# Multipendulum mechatronic setup: Design and experiments[☆]

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## Abstract

In the paper a novel multipendulum mechatronic setup is described. It allows implementing different algorithms of estimation, synchronization and control. The set-up is intended for solving various research and educational tasks in hybrid modeling, analysis, identification and control of mechanical systems. It allows one to study the data communication processes in distributed mechatronic complexes.

*Keywords:* nonlinear dynamics, communication constraints, mechatronic set-up

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## 1. Introduction

Problems of oscillatory mechanical systems control and synchronization have significant theoretical interest and practical value. For the purposes of research and control engineering education it is important to build up appropriate laboratory equipment and software to work for investigation of this kind of system. There are many papers where this problem was considered and significant results have been achieved [13, 43, 25, 16, 17]. In the

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last decades various mechatronic laboratory set-ups have been described in the literature: inverted pendulums [27, 12], a reaction-wheel pendulum [44], cart-pendulum [28, 29], Furuta pendulum [46, 47, 30], coupled two-pendulum systems [9, 24, 32, 25, 51], metronomes on a moving base [40, 39], pendulum-like juggling system [45], etc. A promising design described in [42] provides a possibility to model a variety of different oscillatory systems in single set-up.

However there is still a demand for equipment useful for research as testbeds for testing new control and data exchange algorithms under real world constraints, as well for education, allowing students to enhance their skills in control systems design.

In the present paper the novel multipendulum mechatronic set-up – the *Multipendulum Mechatronic Set-up of the Institute for Problems of Mechanical Engineering* (MMS IPME) is described.

The MMS IPME includes a modular multi-section mechanical oscillating system, electrical equipment (with the computer interface facilities) and the personal computer for experimental data processing, representation of the results the real-time control. The mechanical system consists of the pendulums, connected by means of the springs. The key feature of the MMS IPME is the possibility to change the number of DOF in a broad range. Starting from a single pendulum one may increase the number of the pendulums in the chain up to 50. It is also possible to shape pendulums into several chains. Connecting then the chains by rigid rods between the centers of the springs it is possible to model 2D-lattices.

The MMS IPME allows to implement different algorithms of estimation, synchronization and control. Particularly, it may be used as a multi-degree of freedom mechanical model of controlled physical systems [2] and power system networks [41, 31].

In Section 2, a brief description of the construction is presented. For making laboratory experiments and on-line control, electrical design, data exchange interface and software tools were created. Their description is given in Section 3. Possible ways for improving the data exchange in multiagent mechatronic complexes are outlined in Section 4, where both hardware-software and algorithmic solutions are presented. Some results of experiments with the MMS IPME are described in Section 5.

## 2. Design of mechanical part

The schematic of a pendulum section is presented in Fig. 1 (see also pictures in Figs. 2, 3). The foundation of the section is a hollow rectangular body. Inside the body an electrical magnet and electronic controller board are mounted. On the foundation the support containing the platform for placing the sensors in its middle part is mounted. The pendulum itself has a permanent magnet tip in the bottom part. The working ends of the permanent magnet and the electrical magnet are posed exactly opposite each other and separated with a non-magnetic plate in a window of the body. The idea behind control of the pendulum is changing the poles of the electrical magnet by means of switching the direction of the current in the windings of the electrical magnet. Additionally, two computer-controlled electric motors may be connected with the first and the last pendulums of the chain via the torsion springs for changing the boundary conditions on the chain.[1] In order to allow changes of the eigenfrequency of the pendulum oscillations the pendulum is endowed with additional plummets and counterparts changing its effective length (the distance between the suspension point and the center of mass. On the rotation axis of the pendulum the optical encoder disk for measuring the angle (phase) of the pendulum is mounted. It has 90 slits. The peripheral part of the disk is posed into the slit of the sensor support. The sensor consists of a radiator (emitting diode) and a receiver (photodiode). The obtained sequences of signals allow to measure the pendulum angle, evaluate amplitude of oscillations and register time instants of crossing the lower equilibrium point.

Axes of the neighbor sections are connected with the torsion springs, arranging force interaction and allowing exchanging energy between neighbor sections. The set of interconnected pendulum sections represents a complex oscillatory dynamical system, characterized by nonlinearity and high number of degrees of freedom. Such a mechanical system can serve as a basis for numerous educational and research experiments related to dynamics, control and synchronization in the networks of multidimensional nonlinear dynamical systems. In principle, any number of sections can be connected. At the moment mechanical parts of 50 sections are manufactured.

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[1] At present, only the “left-side” motor is in service. The boundary conditions at the “right” end of the chain are free.

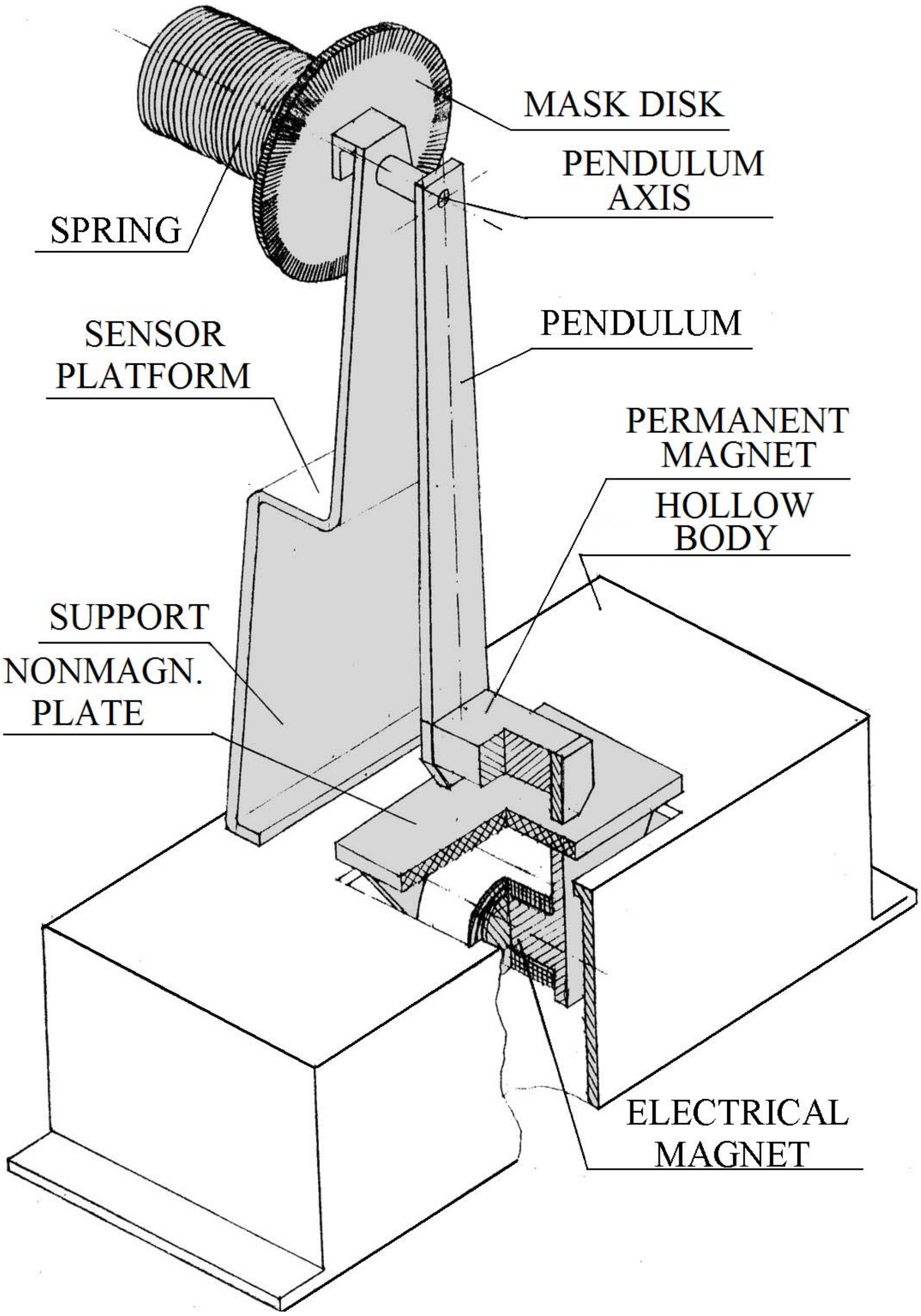

Figure 1: Schematics of the pendulum section.

Figure 2: Photo of the chain from twelve pendulum sections and the motor.

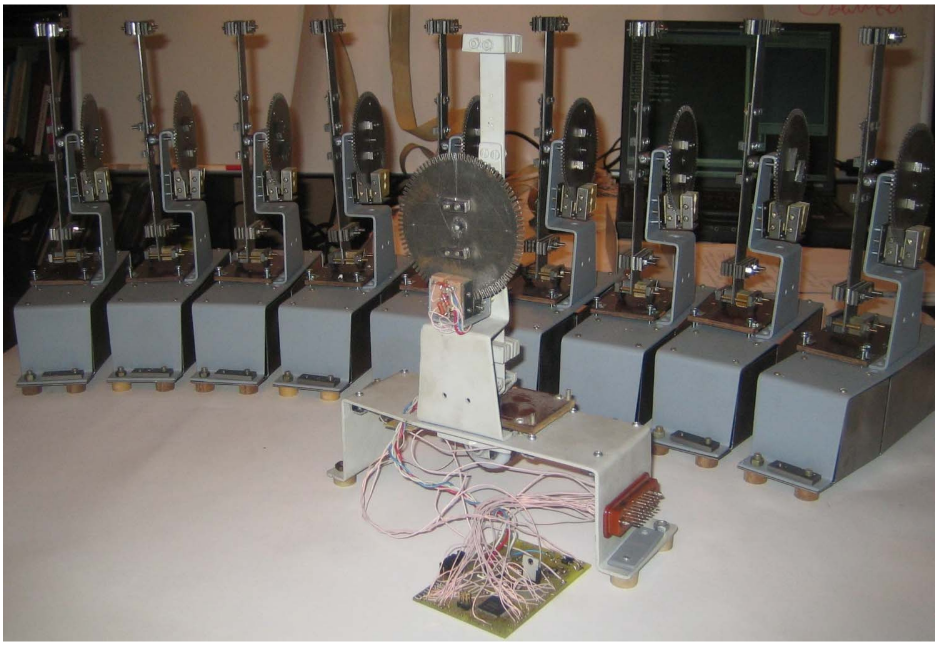

Figure 3: Photo of ten pendulum sections and the universal logic board.

## 3. Hardware and software of the MMS IPME

The control system of the complex consists of: controllers for pendulum modules and motors; the supervisory computer and the interface unit; the electric power devices (power amplifiers, power supply units); the communication channel. Below, these components are described in more detail.

### *3.1. Local controllers of modules*

Controllers of the pendulum modules and the motors are mounted at the *universal logic board* (ULB) developed for the MMS IPME for unification of modules schematic and achievement the highest possible controller speed. The kernel of the ULB is the erasable programmable logic device *EPM240T100C5* manufactured by *Altera Corporation*. Each controller is a specialized firmware machine with a built-in microprogram for control of pendulums or DC motors, and for measuring the shaft rotation angles. The ULB also includes three input signal shapers (comparators), mounted on the mechanical parts of the modules, the secondary supply sources and the timer. The firmware is designed with the help of the design system *Quartus II* [1].

The local controllers are used for offloading the upper level supervisory computer, for formation of the pulse-width modulated control signal, for measurement the shaft angle, for event service of sensors, requiring immediate response, for formation of the protocol for information exchange with the main supervisory computer and for maintaining high data rate communication over the channel.

### *3.2. Communications protocol*

The communications protocol secures transferring the instructions and command qualifiers to the interface board of the pendulum sections and pickup the data from the interface board sensors. The communications protocol uses three kinds of passing: address passing, instruction (mode) passing and data passing. The communications protocol is based on explicit addressing with use the read-write registers of the Enhanced Parallel Port. Logical separation between the data and instruction flows is made with use of two highest stages of the address register. The rest five stages of this register contain the module address for a read-write cycle.

Described data exchange protocol ensures exchange rate up to 9615 Hz for simultaneous operation over the bus with the total number (52) of modules, and up to 500 kHz for operation with a single module.

### *3.3. The supervisory computer and the interface unit*

The personal computer with *Intel Celeron* processor with operating system *GNU/Linux* is chosen for upper-level control of the local controllers, processing and visualization of experimental data. Operating system *GNU/Linux* allows to optimize the CPU time allocation in favor of the priority tasks of query and control.

For supporting the data exchange operations, the packages *comedi 0.7.76* and *comedilib 0.8.1* are installed additionally to OS *ALT Linux Desktop 4.0 Personal*. These packages represent a specialized set of software for abstraction from the hardware peculiarities by means of a virtual file system. Package *comedi 0.7.76* is a set of drivers supporting the universal analog-to-digital and digital input/output devices of the leading manufacturers such as *Advantech*, *National Instruments*, *ComputerBoards*.

The experimental results may be postprocessed using the packages MATLAB (running under *GNU / Linux*) or its the open-source analog SCILAB [11]. Alternatively, one can process experimental data using package *Octave* [15] or package *gnuplot* for visualization.

## 4. Directions for optimization of communication channel for multiagent mechatronic complexes

The first experiments, which have been performed with the MMS IPME, consisting from five pendulum modules and one motor, showed correctness of the technical solutions used for control of a multiagent distributed mechatronic system, but some disadvantages have been also revealed.

Identified problems are caused by a single-computer implementation of the hardware-software system, where the single PC is used both for equipment control and for the real-time visualization of the experiments. Visualization requires loading the graphics adapter (*X Server* in the case of the OS GNU/Linux). This imposes considerable demands to the system resources. Additionally, Window Manager (a graphical shell which provides convenience of the interactive graphical user interface) should be also loaded. An interception of the system's priorities from the side of the graphical components destabilizes the driver package *comedi 0.7.76*. This reduces the absolute sampling rate of the bus. In some cases (if a high-precision control is necessary), the experiment may not be even implemented. Consequently, in present time, the data input requests and acquisition of the experimental data are performed from the console without launching the graphical user interface support.

### 4.1. Hardware-software solutions

A partial way for improving the system performance would be using the real-time support modules of higher execution priority such as *rtai* or *rtLinux* [14]. This decision will help stabilize the data acquisition process. However, in the case of programmable input/output, the problem of increasing the sampling rate to the possible maximum is still a significant one.

A promising solution for modernization of the mechatronic set-up would be changeover from the present single-computer architecture to the multi-computer complex. In this case, the different nodes of the complex may be linked over the network *Ethernet* based on a synchronous access protocol, without any association of the complex' network with the existing computer networks to avoid interfering on the order of messaging for protocols *ARP* or *ICMP*. Hardware solution of the controller for data exchange with the stand modules (assuming allocation one controller for 3 – 5 modules) is reasonable to realize on the base of processor modules *x86* for ensuring a uniform cycle of support and developing the software of the complex.

Operating systems *microLinux* (*ucLinux*), *Embedded Linux*, or *OpenWrt* with built-in support for the real-time mode are planned to use for exchange controllers. An advantage of modules *CPC150* and *CPC109* is a compact solution (single-board computer with integrated digital I/O). There is an additional opportunity connect standard monitor and keyboard to the modules *CPC150* and *CPC109* up to stand-alone operation without the upper level computer. Transition from a centralized architecture to the distributed one significantly increases the computational power of the complex, allowing to flexibly resource allocation between computing modules.

### 4.2. Algorithmic solutions. State estimation under the data rate limitations

The abovementioned directions for increasing the performance capability and the data exchange rate of the complex may be reffered to as hardware-software solutions. During the last decades the considerable attention was riveted to the *algorithmic* solutions for improving the system performance under the data-rate limitations, see the surveys [38, 8, 7], the monograph [34] and the references therein. Particularly, it has been shown that the control/observation of linear systems is possible if and only if the capacity of the information channel exceeds the entropy production of the system at the equilibrium (the *Data Rate Theorem*) [35, 36, 37]. The coding-decoding schemes were proposed giving an opportunity to get closer to the minimum possible data rate. Two ideas are basically applied for this purpose: using a smart sensors, which incorporate the model of the observed plant dynamics, and also applying the zooming strategy, where the range of the encoder is updated during the control or observation process [10, 33, 48].

First results on synchronization of nonlinear systems under information constraints were presented in [20, 21], where so called observer-based synchronization scheme has been employed. It is shown that for the first-order coder-decoder scheme the upper bound of limit synchronization error is proportional to the maximum rate of the coupling signal and inversely proportional to the information transmission rate (channel capacity). The controlled synchronization problem was analyzed in [3]. It was shown that in the case of an ideal channel and non-corrupted measurements, the output feedback controlled synchronization strategy with the full order encoder/decoder pair, ensures exponentially vanishing synchronization error if the channel capacity exceeds a certain threshold. This concurs with the known results, obtained for linear systems in [49, 50, 10].

Further on, the approach of [20, 21] has been applied to the observation of nonlinear systems over the limited-band communication channel in [22, 19, 23]. Let us briefly recall the mentioned observation schemes.

Consider the following nonlinear plant model:

$$\dot{x}(t) = Ax(t) + B\psi(y),\ y(t) = Cx(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the plant state variables vector; $y(t)$ is a scalar output variable; $A$ is an $(n\times n)$-matrix; $B$ is an $(n\times 1)$-matrix; $C$ is an $(1\times n)$-matrix, $\psi(y)$ is a continuous nonlinearity.

The problem is to obtain the state estimation of (1) over the digital communication channel with the limited bandwidth. The measured data are sampled with a certain sampling rate $T_s$ and are represented by finite-length codewords to be transmitted over the channel. The following uniform memoryless (static) quantizer is employed:

$$q_{\nu,M}(y) = \begin{cases} \delta\cdot\langle\delta^{-1}y\rangle, & \text{if } \ |y|\le M, \\ M\,\mathrm{sign}(y), & \text{otherwise}, \end{cases} \tag{2}$$

where $M > 0$ is a real number (the *quantizer range*), $\nu \in \mathbb{Z}$ is the positive integer, $\delta = 2^{1-\nu}M$; $\langle\cdot\rangle$ denotes the round-up to the nearest integer, $\mathrm{sign}(\cdot)$

is the signum function. The quantization interval $[-M, M]$ is equally split into $2^\nu$ parts. Therefore, the cardinality of the mapping $q_{\nu,M}$ image is equal to $2^\nu + 1$ and each codeword contains $R = \log_2(2^\nu + 1) = \log_2(2M/\delta + 1)$ bits.

The *one-step memory* coder uses the *central number* $c[k]$, $k = 0, 1, \ldots$ with the initial condition $c[0] = 0$ [49, 50]. At step $k$, the coder compares the current measured output $y[k]$ with the number $c[k]$, forming the deviation signal $\partial y[k] = y[k] - c[k]$. Then $\partial y[k]$ is discretized with a given $\nu$ and $M = M[k]$ according to (2). The quantized output signal

$$\bar{\partial} y[k] = q_{\nu,M[k]}(\partial y[k]) \tag{3}$$

is represented as an $R$-bit codeword and transmitted over the communication channel to the decoder. At the next step, the central number $c[k+1]$ and the quantizer range $M[k]$ are renewed by the following update algorithms [22]:

$$c_[k + 1] = c[k] + \bar{\partial} y[k], \quad c[0] = 0, \quad k = 0, 1, \ldots, \tag{4}$$

$$M_[k] = (M_0 - M_\infty)\rho^k + M_\infty, \quad k = 0, 1, \ldots, \tag{5}$$

where $0 < \rho \leq 1$ is the decay parameter, $M_\infty$ stands for the limit value of $M[k]$. The initial value $M_0$ should be large enough to capture all the region of possible values of $y[0]$.

The coder of *full order* embeds the observer. In [19, 23], the *observer error* (*innovation signal*) is transmitted over the channel rather than a measured plant output. For describing a such kind of the coders, let us introduce the error between the the plant and observer outputs as $\varepsilon(t) = y(t) - \hat{y}(t) = Ce(t)$. This signal is subjected to the coding procedure (2) – (5) instead of $y(t)$, forming the quantized signal $\bar{\varepsilon}[k]$. The following state estimation algorithm is realized by the coder:

$$\begin{aligned} &\dot{\hat{x}}(t) = A\hat{x}(t) + B\psi(\hat{y}) + L\bar{\varepsilon}(t), \; \hat{y}(t) = C\hat{x}(t), \\ &\bar{\varepsilon}(t) = \bar{\varepsilon}[k] \;\text{ as }\; t \in [t_k, t_{k+1}), \quad t_k = kT_s, \quad k = 0, 1, 2, \ldots. \end{aligned} \tag{6}$$

where $\hat{x} \in \mathbb{R}^n$ stands for the estimate of the plant state vector $x(t)$, $(n \times 1)$-matrix $L$ is the observer gain (the design parameter).

## 5. Experiments with the MMS IPME

The system is currently being tested and tuned. Five active and up to 46 passive sections are ready for connection. Already at this stage the system can be used for demonstration and for research. In one of the series of experiments, either inphase and antiphase synchronization in the chain of pendulums, excited by the external harmonic torque, is demonstrated. Another series was carried out for experimental evaluation of the data transmission schemes of Section 4.2. Some results are presented below.

### *5.1. Modeling the chain of pendulums*

Following [4], let us omit retroaction from the pendulum chain to the drive motor and consider the *rotation angle* of the drive shaft, connected with the first pendulum of the chain, as the control action. The last ($N$th) pendulum in the chain is mechanically connected only with the previous one, no boundary conditions for $N$th pendulum are specified. This leads to the following chain dynamics model:

$$\begin{cases} \ddot{\varphi}_1 + \rho\dot{\varphi}_1 + \omega_0^2 \sin\varphi_1 - k(\varphi_2 - 2\varphi_1) = ku(t), \\ \ddot{\varphi}_i + \rho\dot{\varphi}_i + \omega_0^2 \sin\varphi_i - k(\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}) = 0, \\ \qquad\qquad\qquad (i = 2, 3, \ldots, N-1), \\ \ddot{\varphi}_N + \rho\dot{\varphi}_N + \omega_0^2 \sin\varphi_N - k(\varphi_N - \varphi_{N-1}) = 0, \end{cases} \tag{7}$$

where $\varphi_i = \varphi_i(t)$ ($i = 1, 2, \ldots, N$) are the pendulum deflection angles; $u = u(t)$ is the controlling action (the rotation angle of the drive shaft). The values $\rho$, $\omega_0$, $k$ are the system parameters: $\rho$ is the viscous friction parameter; $\omega_0$ is the natural frequency of small oscillations of the isolated pendulum; $k$ is the coupling strength parameter, which depends on the stiffness of the connecting spring. The model (7) parameters have been estimated by means of calculation on the basis of mass-geometry properties of the mechanical system, and then were qualified by means of the trial-and-error procedure, applyed to the experimental data sets. The following parameter estimates were finally obtained: $\omega_0 = 5.5$ s$^{-1}$, $\rho = 0.95$ s$^{-1}$, $k = 5.8$ s$^{-2}$.

### *5.2. Synchronization in the harmonically excited chain of pendulums*

The series of experiments has been fulfilled for experimental validation of analysis of synchronization in the chain of pendulums, excited by means of the harmonic torque, given in [6, 4].

The chain of four pendulums was taken. Following [6, 4], model (7) was linearized in the neighborhood of the equilibrium. The frequency magnitude responses $A_i(\omega_m)$ $(i = 1, \ldots, 4)$ from the drive shaft rotation angle $u(t)$ to angular deflections $\varphi_i(t)$ are plotted in Fig. (4). The phase shifts $\Delta\psi_{i,i+1}(\omega_m)$ between the adjacent pendulums are depicted in Fig. (5). It is seen from the plots, that there exist certain frequencies $\omega_1$, $\omega_2$ such that the pendulums motion demonstrates inphase synchronization if the excitation frequency $\omega_m$ is less than $\omega_1$ and approximately antiphase synchronization if $\omega_m > \omega_2$. It is also seen that the pendulum chain may play a role of a mechanical band pass filter with the bandwidth $[\omega_1, \omega_2]$.

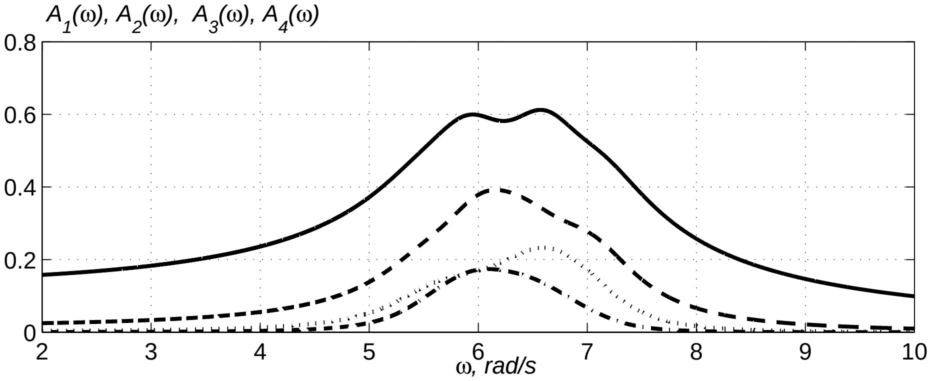

Figure 4: Frequency magnitude responses for pendulum rotation angles in the chain (analytical evaluation). $A_1(\omega_m)$ – solid line, $A_2(\omega_m)$ – dashed line, $A_3(\omega_m)$ – dotted line, $A_4(\omega_m)$ – dash-dot line.

The corresponding exerimental results are demonstrated in Figs. 6–8, where the time histories of the pendulums' rotation angles in the steady-state mode are plotted. An inphase synchronization in the chain of pendulums for the case $\omega_m = 4.0$ rad/s is presented in Fig. 6. An antiphase synchronization for $\omega_m = 8.9$ rad/s is seen in Fig. 7. The running wave for the case $\omega_m = 6$ rad/s is demonstrated in Fig. 8. The similar results have been obtained by experiments with the chain of twelve passive pendulum sections.

### *5.3. Testing the data exchange and state estimation algorithm*

The state estimation scheme of Section 4.2 (Eqs. (2)–(6)) has been modified for taking into account presence of the external excitation signal $u(t)$, applied to the chain. In our experiments, the signal $u(t)$ has been measured

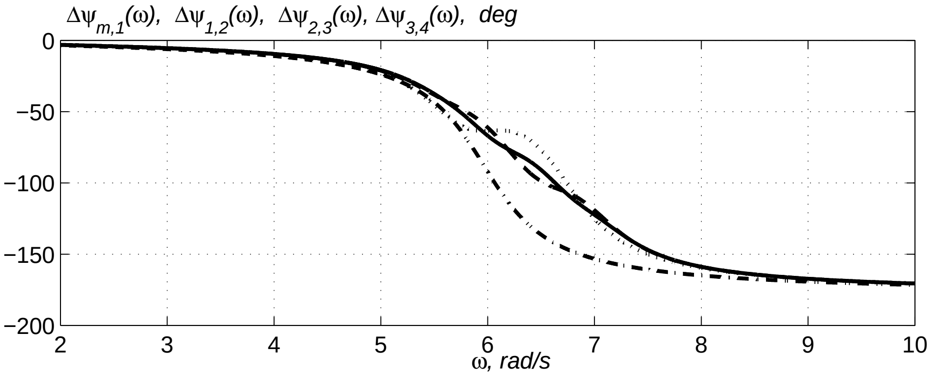

Figure 5: Phase shifts between the pendulum rotation angles in the chain (analytical evaluation). $\Delta\psi_{m,1}(\omega_m)$ – solid line, $\Delta\psi_{1,2}(\omega_m)$ – dashed line, $\Delta\psi_{2,3}(\omega_m)$ – dotted line, $\Delta\psi_{3,4}(\omega_m)$ – dash-dot line.

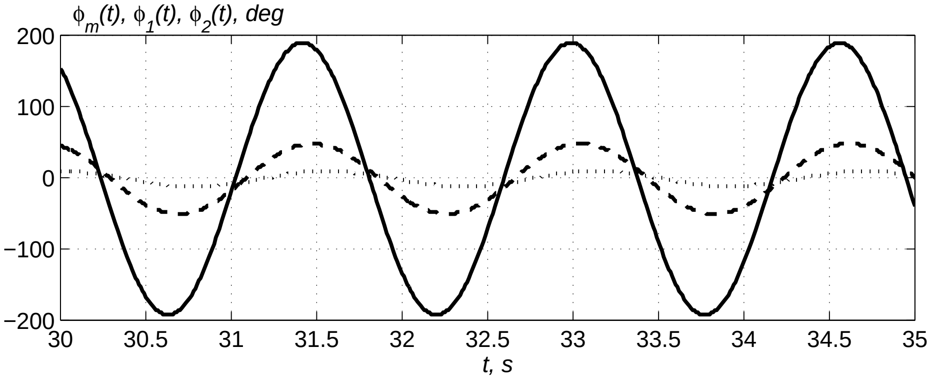

Figure 6: Experimental result. Inphase synchronization in the chain of pendulums. $\varphi_m(t)$ – solid line, $\varphi_1(t)$ – dashed line, $\varphi_2(t)$ – dotted line; $\omega_m = 4.0$ rad/s.

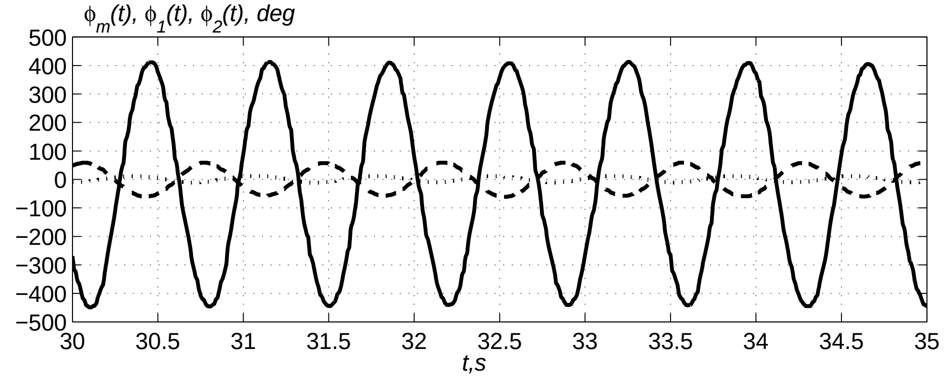

Figure 7: Experimental result. Antiphase synchronization in the chain of pendulums. $\varphi_m(t)$ – solid line, $\varphi_1(t)$ – dashed line, $\varphi_2(t)$ – dotted line; $\omega_m = 8.9$ rad/s.

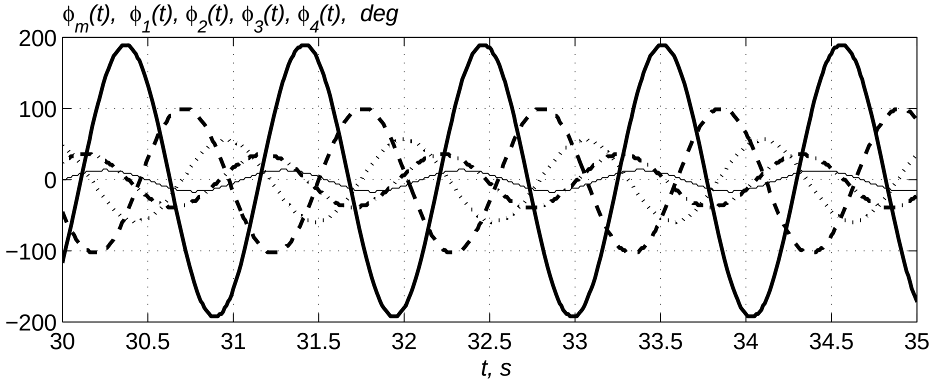

Figure 8: Experimental result. Running wave in the chain of four pendulums. $\varphi_m(t)$ – solid line, $\varphi_1(t)$ – dashed line, $\varphi_2(t)$ – dotted line, $\varphi_3(t)$ – dash-dot line, $\varphi_4(t)$ – thin line; $\omega_m = 6.0$ rad/s ($\omega_m$ lies inside the pass band).

by the the optical encoder with 2-degrees accuracy and 100 Hz sampling frequency. The measured data were transmitted over the communication link without any restrictions on the data rate. Therefore, for the considered system, the autonomus plant model (1) was replaced by the following exogenous model:

$$\dot{x}(t) = Ax(t) + B\psi(y) + Du(t), \; y(t) = Cx(t), \tag{8}$$

where $u(t) \in \mathbb{R}^m$ stands for the external input, $D$ is an $(n \times m)$ matrix, the other notations are the same as in (1). Respectively, the modified observer (6) equation reads as

$$\begin{aligned} &\dot{\hat{x}}(t) = A\hat{x}(t) + B\psi(\hat{y}) + D\tilde{u}(t) + L\bar{\varepsilon}(t), \; \hat{y}(t) = C\hat{x}(t), \\ &\bar{\varepsilon}(t) = \bar{\varepsilon}[k] \;\text{ as }\; t \in [t_k, t_{k+1}), \;\; t_k = kT_s, \;\; k = 0, 1, 2, \ldots. \end{aligned} \tag{9}$$

where $\tilde{u}(t)$ denotes the measured values of the exogenous input $u(t)$. The errors of the plant input $u(t)$ and output $y(t)$ measurements introduce imperfections into data transmission procedure and impose limitation on the estimation accuracy, achievable in the real-world systems.

The state estimation procedure (2)–(5), (9) has been tested as applied to the chain of four pendulum sections, excited by the motor, which is connected with the pendulum #1 via the torsion spring. The harmonical and the irregular control voltages have been applied to the motor. The first-order data transmission scheme (2)–(5) has been used for transferring the rotation angles of the motor and the pendulum # 4 to the central computer. For obtaining the estimates of the angles and angular velocities for the pendulums ##1–3, the full-order coding algorithm (2),(4),(5),(6) was applyed. The following coder/decoder parameters have been taken: $T_s \in [0.01, 0.05]$ s, $\nu \in \{4, 6, 8\}$, $M_{0,\mathrm{m}} = 15$, $M_{\infty\mathrm{m}} = 0.1$, $M_{0,\varepsilon} = 2$, $M_{\infty,\varepsilon} = 5e^{-4}$, $\rho = 0.99$, $L = [27.3, 362]^{\mathrm{T}}$. The estimation accuracy has been calculated as the mean-square relative error $Q(R,\nu) = \sqrt{\dfrac{\int_0^T (\varphi_1 - \hat{\varphi}_1)^2\,dt}{\int_0^T \varphi_1^2\,dt}}$, $T = 30$ s. The experimental results are depicted in Figs. 9–11. It seen, that the data transmision rate may be taken about 200 bit/s, ensuring the appropriate accuracy of the state transmission in approximately 3 % of the relative mean-square error. It is also seen, that there exists a threshold, bounding the secure data transmission rate, confirming the results of [19, 23].

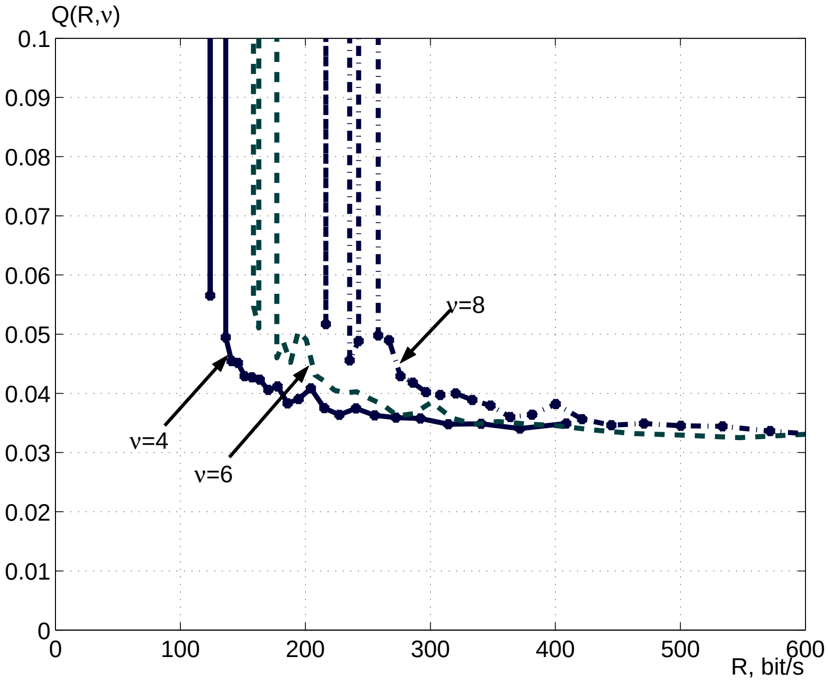

Figure 9: Relative estimation error vs transmission rate $R$ for different $\nu$. Solid line – $\nu = 4$, dashed line – $\nu = 6$, dash-dot line – $\nu = 4$.

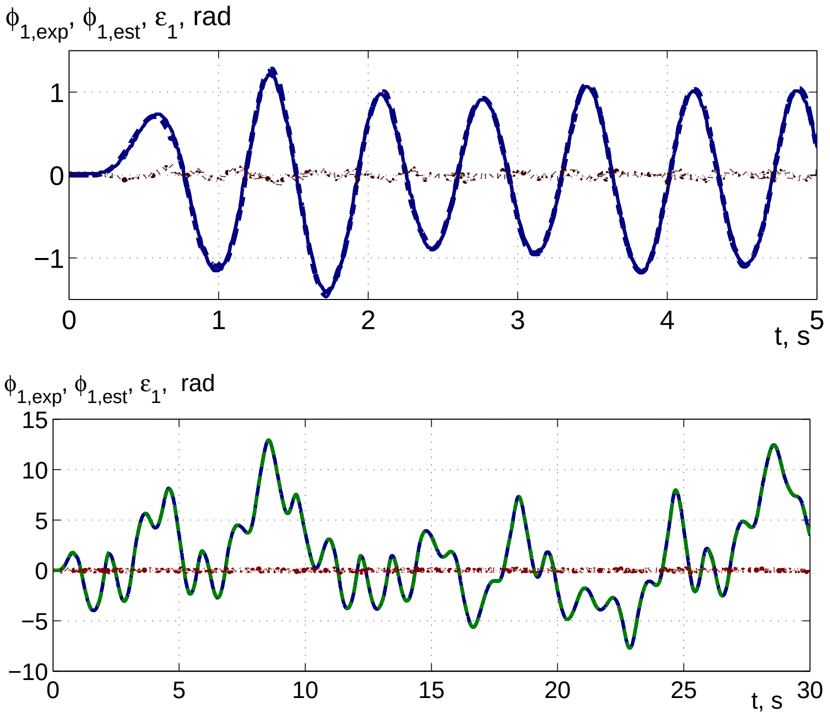

Figure 10: Measured process $\varphi_1(t)$ (dashed line), process estimate $\hat{\varphi}_1(t)$, obtained by the decoder (solid line), estimation error $\varepsilon_1(t) = \varphi_1(t) - \hat{\varphi}_1(t)$ (dotted line). $T_s = 0.020$ s, $\nu = 4$, $R = 200$ bit/s. *a*) – harmonic excitation, *b*) – irregular excitation.

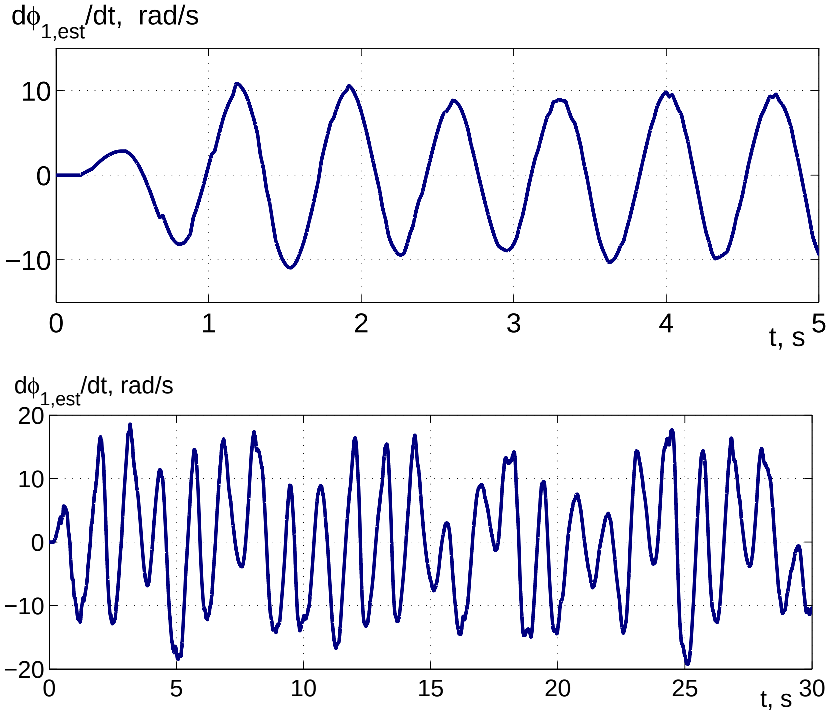

Figure 11: Estimate of the angular rate $\dot{\varphi}_1(t)$, obtained by the decoder. $T_s = 0.020$ s, $\nu = 4$, $R = 200$ bit/s. *a*) – harmonic excitation, *b*) – irregular excitation.

## Conclusions

A novel multipendulum mechatronic set-up is designed, allowing to implement different algorithms of estimation, synchronization and control. The set-up is aimed at solving various research and educational tasks in the areas of hybrid modeling, analysis, identification and control of mechanical systems as well the data communication in distributed mechatronic complexes.

The system is currently being tested and tuned. Four active and up to 46 passive sections are ready for connection. Already at this stage the system can be used for demonstration and for research. Connecting up to twelve pendulums and one electric motor, harmonically exciting the system, has shown that the type of the synchronization mode (inphase or antiphase) depends on the excitation frequency, that agree with the conclusions of [18, 4]. Possible ways for improving the data exchange in multiagent mechatronic complexes are outlined in the paper. The data transmission scheme of [19, 23] has been modified for non-autonomous systems and experimentally tested. It is seen, that the data transmision rate may be taken about 200 bit/s, ensuring the appropriate accuracy of the state transmission over the digital communication channel. that there demonstrated that there exists a threshold, bounding the secure data transmission rate, which confirms the results of [19, 23].

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