Synchronization of Nonlinear Systems Over Intranet: Cart-pendulum Case Study*

M.S. Ananyevskiy[1,2], R.E. Seifullaev[1], D.A. Nikitin[1], A.L. Fradkov[1,2,3]

***Abstract*— The master-slave synchronization problem for two cart-pendulum systems when both measurement and control signals are transmitted via intranet communication channel is examined. The speed-gradient method for exciting master system and linear state feedback for slave system is used. Theoretical analysis is performed by the nonlinear extension of Fridman's method. Experimental results are presented for a cart-pendulum system constructed from Lego. Theoretical bound for sampling interval guaranteeing synchronization is about h=0.1 sec. Such bounds are positive for control over intranet and very negative for control over internet. However experiments show that real values of the communication delays are even bigger.**

I. INTRODUCTION

With the development of digital communication channels the problem of control over network became actual. If there are several control plants, several regulators it is useful to use computer networks (wired, wireless or mixed) and the Internet for control. Control over network brings some new difficulties to the problem – the existence of non-stationary delays in channel. Mass-production and reduction of price leads to decreasing accuracy of sensors and regulators, lowering the quality of control plants. Therefore the problem is to develop the control system robust to

- non-stationary delays in channel,
- inaccuracy of mathematical model,
- roughness of sensors and regulators.

What factor is worst for control system? To find an answer several experiments were executed, results are presented at this paper.

Control over network is a relatively new yet rapidly expanding area of research. There are some publications devoted to control of manipulators over Internet [1], [2], [3], In paper [4] several experiments for control of tele-operators over different communication channels are presented: commercial internet, wireless lan, ethernet lan, the feedback was based on a video-signal from different applications (iChat/Skype, HaiVision, Hai1000, Hai200 etc.). The problem of stabilizing inverted pendulum over localnet was studied in [5].

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[1] Saint-Petersburg State University, Russia

[2] Institute of Problems in Mechanical Engineering Russian Academy of Sciences, Russia

[3] Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics, Russia

In this paper the master-slave synchronization problem for two cart-pendulum systems when both measurement and control signals are transmitted via intranet communication channel is examined. In section II a general control problem over network and speed-gradient method are described. In section III a theoretical result for partial case: for sample-data control of Lurie systems is presented. Equations and simplified equations for pendulum-cart system one can find in section IV. In section V the control algorithm for exciting master system and control algorithm for master-slave synchronization are presented. The theoretical analysis of synchronization algorithm is performed in section VI. Section VII is devoted to experiments for a cart-pendulum system constructed from Lego. Conclusions are in section VIII.

II. NONLINEAR CONTROL PROBLEM

Consider a control plant described by a system of state equations

$$\dot{x} = f(x,u,t), \quad x(0) = x_0, \qquad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \qquad (1)$$

here x is the state vector, $f(x,u,t)$ is some function, u is the control input, t is time. Control goal is specified by means of a non-negative function $Q(x,t)$ (further it is called goal function)

$$\lim_{t\to+\infty} Q(x(t),t) = 0, \qquad (2)$$

where $x(t)$ is the solution of (1) with some admissible $u(t)$.

Feedback control over network realized with some non-static delay and with measurements sampling

$$u(t) = u(x(t_k)), \quad t \in [t_k + \Delta_k, t_{k+1} + \Delta_{k+1}), \quad k \in \mathbb{N}, \qquad (3)$$

here t_k is a timestamp of k-th measurement, Δ_k is a delay ($\Delta_k > 0$).

In order to design control algorithm the scalar function $w(x,u,t)$ is calculated that is the speed of changing $Q(x,t)$ along trajectories $x(t)$ of (1)

$$w(x,u,t) = \frac{\partial Q(x,t)}{\partial t} + \frac{\partial Q}{\partial x} f(x,u,t). \qquad (4)$$

Then it is needed to evaluate the gradient of $w(x,u,t)$ with respect to input variables

$$\nabla_u w(x,u,t) = \nabla_u \frac{\partial Q}{\partial x} f(x,u,t). \qquad (5)$$

Finally the algorithm of changing $u(t)$ is determinated according to the differential equation (differential form)

$$\dot{u} = -\Gamma \nabla_u w(x,u,t), \quad u(0) = u_0 \qquad (6)$$

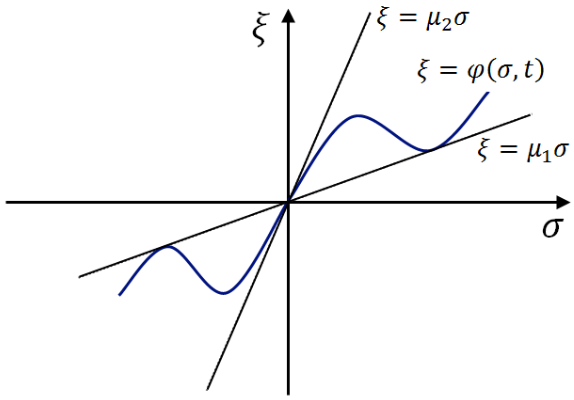

Fig. 1. Sector bounded nonlinearity

or to the algebraic equation (finite form)

$$u = u_0 - \Gamma\nabla_u w(x,u,t), \tag{7}$$

where $\Gamma = \Gamma^T > 0$ is the positive definite gain matrix, u_0 is some initial value of control algorithm. It can be also introduced a speed-pseudogradient algorithm

$$u = u_0 - \Gamma\psi(x,u,t), \tag{8}$$

where $\psi(x,u,t)$ satisfies the pseudogradient condition

$$\psi(x,u,t)^T\nabla_u w(x,u,t) \geqslant 0. \tag{9}$$

The algorithm (6) is called speed-gradient algorithm [6], since it suggests to change $u(t)$ proportionally to the gradient of the speed of changing $Q(x(t),t)$.

III. SAMPLED-DATA CONTROL OF LURIE SYSTEM

Consider a nonlinear system

$$\begin{aligned} &\dot{x}(t) = Ax(t) + q\xi(t) + (B + B_0\xi_0(t))\,u(t),\\ &\sigma(t) = r^T x(t), \qquad \sigma_0(t) = r_0^T x(t),\\ &\xi(t) = \varphi(\sigma(t),t), \quad \xi_0(t) = \varphi_0(\sigma_0(t),t), \end{aligned} \tag{10}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control function, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $B_0 \in \mathbb{R}^{n\times m}$ are constant matrices, $q \in \mathbb{R}^n, r \in \mathbb{R}^n, r_0 \in \mathbb{R}^n$ are constant vectors.

Suppose that $\xi(t) = \varphi(\sigma(t),t)$ is a nonlinear function (Fig. 1) satisfying

$$\mu_1\sigma^2 \leqslant \sigma\xi \leqslant \mu_2\sigma^2, \tag{11}$$

for all $t \geqslant 0$ where $\mu_1 < \mu_2$ are real numbers, and nonlinear function $\xi_0(t) = \varphi_0(t)$ is bounded for all $t \geqslant 0$

$$\varphi_0^- \leqslant \xi_0(t) \leqslant \varphi_0^+.$$

Given a sequence of sampling times $0 = t_0 < t_1 < \ldots < t_k < \ \ldots$ and a piecewise constant control function

$$u(t) = u_d(t_k), \ \ t_k \leqslant t < t_{k+1},$$

where $\lim\limits_{k\to\infty} t_k = \infty$.

Assume that $h \in \mathbb{R}\,(h > 0)$ and

$$t_{k+1} - t_k \leqslant h, \ \ \forall k \geqslant 0,$$

and consider a sampled-time control law

$$u(t) = Kx(t_k), \ \ t_k \leqslant t < t_{k+1}, \tag{12}$$

where $K \in \mathbb{R}^{m\times n}$. Control function (12) can be rewritten as follows:

$$u(t) = Kx(t - \tau(t)), \tag{13}$$

where $\tau(t) = t - t_k, t_k \leqslant t < t_{k+1}$.

It is required to analyze the influence of the upper bound h of sampling intervals on the closed-loop system exponential stability:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + (B + B_0\xi_0(t))\,Kx(t - \tau(t)) + q\xi(t),\\ &\sigma(t) = r^T x(t), \qquad \sigma_0(t) = r_0^T x(t),\\ &\xi(t) = \varphi(\sigma(t),t), \quad \xi_0(t) = \varphi_0(\sigma_0(t),t),\\ &\tau(t) = t - t_k, \ t \in [t_k, t_{k+1}). \end{aligned} \tag{14}$$

The key tools to obtain the results are application of Yakubovich's S-procedure [8] and Fridman's method for linear system [7], based on a new general time-dependent Lyapunov-Krasovskii functional. The following theorem is based on the results proposed in [9].

Assume that P, Q are symmetric positive definite $n\times n$ matrices, P_2, $P_3, X, X_1, Y_1, Y_2, Y_3, T$ are some $n\times n$ matrices, $\varkappa_0^-$, $\varkappa_0^+$, $\varkappa_1^-$, $\varkappa_1^+$ are positive real scalars.

Consider the following matrices linearly depending on their arguments:

$$\begin{aligned} &\Theta = \Theta(h, P, X, X_1), \quad \Theta \in \ \mathbb{R}^{2n\times 2n},\\ &\Psi_{H0}^- = \Psi_{H0}^-(P, Q, P_2, P_3, X, X_1, Y_1, Y_2, Y_3,\\ &\qquad \varkappa_0^-, \varkappa_0^+, \varkappa_1^-, \varkappa_1^+), \ \Psi_{H0}^- \in \ \mathbb{R}^{(3n+N)\times(3n+1)},\\ &\Psi_{H0}^+ = \Psi_{H0}^+(P, Q, P_2, P_3, X, X_1, Y_1, Y_2, Y_3,\\ &\qquad \varkappa_0^-, \varkappa_0^+, \varkappa_1^-, \varkappa_1^+), \ \Psi_{H0}^+ \in \ \mathbb{R}^{(3n+N)\times(3n+1)},\\ &\Psi_{H1}^- = \Psi_{H1}^-(h, \alpha, P, Q, P_2, P_3, X, X_1, Y_1, Y_2, Y_3),\\ &\qquad \varkappa_0^-, \varkappa_0^+, \varkappa_1^-, \varkappa_1^+, \ \Psi_{H1}^- \in \ \mathbb{R}^{(4n+N)\times(4n+1)},\\ &\Psi_{H1}^+ = \Psi_{H1}^+(h, \alpha, P, Q, P_2, P_3, X, X_1, Y_1, Y_2, Y_3),\\ &\qquad \varkappa_0^-, \varkappa_0^+, \varkappa_1^-, \varkappa_1^+, \ \Psi_{H1}^+ \in \ \mathbb{R}^{(4n+N)\times(4n+1)}, \end{aligned}$$

where the explicit forms of these matrices can be found in [9]).

Theorem 1: Given $\alpha > 0$, let there exist matrices $P \in \mathbb{R}^{n\times n}$ $(P > 0)$, $Q \in \mathbb{R}^{n\times n}$ $(Q > 0)$, $P_2 \in \mathbb{R}^{n\times n}$, $P_3 \in \mathbb{R}^{n\times n}$, $X \in \mathbb{R}^{n\times n}$, $X_1 \in \mathbb{R}^{n\times n}$, $T \in \mathbb{R}^{n\times n}$, $Y_1 \in \mathbb{R}^{n\times n}$, $Y_2 \in \mathbb{R}^{n\times n}$, Y_3 and positive real scalars $\varkappa_0^-$, $\varkappa_0^+$, $\varkappa_1^-$, $\varkappa_1^+$, such that the following LMIs

$$\Theta > 0, \quad \Psi_{H0}^- < 0, \quad \Psi_{H0}^+ < 0, \quad \Psi_{H1}^- < 0, \quad \Psi_{H1}^+ < 0$$

are feasible. Then system (14) is exponentially stable with decay rate α.

Remark 1: Since LMIs $\Psi_{H0}^- < 0, \Psi_{H0}^+ < 0$ do not depend on h, the necessary condition on K for exponential stability with sampled-data control is the exponential stability of the system with continuous control $u = Kx(t)$.

IV. PENDULUM ON A CART

Consider two independent systems type "pendulum on a cart" with equal parameters. The systems equations are as follows [10]

$$\begin{cases} (M+m)\ddot{x}_k + ml\ddot{\phi}_k \cos\phi_k = \\ \qquad = u_k + ml(\dot{\phi}_K)^2 \sin\phi_k + f(t,x_k) \\ ml\ddot{x}_K \cos\phi_k + (ml^2+J)\ddot{\phi}_k = \\ \qquad = -mgl\sin\phi_k + \mu(t,\phi_k) \end{cases}, \tag{15}$$

x_k is coordinate of a cart, ϕ_k is angular coordinate of k-th pendulum, l is pendulum length, J is the inertia of pendulum relative to center of inertia, $f(t,x_k)$ is the friction for cart, $\mu(t,\phi)$ is the inertia corresponding to friction, M is cart mass, m is pendulum mass, g is the gravity acceleration, u_k is the control input for k-th pendulum, $k=1,2$.

Energy (E_k) of k-th pendulums is the following

$$\begin{aligned} E_k = \tfrac{1}{2}((M+m)\dot{x}_k^2 + (ml^2+J)\dot{\phi}_K^2 + \\ +2ml\dot{x}\dot{\phi}_k\cos\phi_k) + mgl(1-\cos\phi_k). \end{aligned} \tag{16}$$

For experiments the simplified model was used:

$$l\ddot{\phi}_k(t) = -g\sin\phi_k(t) - u_k\cos\phi_k(t), \quad k=1,2, \tag{17}$$

here u_k is the new control input (cart acceleration). This means that the mass of cart is greatly bigger than the mass of pendulum, pendulum mass is concentrated in point and there is no friction.

Control goal is the phase synchronization:

$$\lim_{t\to+\infty} (\phi_1(t)-\phi_2(t))^2 = 0. \tag{18}$$

V. CONTROL ALGORITHMS

According to the section II the speed-gradient algorithm for energy stabilization of master system was used

$$u_1(t) = -\alpha\nabla_{u_1}\frac{d}{dt}(E(\phi_1(t),\dot{\phi}_1(t)) - E_*)^2, \tag{19}$$

where E_* is the goal energy. So the excitation algorithm was the following

$$u_1(t) = -\gamma(E(\phi_1(t),\dot{\phi}_1(t)) - E_*)\cos(\phi_1(t))\dot{\phi}_1(t), \tag{20}$$

here $\gamma > 0$ is the parameter.

For synchronization (slave-to-master) the PD-regulator was used.

$$u_2 = \beta_1(\phi_1(t)-\phi_2(t)) + \beta_2(\dot{\phi}_1(t)-\dot{\phi}_2(t)), \tag{21}$$

where β_1, β_2 are parameters. Analysis of this algorithm is presented in next section.

VI. CART-PENDULUM SYSTEM SYNCHRONIZATION

Consider the simplified model of two cart-pendulum systems described by two equations:

$$\begin{cases} \ddot{\varphi}_1(t) &= -\dfrac{g}{l}\sin\varphi_1(t) - \dfrac{1}{Ml}\cos\varphi_1(t)u(t), \\ \ddot{\varphi}_2(t) &= -\dfrac{g}{l}\sin\varphi_2(t), \end{cases} \tag{22}$$

where $\varphi_i(t)$ $(i=1,2)$ is the deviation angle of the i-th pendulum from the vertical, $u(t)$ is a control (traction force of the first cart), M and l are the masses and lengths (it is supposed that the carts have the same masses, and the pendulums have the same lengths).

System (22) can be rewritten as follows

$$\begin{aligned} \dot{x}(t) &= Ax(t) + q_1\xi_1(t) + (B + B_0\xi_0(t))\,u(t), \\ \xi_0(t) &= \cos\varphi_1(t), \quad \sigma_1(t) = r_1^T x(t), \\ \xi_1(t) &= \sin\sigma_1(t)\cos\frac{\varphi_1(t)+\varphi_2(t)}{2}, \end{aligned} \tag{23}$$

where

$$x(t) = \begin{bmatrix} \varphi_1(t)-\varphi_2(t) \\ \dot{\varphi}_1(t)-\dot{\varphi}_2(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 \\ -\dfrac{1}{Ml} \end{bmatrix}, \quad q_1 = \begin{bmatrix} 0 \\ -\dfrac{2g}{l} \end{bmatrix}, \quad r_1 = \begin{bmatrix} \dfrac{1}{2} \\ 0 \end{bmatrix}.$$

ξ_1 satisfies

$$-0.2173\cdot\sigma_1^2 \leqslant \sigma_1\,\xi_1 \leqslant \sigma_1^2$$

for all $t \geqslant 0$.

Suppose that the oscillation amplitude is not greater than $60°$. Hence, ξ_0 satisfies

$$0.5 \leqslant \xi_0 \leqslant 1$$

for all $t \geqslant 0$.

Consider the following control law

$$u(t) = Kx(t_k), \;\; t_k \leqslant t < t_{k+1}, \tag{24}$$

where $t_{k+1} - t_k \leqslant h, \;\; \forall k \geqslant 0$.

For the system parameters $g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$, $l = 0.25\,\mathrm{m}$, $M = 1\,\mathrm{kg}$, $K = [17.7, 3.875]$ the simulation results show that the upper bound of sampling (23) lies within $h = 0.174$ and $h = 0.175$. Figures 2-4 illustrate the plots of pendulum angles, angular speeds and control for $h = 0.09$ (stable case), and figures 5-7 show the same ones for $h = 0.176$ (unstable case).

Comparison the estimates obtained by Theorem 1 with simulation results is given in Table *I* .

Theorem 1	Simulation	Quality of Estimates
$h \leqslant 0.099$	$h \leqslant h_*, 0.174 < h_* < 0.175$	56%

TABLE I

UPPER BOUNDS (FOR $\alpha = 0$) FOR WITCH SYSTEM (23) IS EXPONENTIALLY STABLE

The dependence of upper bound h on decay rate α is given on Fig. 9.

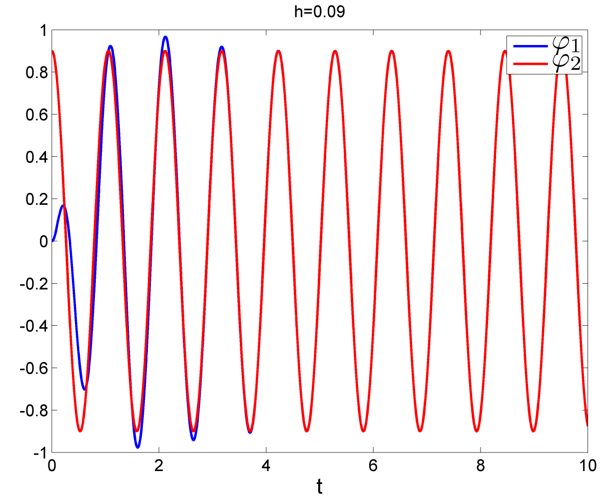

Fig. 2. Deviation angles. $h = 0.09$

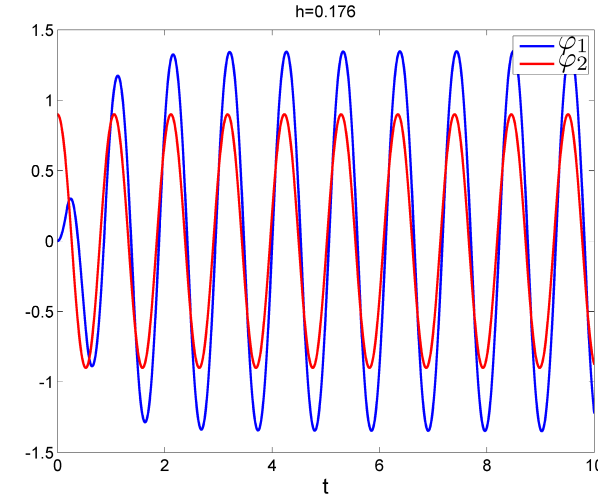

Fig. 5. Deviation angles. $h = 0.176$

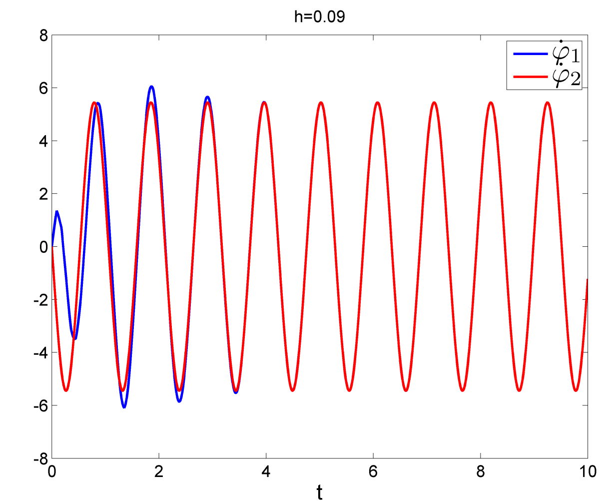

Fig. 3. Angular speeds. $h = 0.09$

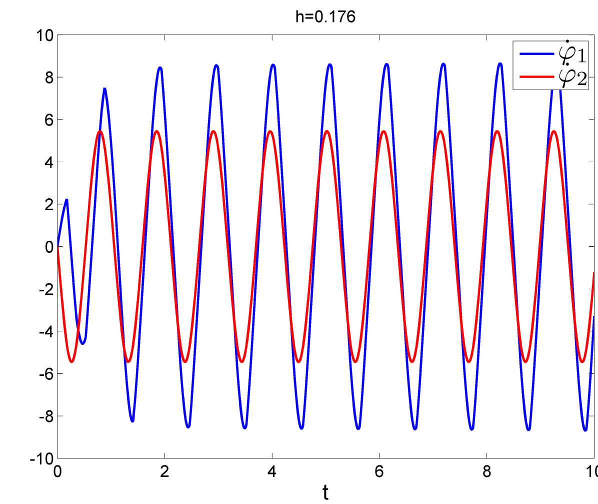

Fig. 6. Angular speeds. $h = 0.176$

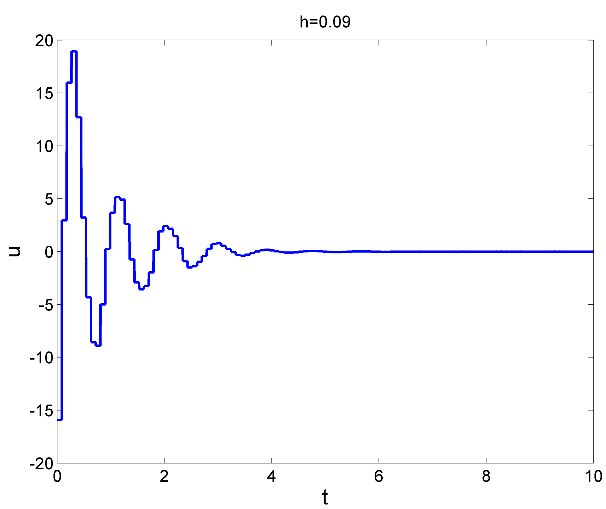

Fig. 4. Control. $h = 0.09$

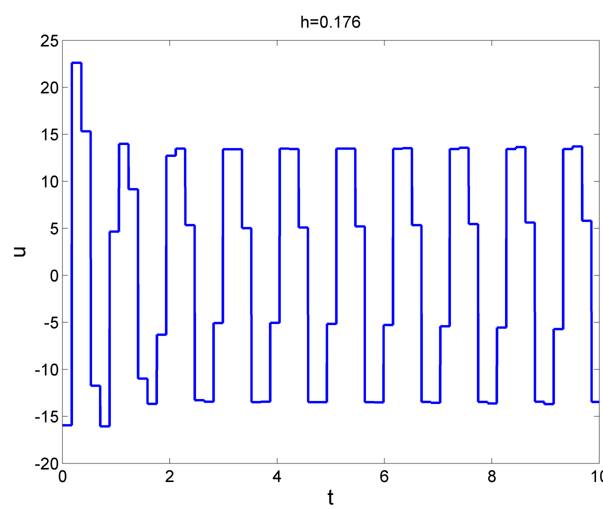

Fig. 7. Control. $h = 0.176$

VII. EXPERIMENT

For experiment two pendulums on a cart set-up has been constructed from Lego Mindstoms NXT2.0 (fig. 8). Each pendulum system had two motors, one angular sensor and was connected to computer with usb-cable.

Angular sensor accuracy is one degree. Backslash of motors is around ten degrees. Friction and period of oscillations could be obtained from free dynamics: both pendulums were placed in horizontal position manually and released 11. The period of oscillations of each pendulum was around one second. Pendulums have same construction, but one has bigger friction.

Pendulums were connected by local net. According to architecture of our software sending controls to actuators and receiving data from sensor was not parallel (alfa-version of software "Cloud Mechatronic Laboratory"). So each round consists of six sequential procedures: get angular from sensor one (1-st pendulum), get angular from sensor two (2-nd pendulum), send pulse to motor one (1-st pendulum), send pulse to motor two (1-st pendulum), send pulse to motor third (2-nd pendulum), send pulse to motor forth (2-nd pendulum). Each operation has an unpredictive delay. Some statistics was collected 10.

Time history of synchronization for algorithm (21) with parameters $[17.7, 3.875]$ (from previous section) is presented on fig. 12. Since the model of actuators (motors) is unknown, an unknown gain parameter $\gamma > 0$ is introduced. Sampling time was around 0.7 sec. It is seen that the synchronization is achieved with reasonable accuracy.

VIII. CONCLUSIONS

In this paper the theoretical and experimental analysis of two cart-pendulum systems synchronization is performed for the case when both measurement and control signals are transmitted via intranet communication channel. The master-slave synchronization problem is examined. Oscillations of the master system are excited by means of the speed-gradient method [11]. The linear state feedback controller is used for the slave system. Experiments demonstrate that the communication delays for this system are rather large and may vary significantly depending on connection type. Therefore they should be taken into account for stability analysis.

Theoretical evaluation of the upper bound for the sampling intervals ensuring synchronization (convergence of synchronization error to zero) is performed by the nonlinear extension of Fridman's method [9]. Theoretical bound for sampling interval guaranteeing synchronization is about h=0.1 sec., while simulations result in the threshold value h=0.175 sec. For control over internet delays may be up to 1.0 sec. and this is normal situation. So the worst factor for control over network in our experiment are non-stationary delays. However experiments show that real values of the communication delays are even bigger. It means that the experiments may provide only approximate synchronization. Further research will be aimed at studying approximate synchronization taking into account actuator (motor) dynamics and studying more sophysticated synchronization algorithms.

Fig. 8. Two pendulums on a cart.

REFERENCES

[1] W.J. Book, H. Lane, L.J. Love, et al. A novel teleoperated long-reach manipulator testbed and its remote capabilities via the internet. IEEE International Conference on Robotics and Automation (ICRA). Minneapolis MN. April 22–28, 1996. P. 1036-1041.

[2] T. Debus, J. Stoll, R.D. Howe, P. Dupont. Cooperative human and machine perception in teleoperated assembly. Experimental Robotics VII. The Fifth International Symposium. Honolulu, HI. December 10–13, 2000.

[3] D. Andreu, P. Fraisse. Internet enhanced teleoperation: Toward a remote supervised delay regulator. IEEE International Conference on Industrial Technology. Maribor, Slovenia. December 10–12, 2003.

[4] M.J.H. Lum, J. Rosen, H. King, et al. Teleoperation in surgical robotics - network latency effects on surgical performance. 31st Annual International Conference of the IEEE EM Minneapolis. Minnesota, USA. September 2–6, 2009.

[5] O.R. Natale, O. Sename, C. Canudas-de-Wit. Inverted pendulum stabilization through the ethernet network, performance analysis. Proceedings of the American Control Conference (ACC'04). Boston, Massachussets, June 30 – July 2, 2004.

[6] Fradkov A.L. Speed-gradient scheme and its applications in adaptive control. Automation and Remote Control, Vol. 40, 1979. P. 1333–1342.

[7] E.M. Fridman. A refined input delay approach to sampled-data control. Automatica, Vol. 46, 2010. P. 421–427.

[8] V.A. Yakubovich, G.A. Leonov, A.Kh. Gelig. Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities. Singapore, World Scientific, 2004.

[9] R.E. Seifullaev, A.L. Fradkov. Sampled-Data Control of Nonlinear Oscillations Based on LMIs and Fridman's Method. 5th IFAC International Workshop on Periodic Control Systems. Caen, France. 2013. P. 95–100.

[10] F.L. Chernousko, I.M. Ananievski, S.A. Reshmin, Control of Nonlinear Dynamical Systems. Methods and Applications. Berlin, Heidelberg: Springer. 2008. 396 p.

[11] B.R. Andrievskii, P.Yu. Guzenko, A.L. Fradkov, Control of Nonlinear Vibrations of Mechanical Systems via the Method of Velocity Gradient, Automation and Remote Control, Vol. 57, No. 4, 1996. P. 456–467.

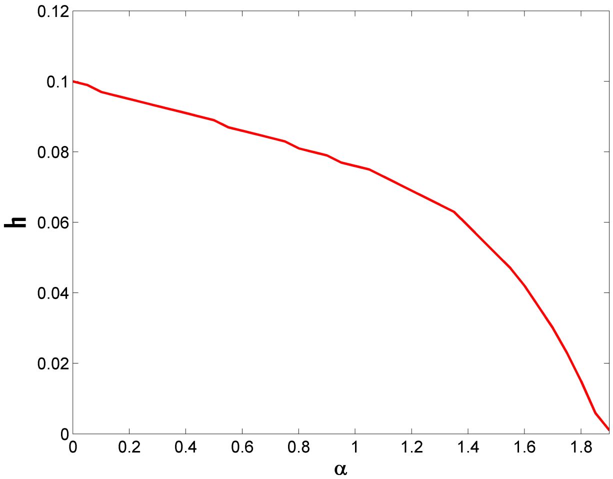

Fig. 9. The dependence of the upper bound on decay rate α

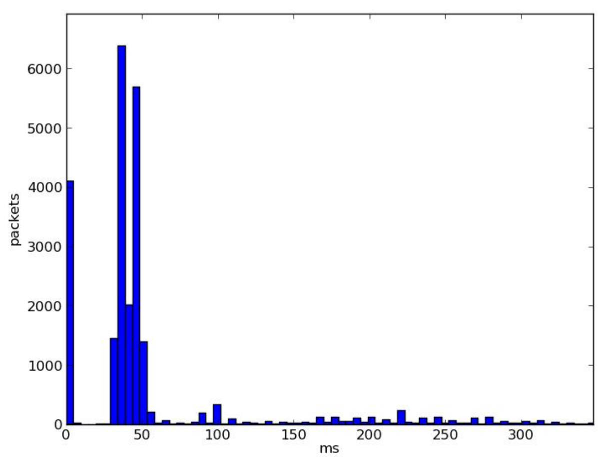

Fig. 10. Histogram of delays ("packet" is a single data transfer between plant and regulator).

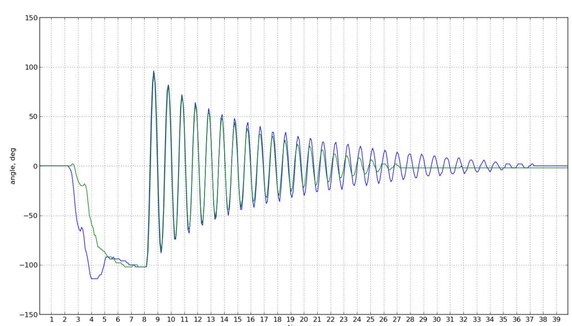

Fig. 11. Free dynamics of pendulum systems. Period of oscillations is around one second. Pendulums are quite the same, but one has bigger friction.

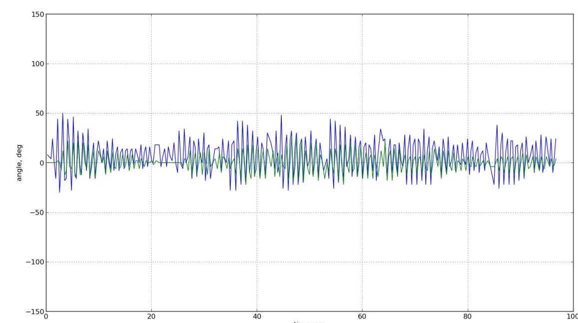

Fig. 12. Synchronization of pendulum systems with PD-regulator. Sampling time was around 0.7 sec.