Controlled Passage Through Resonance for Two-Rotor Vibration Unit

A. L. Fradkov and D. A. Tomchin and O. P. Tomchina

Abstract The problem of controlled passage through resonance zone for mechanical systems with several degrees of freedom is studied. Control algorithm design is based on speed-gradient method and estimate for the frequency of the slow motion near resonance (Blekhman frequency). The simulation results for two-rotor vibration units illustrating efficiency of the proposed algorithms and fractal dependence of the passage time on the initial conditions are presented.

Key words: Passage Through Resonance, Control, Vibration units.

1 Introduction

The usage of mechatronic elements sets up broad prospects in designing of modern vibration units. One of the main task for control of mechatronic vibration unit is passing through resonance zone at start up and speed up modes of vibroactuators, when range of operating modes belongs to a post-resonance zone. Such a problem occurs e.g. if the power of a motor is not sufficient for passing through resonance zone due to Sommerfeld effect deeply investigated by I. I. Blekhman.

Historically the first approach to the problem of passing through resonance zone was the so-called "double start" method due to V. V. Gortinskiy [1]. The method is

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based on the insertion of time relay into motor control circuit for repeatedly switching on and off motor at pre-calculated time instants. Basically, this and other feedforward (nonfeedback) methods are characterized by difficulties in calculation of switching instants of a motor and sensitivity to inaccuracies of model and to interferences. A prospective approach to the problem is based on feedback control algorithms. Passing through resonance zone control algorithms of mechanical systems with feedback measurements were considered in [2]– [5]. However the early algorithms [2] did not have enough robustness under uncertain conditions and were hard to design.

For practical implementation its important to develop reasonably simple passing through resonance zone control algorithms, which have such robustness property: keeping high quality of the controlled system (vibration unit) under variation of parameters and external conditions. A number of such algorithms based on a speed-gradient method, were suggested in works [3]– [5].

This work is dedicated to extension of the results of [3]– [5]. Problem statement for control of passage through resonance zone for mechanical systems with several degrees of freedom is adopted from [5]. The control algorithms based on the speed-gradient method for two-rotor vibration units with unbalanced rotors are described. The simulation results illustrating efficiency of the proposed algorithms and fractal dependence of the passage time on the initial conditions are presented.

2 Problem statement and approach to solution.

According to [5] the problem of passing through resonance is defined as design of control algorithm providing achievement of the prespecified energy level of the rotating subsystem under limited energy of the supporting body oscillations at a resonance frequency.

Approach to solution of a problem proposed in this paper, is based upon usage of the speed gradient algorithms design [5] and a motion separation into fast and slow components, which occurs near resonance zone. Quantitative analysis of slow "pendular" movements $\psi(t)$ was performed by I. I. Blekhman in [6] where the frequency of an "internal pendulum" was evaluated. It will be further called the "Blekhman frequency".

The idea of control algorithms described below, is in that slow motion $\psi(t)$ is being isolated and then "swinging" starts to obtain rise of energy of a rotating subsystem. To isolate slow motions, low-pass filter is being inserted into energy control algorithms. Particularly, if slow component appears in oscillations of angular velocity of a rotor $\dot{\varphi}$, then the control algorithm proposed in [3] is used:

$$u = -\gamma\,\mathrm{sign}\Big((H - H^*)\,\dot{\psi}\Big)\,, \quad T_\psi \dot{\psi} = -\psi + \dot{\varphi}\,, \tag{1}$$

where $H = H(p,q)$ denotes the Hamiltonian (total energy of the system), ψ is the variable of a filter, that corresponds to slow motions of $\psi(t)$, T_ψ is the time constant of a filter. At low damping, slow motions also fade out slowly, what gives control algorithm an opportunity to create suitable conditions to pass through resonance zone, and after that "swinging" can be turned off and then algorithm switches to controlling with constant momentum. For a proper work of a filter, it should suppress fast oscillations with frequency ω and pass slow oscillations with ω_B frequency, where ω_B is the Blekhman frequency. I.e. time constant of a filter T_ψ should be chosen from the inequality

$$T_\psi < 2\pi/\omega_B \,. \tag{2}$$

Algorithms of passing through resonance zone for the two-rotor vibration units are described and studied below.

3 Passing through resonance control algorithm of two-rotor vibration unit.

Consider the problem of two-rotor vibration unit start-up (spin-up), the unit consists of two rotors, installed on the supporting body elastically connected with fixed basis. Assume that the system dynamics may be considered in the vertical plane (see Fig. 1). Then the equations of dynamics have the following form [7]:

$$\begin{aligned}
&m_0\ddot{x}_c - \\
&-m\rho\Big\{\Big[\sin(\varphi+\varphi_1)+\sin(\varphi+\varphi_2)\Big]\ddot{\varphi}+\Big[\cos(\varphi+\varphi_1)+\cos(\varphi+\varphi_2)\Big]\dot{\varphi}^2+ \\
&+\sin(\varphi+\varphi_1)\ddot{\varphi}_1+\sin(\varphi+\varphi_2)\ddot{\varphi}_2+\cos(\varphi+\varphi_1)\dot{\varphi}_1^2+\cos(\varphi+\varphi_2)\dot{\varphi}_2^2+ \\
&+2\dot{\varphi}\Big(\cos(\varphi+\varphi_1)\dot{\varphi}_1+\cos(\varphi+\varphi_2)\dot{\varphi}_2\Big)\Big\}+\beta\dot{x}_c+2c_{01}x_c=0\,;
\end{aligned} \tag{3}$$

$$\begin{aligned}
&m_0\ddot{y}_c + \\
&+m\rho\Big\{\Big[\cos(\varphi+\varphi_1)+\cos(\varphi+\varphi_2)\Big]\ddot{\varphi}-\Big[\sin(\varphi+\varphi_1)+\sin(\varphi+\varphi_2)\Big]\dot{\varphi}^2+ \\
&+\cos(\varphi+\varphi_1)\ddot{\varphi}_1+\cos(\varphi+\varphi_2)\ddot{\varphi}_2-\sin(\varphi+\varphi_1)\dot{\varphi}_1^2-\sin(\varphi+\varphi_2)\dot{\varphi}_2^2- \\
&-2\dot{\varphi}\Big(\sin(\varphi+\varphi_1)\dot{\varphi}_1+\sin(\varphi+\varphi_2)\dot{\varphi}_2\Big)\Big\}+\beta\dot{y}_c+2c_{02}y_c+m_0g=0\,;
\end{aligned} \tag{4}$$

$$
\begin{aligned}
&-m\rho\left\{\Big[\sin(\varphi+\varphi_1)+\sin(\varphi+\varphi_2)\Big]\ddot{x}_c-\Big[\cos(\varphi+\varphi_1)+\cos(\varphi+\varphi_2)\Big]\ddot{y}_c\right\}+\\
&\quad+\Big[J+J_1+J_2-2m\rho r(\cos\varphi_1-\cos\varphi_2)\Big]\ddot{\varphi}+\\
&\quad+\Big[J_1-mr\rho\cos\varphi_1\Big]\ddot{\varphi}_1+\Big[J_2+mr\rho\cos\varphi_2\Big]\ddot{\varphi}_2+\\
&\quad+mr\rho\left\{2\sin\varphi_1\,\dot{\varphi}\dot{\varphi}_1-2\sin\varphi_2\,\dot{\varphi}\dot{\varphi}_2+\sin\varphi_1\,\dot{\varphi}_1^2-\sin\varphi_2\,\dot{\varphi}_2^2\right\}+\\
&\quad+m\rho g\Big[\cos(\varphi+\varphi_1)+\cos(\varphi+\varphi_2)\Big]+\beta\dot{\varphi}+c_{03}\varphi=0\,;
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&J_1\ddot{\varphi}_1-m\rho\Big[\sin(\varphi+\varphi_1)\ddot{x}_c-\cos(\varphi+\varphi_1)\ddot{y}_c\Big]+\Big[J_1-mr\rho\cos\varphi_1\Big]\ddot{\varphi}-\\
&\quad-mr\rho\sin\varphi_1\,\dot{\varphi}^2+mg\rho\cos(\varphi+\varphi_1)+k_c\dot{\varphi}_1=M_1\,;\\
&J_2\ddot{\varphi}_2-m\rho\Big[\sin(\varphi+\varphi_2)\ddot{x}_c-\cos(\varphi+\varphi_2)\ddot{y}_c\Big]+\Big[J_2+mr\rho\cos\varphi_2\Big]\ddot{\varphi}+\\
&\quad+mr\rho\sin\varphi_2\,\dot{\varphi}^2+mg\rho\cos(\varphi+\varphi_2)+k_c\dot{\varphi}_2=M_2\,;
\end{aligned}
\tag{6}
$$

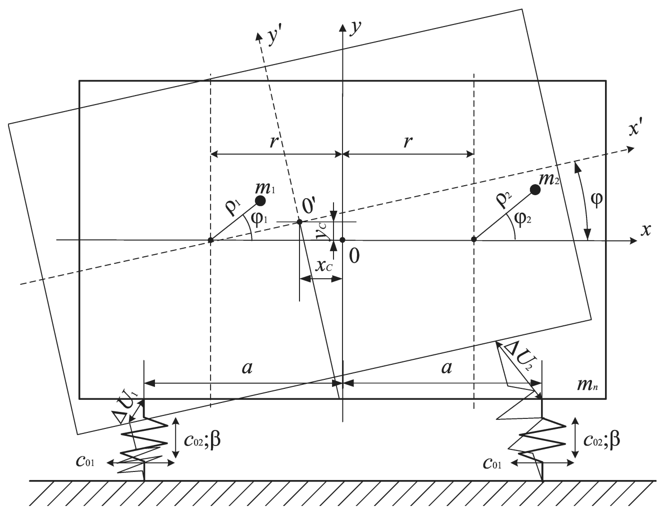

Fig. 1 Two-rotor vibration unit with horizontal supporting body.

Here φ, φ_1, φ_2 are angle of the support and rotation angles of the rotors, respectively, measured from the horizontal position, x_c, y_c are the horizontal and vertical displacement of the supporting body from the equilibrium position, $m_i=m$, $i=1,2$ and m_n are the masses of the rotors and supporting body, J_1, J_2 are the inertia moments of the rotors, $\rho_i=\rho$, $i=1,2$ are the rotor eccentricities, c_{01}, c_{02} are the horizontal and vertical spring stiffness, g is the gravity acceleration, m_0 is the total mass of the unit , $m_0=2m+m_n$, β is the damping coefficient, k_c the friction coefficient in the bearings, $M_i = u_i(t)$ are the motor torques (controlling variables). It is assumed that rotor shafts are orthogonal to the motion of the support.

At the low levels of constant control action $u_i(t) \equiv (-1)^i M_0$, $i = 1,2$ at the nearresonance zone the rotor angle is "captured", while increase of the control torque leads to passage through resonance zone towards the desired angular velocity. The simulation results for the system (3, 4, 5, 6) are shown on Fig. 2 with basic system parameters: $J_i = 0.014\,[\mathrm{kg\,m^2}]$, $m = 1.5\,[\mathrm{kg}]$, $m_n = 9\,[\mathrm{kg}]$, $\rho = 0.04\,[\mathrm{m}]$, $k_c = 0.01\,\mathrm{J/s}$, $\beta = 5\,[\mathrm{kg/s}]$, $c_{02} = 5300\,[\mathrm{N/m}]$, $c_{01} = 1300\,[\mathrm{N/m}]$ and constant torque $M_0 = 0.65\,[\mathrm{N\,m}]$ (internal curves, capture) and $M_0 = 0.66\,[\mathrm{N\,m}]$ (external curves, passage).

Synthesis of the control algorithm $u = \mathbf{U}(z)$ is needed for acceleration of the unbalanced rotors, before the system passes through the resonance zone, where $z = [x,\dot{x},y,\dot{y},\varphi,\dot{\varphi},\varphi_1,\dot{\varphi}_1,\varphi_2,\dot{\varphi}_2]^T$ is the state vector of the system. It is assumed that the level of control action is limited and does not allow system to pass through the resonance zone using constant action.

As seen in Fig. 2 the rotational motions separate into fast and slow ones i.e. the proposed approach applies. The following modification of the control algorithm (1) is proposed for passing through resonance zone:

$$\begin{cases} u_i = \begin{cases} (-1)^i M_o\,, & if \quad \gamma = 1\,, \\ (-1)^i M_o\,, & if \quad \gamma = 0 \;\&\; (H - H^*)(\dot{\varphi}_i - \psi_i) > 0\,, \\ \qquad 0\,, & else\,, \end{cases} \\ T_\psi \dot{\psi}_i = -\psi_i + \dot{\varphi}_i\,, \quad i = 1,2 \end{cases} \tag{7}$$

where $H = T + \Pi$, $\psi_i(t)$ are the variables of the filters, $T_\psi > 0$, $T_\psi = \mathrm{const}$ and H^* are the parameters of the algorithm,

$$\gamma(t) = \max_{0 \leqslant \tau < t} \mathrm{sgn}\Big(H(\tau) - H^*\Big)$$

where $\mathrm{sgn}[z] = 1$ with $z > 0$, $\mathrm{sgn}[z] = 0$ with $z \leqslant 0$. Time constant of the filters T_ψ should satisfy the relation (2). At the same time, the values that are too big lead to decrease in average power of the control signal and to slow down the algorithm.

Efficiency of the proposed control algorithm was studied in the MATLAB environment. The relative simulation error does not exceed 5 %. Calculations were made with the same values of the basic parameters of the system. Then the lowest constant control action providing passing through the resonance is $0.66\,[\mathrm{N\,m}]$.

In the simulation with the proposed algorithm (7), the minimum value of a control torque M_0 to pass through resonance, was being calculated.

Simulation results have shown that the minimum value is $M_0 = 0.42\,[\mathrm{N\,m}]$ (with the precision of $0.01\,[\mathrm{N\,m}]$), see Fig. 3. In comparison to constant control action, proposed algorithm may reduce the control action level by 1.5 times.

Study of the algorithm with different asymmetric initial positions of rotors [4] has confirmed its efficiency. The graphs are similar to the previous ones, except for

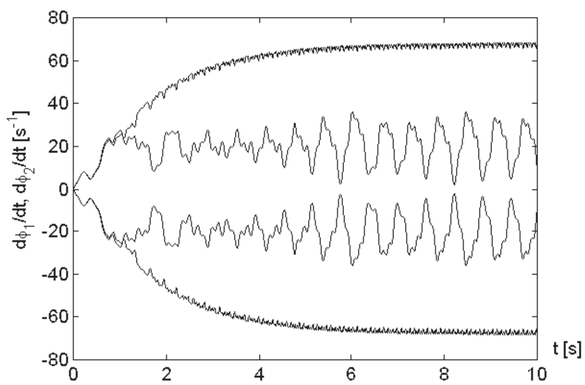

Fig. 2 Constant control action, $u_i(t) \equiv (-1)^i M_0$, $M_0 = 0.65\,[\mathrm{N\,m}]$ (internal curves, capture) and $M_0 = 0.66\,[\mathrm{N\,m}]$ (external curves, passing).

the horizontal support deviation, which differs in the initial area. Fig. 4 shows the minimum M_0 value change depending on T_ψ in the algorithm (7).

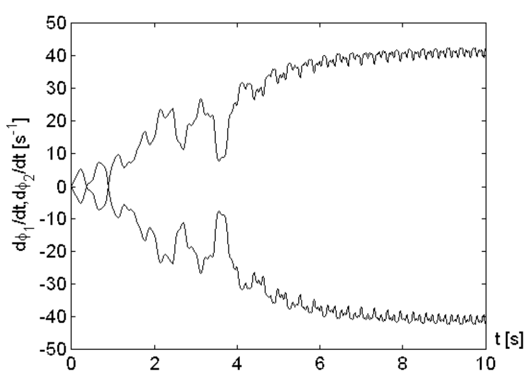

Fig. 3 Passing through resonance with algorithm (7). $M_0 = 0.42\,[\mathrm{N\,m}]$, $T_\psi = 0.35\,[\mathrm{s}]$.

Thus, using the proposed control algorithm, the level of control action required for passing through resonance can be significantly reduced. The algorithm only has two tunable parameters and is simple to use, despite the complicated behavior of the system. The closed control system is low sensitive to an asymmetry of the initial conditions of the unit. However, it is sensitive to the initial conditions themselves. Even more, the dependence of passage time on initial conditions has a fractal shape, see Fig. 5.

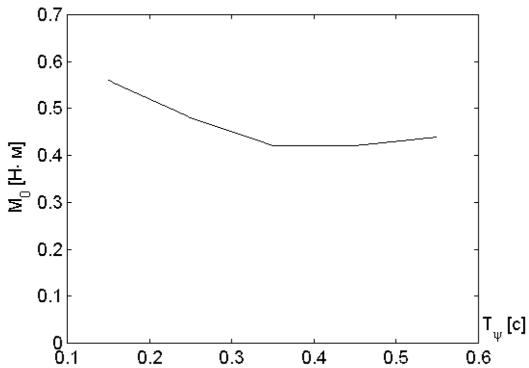

Fig. 4 Algorithm efficiency as a function of the time constant T_{ψ}.

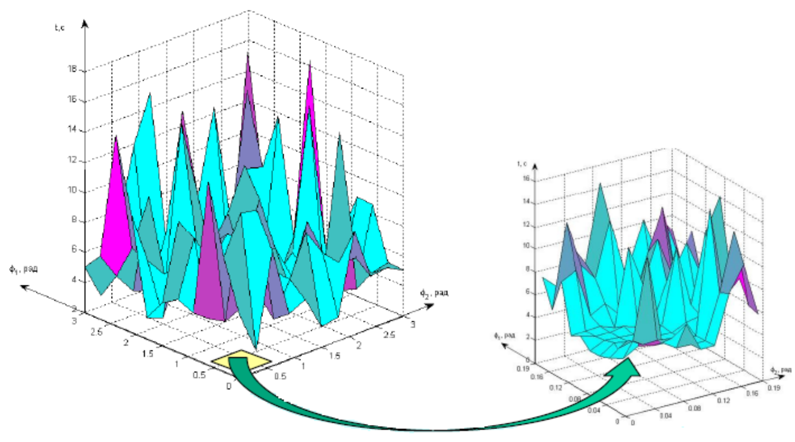

Fig. 5 Time of passing through resonance for different initial conditions.

4 Conclusion

This paper systematically shows the approach for synthesis of simple control algorithms of passing through resonance zone in vibration units, proposed in [3]– [5] and based on "swinging" of slow oscillations of an angular velocity of the rotors. Algorithms are based on the speed-gradient method and the filtering the fast oscillations. The efficiency of the algorithms and fractal dependence of the passage time on the initial conditions is demonstrated by computer simulation.

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