the auxiliary error $\Xi_\delta = \varepsilon + \overline{\eta} - \overline{\Omega}\widetilde{\theta}$, we obtain with regard for (A.4), (4.12), and (4.10) that

$$\begin{aligned}\dot{\Xi}_y = \overline{G}(t)\Xi_y + \varphi_0(y) - \varphi_0(y_r) + b\Big(\varphi(y) - \varphi(y_r)\Big)^{\mathrm{T}}\theta\\ - k(y_r)\xi + \Big(A(y) - A(y_r)\Big)x, \quad \xi_y = c^{\mathrm{T}}\Xi_y.\end{aligned}$$

Therefore, the extended error signal $\overline{e}$ obeying (4.8) is representable as $\overline{e} = \overline{\omega}^{\mathrm{T}}\widetilde{\theta} + \xi_\delta + \xi$ where $\xi_\delta = c^{\mathrm{T}}\Xi_\delta$. By applying now the approach used to prove Theorem 4, we see that Theorem 5 is true.

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This paper was recommended for publication by B.T. Polyak, a member of the Editorial Board