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Adaptive Control of Systems with Fast Varying Unknown Delay in Measurements

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Abstract— **An adaptive output feedback controller with a fast varying unknown delay in the measurements is considered. Sufficient conditions for the resulting closed-loop system to be asymptotically stable with respect to state variables within some domain of attraction are derived. It is shown that the adaptive gain tends to a constant value. The proof is based on Passification lemma and a Lyapunov-Krasovskii method. The result is applicable to systems with polytopic-type uncertainties. Advantages of the adaptive control over the static output feedback are demonstrated on a pitch angle control of a flying vehicle.**

I. INTRODUCTION

An adaptive control of time delay systems has been known as an important and challenging problem [1]. In one possible formulation it is required to construct an adaptive controller that stabilizes uncertain system with a state delay [2], [3], [4], [5]. More difficult case (see Remark 4) arises when an adaptive controller experiences output/input delay [6], [7], [8]. One of the classical approaches to this problem is based on a predictor method for known time delay and adaptive predictor method for unknown time delay [1]. However, these methods work only with slowly varying delay, i. e. derivative of the time delay should be smaller than one. Another way to resolve this issue is to assume that the difference between current and delayed control signal is not large [8], but this assumption is difficult to verify. Differently from [1] our approach allows to control systems with fast-varying unknown delay and differently from [8] we do not impose any assumptions on the control signal.

As it was shown in [9], any hyper-minimum-phase linear time-invariant system can be stabilized by a static output feedback $u(t) = -ky(t)$ if k is large enough (for more established description of this approach see [10]). For the case of uncertain systems an adaptive control law can be derived via speed gradient method [11]. Meanwhile, it has not been studied yet how this adaptive controller will operate in the presence of unavoidable measurement delay that arises due to the delays in signal transmission and processing.

The objective of this paper is to analyze an adaptive controller in the presence of time-varying measurement delay. We derive an upper bound for time delay such that in some domain of initial conditions the states of the closed-loop system tend to zero, whereas an adaptive controller gain tends to a constant value. The proof extends passification based adaptive control to systems with time-varying unknown delay.

For a given class of uncertain hyper-minimum-phase linear systems, one can show that if the measurement time delay is small enough then for a *big enough* controller gain the static output feedback ensures global asymptotic stability of all closed-loop systems from a given class of uncertainty (to derive that Theorem 1 with $\mu = 0$ may be applied). An advantage of the adaptive controller over the static one is that while stabilizing a particular system from a class of uncertainties it yields a smaller control gain. Moreover, the limiting value of an adaptive gain depends on the initial state of the system, i. e. the closer to zero initial state is the smaller limiting value will be achieved. More specifically these ideas are demonstrated on an example of a pitch angle control of a flying vehicle in Sec. IV.

The remainder of the paper is organized as follows. In Sec. I-A we give an auxiliary result that is heavily used throughout the paper. In Sec. II we introduce an adaptive controller under consideration. In Sec. III we formulate main results. In Sec. IV we demonstrate advantages of the adaptive control over the static output feedback on a pitch angle control of a flying vehicle. Some conclusions are drawn in Sec. V.

A. Preliminaries: Passification lemma

Definition 1: A linear system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = g^T Cx(t)$ with a transfer function $W(\lambda) = g^T C(\lambda I - A)^{-1}B$ is called *hyper-minimum-phase* if the polynomial $\varphi(\lambda) = W(\lambda)\det(\lambda I - A)$ is Hurwitz and $g^T CB = lim_{\lambda\to\infty}\lambda W(\lambda)$ is a positive number.

The proof of the main results is based on Passification lemma [12].

Lemma 1 (Passification lemma): Let the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{l\times n}$, $g \in \mathbb{R}^{l\times 1}$ be given. Then for existence of a positive-definite $n \times n$-matrix P and $k_* \in \mathbb{R}$ such that

$$PA_* + A_*^T P < 0, \quad PB = C^T g, \quad A_* = A + k_* Bg^T C$$

it is necessary and sufficient that the system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = g^T Cx(t)$ is hyper-minimum-phase.

Remark 1: If a transfer matrix $W(\lambda) = g^T C(\lambda I - A)^{-1}B$ is hyper-minimum-phase then there exists k_* such that a control law $u = k_* y + v$, where v is a new control signal,

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makes the system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = g^T Cx(t)$ *strictly passive* with respect to a new input v, i. e. there exists a nonnegative scalar function $V(x)$ and a scalar function $\mu(x)$, where $\mu(x) > 0$ for $x \neq 0$, such that the following holds

$$V(x) \leq V(x_0) + \int_0^t \left[y^T(t)v(t) - \mu(x(t)) \right] dt$$

for any solution satisfying $x(0) = x_0$. Appropriate value for k_* is any negative number such that

$$k_* < k_0 = \inf_{\omega \in \mathbb{R}} \operatorname{Re} \left\{ W(i\omega)^{-1} \right\}. \tag{1}$$

II. PROBLEM FORMULATION

Consider the linear system

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \\ &y(t) = Cx(t - r(t)), \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}^l$ is the output ($y(t) = 0$ for $t - r(t) < 0$) and $r(t)$ is an unknown delay that satisfies $0 \leq r(t) \leq h$.

Let $g \in \mathbb{R}^l$ be such that $g^T C(\lambda I - A)^{-1} B$ is hyper-minimum-phase. Consider the adaptive controller:

$$\begin{aligned} &u(t) = 0, \quad t \in [0, h), \\ &u(t) = k(t) g^T y(t), \quad \dot{k}(t) = -\gamma^{-1} \left(g^T y(t) \right)^2, \quad t \geq h \end{aligned} \tag{3}$$

with $k, \gamma \in \mathbb{R}$, $\gamma > 0$. In the next section we investigate the stability of the closed-loop system (2), (3) with respect to $x(t)$.

Remark 2: Note that for $t \geq h$ the second line of (3) is well defined, while for $t \in [0, h)$ the value of $y(t)$ can be undefined. In fact one may assume that there exists a unique t^* such that $t - r(t) < 0$ for $t < t^*$ and $t - r(t) \geq 0$ for $t \geq t^*$ and take $u(t) = 0$ for $t < t^*$ and $u(t)$ from the second line of (3) for $t \geq t^*$. To derive a simple solution we take $t^* = h$. For more detailed discussion on this issue see [13].

Remark 3: Note that the system with a constant control delay and a time-varying measurements delay can be transformed into (2), notably the stability of the system

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t - r_1), \\ &y(t) = Cx(t - r_2(t)), \end{aligned}$$

with the control (3) is equivalent to the stability of the system (2), (3) with $r(t) = r_1 + r_2(t)$.

III. MAIN RESULT

The closed-loop system (2), (3) can be represented in the form:

$$\begin{aligned} &\dot{x}(t) = Ax(t), \quad x(0) = x_0, \quad t \in [0, h), \\ &\dot{x}(t) = Ax(t) + k_* B g^T C x(t - r(t)) + \\ &\qquad + (k(t) - k_*) B g^T C x(t - r(t)), \\ &\dot{k}(t) = -\gamma^{-1} (g^T C x(t - r(t)))^2, \quad t \geq h. \end{aligned} \tag{4}$$

Denote $x_t(\theta) = x(t + \theta)$, $\theta \in [-h, 0]$. For $t \in [0, h)$ we use an estimate:

$$\|x_h\|_{\mathcal{C}}^2 \leq \|x_0\|^2 \max_{s \in [0,h]} \lambda_{\max}(e^{sA^T} e^{sA}) \tag{5}$$

and for $t \geq h$ we will derive results by using the standard Lyapunov-Krasovskii functional

$$V(x_t, \dot{x}_t, k(t)) = V_1(x_t, k(t)) + V_2(x_t) + V_3(\dot{x}_t), \tag{6}$$

where

$$\begin{aligned} V_1(x_t, k(t)) &= x^T(t) P x(t) + \gamma (k(t) - k_*)^2, \\ V_2(x_t) &= \int_{t-h}^t x^T(s) S x(s)\, ds, \\ V_3(\dot{x}_t) &= h \int_{-h}^0 \int_{t+\lambda}^t \dot{x}^T(s) R \dot{x}(s)\, ds\, d\lambda. \end{aligned}$$

Remark 4: Note that analysis of a state delay is more simple than analysis of a measurement delay. Consider the system

$$\begin{aligned} &\dot{x}(t) = A_0 x(t) + A_1 x(t - r(t)) + Bu(t), \\ &y(t) = Cx(t). \end{aligned} \tag{7}$$

Here x, u, y, r have the same meaning as previously and all matrices have appropriate dimensions. To investigate the stability of the closed-loop system (7), (3) functional (6) can be applied. Then

$$\begin{aligned} \dot{V}_1 = 2x^T(t) P \left[A_0 x(t) + A_1 x(t - r(t)) + k_* B g^T C x(t) \right] + \\ 2(k(t) - k_*) x^T(t) P B g^T C x(t) - 2(k(t) - k_*)(g^T y(t))^2. \end{aligned}$$

If $PB = C^T g$ then the last two terms can be canceled and further analysis reduces to the problem of stability of (7) with $u(t) = k_* g^T y(t)$. This problem can be solved by the standard methods, e. g. [17], [18]. In the case of an output delay such cancelation is not possible since the controller does not measure current value of the state. Therefore, we prove that the difference $k(t) - k_*$ will stay bounded and require LMIs (8), (9) to be feasible for all possible values of $k(t) - k_*$.

Theorem 1: Let $g \in \mathbb{R}^l$ be such that $g^T C(\lambda I - A)^{-1} B$ is hyper-minimum-phase. Given M_k, k_* let there exist $n \times n$-matrices $P > 0$, $S > 0$, $R > 0$, and G such that $PB = C^T g$ and the following LMIs hold

$$\Phi|_{\mu = M_k} < 0, \quad \Phi|_{\mu = -M_k} < 0, \tag{8}$$

$$\begin{bmatrix} R & G \\ * & R \end{bmatrix} \geq 0, \tag{9}$$

where

$$\Phi = \begin{bmatrix} \Phi_1 & 0 & 0 & h\Phi_2 & hA^T R \\ * & -R & R & h\Phi_3 & h\Phi_4 \\ * & * & -(S+R) & hG & 0 \\ * & * & * & -h^2 R & 0 \\ * & * & * & * & -R \end{bmatrix},$$

$$\begin{aligned} \Phi_1 &= P[A + k_* B g^T C] + [A + k_* B g^T C]^T P + S, \\ \Phi_2 &= -k_* P B g^T C, \\ \Phi_3 &= \mu C^T g B^T P - G, \\ \Phi_4 &= \mu C^T g B^T R + k_* C^T g B^T R. \end{aligned}$$

Then the closed-loop system (2), (3) is asymptotically stable with respect to $x(t)$ with a domain of stability given by

$$\begin{aligned} &|k(h)-k_*|<M_k,\\ &\|x(0)\|^2<\gamma c\left(M_k^2-(k(h)-k_*)^2\right), \end{aligned} \tag{10}$$

where

$$c=\left[\lambda_{\max}(P)+h\lambda_{\max}(S)+\frac{h^3}{2}\lambda_{\max}(A^TRA)\right]^{-1}\times \left(\max_{s\in[0,h]}\lambda_{\max}(e^{sA^T}e^{sA})\right)^{-1},$$

and $k(t)$ tends to a constant value.

Proof: Let $t\geq h$. Denote $\eta(t)=\frac{1}{h}\int_{t-r(t)}^{t}\dot{x}(s)\,ds$. Differentiating V_1 along (4) we derive

$$\begin{aligned} &\dot{V}_1=2x^T(t)P\left[Ax(t)+k(t)Bg^TCx(t-r(t))\right]-\\ &2(k(t)-k_*)\left(g^Ty(t)\right)^2=2x^T(t)P[A+k_*Bg^TC]x(t)-\\ &2hk_*x^T(t)PBg^TC\eta(t)+2h(k(t)-k_*)\eta^T(t)PBg^TC\times\\ &x(t-r(t))+2(k(t)-k_*)x^T(t-r(t))PBg^TCx(t-r(t))-\\ &2(k(t)-k_*)x^T(t-r(t))C^Tgg^TCx(t-r(t)). \end{aligned} \tag{11}$$

Since $PB=C^Tg$ the last two terms can be canceled. Further

$$\begin{aligned} &\dot{V}_2=x^TSx-x^T(t-h)Sx(t-h),\\ &\dot{V}_3=h^2\dot{x}^TR\dot{x}-h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)\,ds. \end{aligned} \tag{12}$$

Applying Jensen inequality [14] we find that

$$\begin{aligned} &-h\int_{t-h}^{t-r(t)}\dot{x}^T(s)R\dot{x}(s)\,ds\leqslant\\ &\qquad-\frac{h}{h-r(t)}\int_{t-h}^{t-r(t)}\dot{x}^T(s)\,dsR\int_{t-h}^{t-r(t)}\dot{x}(s)\,ds \end{aligned}$$

$$-\,h\int_{t-r(t)}^{t}\dot{x}^T(s)R\dot{x}(s)\,ds\ \leqslant\ -\frac{h}{r(t)}h^2\eta^T(t)R\eta(t).$$

Denote $\alpha_1=\frac{h-r(t)}{h}$, $\alpha_2=\frac{r(t)}{h}$,

$$\begin{aligned} &f_1(t)=\int_{t-h}^{t-r(t)}\dot{x}^T(s)\,dsR\int_{t-h}^{t-r(t)}\dot{x}(s)\,ds,\\ &f_2(t)=h^2\eta^T(t)R\eta(t),\\ &g(t)=h\int_{t-h}^{t-r(t)}\dot{x}^T(s)\,ds\,G\eta(t). \end{aligned}$$

Since (9) is feasible, by applying Park's lemma [15] we find that

$$\begin{aligned} &-h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)\,ds\leqslant-\left[\frac{1}{\alpha_1}f_1(t)+\frac{1}{\alpha_2}f_2(t)\right]\leqslant\\ &-\left[f_1(t)+f_2(t)+2g(t)\right]. \end{aligned} \tag{13}$$

Summing up (11), (12) and using (13) we derive

$$\dot{V}\leq\xi^T\Xi\xi+h^2\dot{x}R\dot{x}, \tag{14}$$

where $\xi(t)=(x(t),x(t-r),x(t-h),\eta(t))^T$,

$$\Xi=\begin{pmatrix}\Phi_1 & 0 & 0 & h\Phi_2\\ * & -R & R & h\Phi_3'\\ * & * & -(S+R) & hG\\ * & * & * & -h^2R\end{pmatrix}.$$

Entries Φ_1, Φ_2 are given in the formulation of the theorem and Φ_3' is the matrix Φ_3 with $\mu=k(t)-k_*$. Substituting $\dot{x}(t)=Ax(t)+k(t)Bg^TCx(t-r(t))$ into (14) and applying the Schur complements formula we find that if $\Phi|_{\mu=k(t)-k_*}<0$ then $\exists\,\varepsilon>0:\dot{V}(t)\leq-\varepsilon\|\xi(t)\|^2$, where $V(t)=V(x_t,\dot{x}_t,k(t))$. If (8), (9) are feasible with the same P, S, R then $\Phi<0$ for $\mu\in[-M_k,M_k]$.

Since (5) is valid, the domain (10) implies $V(h)<\gamma M_k^2$. Moreover, the following implications hold:

$$V(t)\leq\gamma M_k^2\Rightarrow|k(t)-k_*|\leq M_k\Rightarrow\dot{V}(t)\leq 0.$$

From the above it follows that $V(t)<\gamma M_k^2$ for all $t\in[h,\infty)$. Indeed, let $t_*=\min\{t|V(t)=\gamma M_k^2\}$. Then $\forall s\in[h,t_*]$ $\dot{V}(s)\leq 0$. Since $V(t_*)=V(h)+\int_h^{t_*}\dot{V}(s)\,ds$ we have $V(t_*)\leq V(h)<\gamma M_k^2$. The latter contradicts to $V(t_*)=\gamma M_k^2$. As far as $V(t)\leq\gamma M_k^2$ inequality (14) is valid and, therefore, $\exists\,\varepsilon>0:\dot{V}(t)\leq-\varepsilon\|\xi(t)\|^2$. Further,

$$\begin{aligned} V(t)=V(h)+\int_h^t\dot{V}(s)\,ds\leq\\ V(h)-\varepsilon\int_h^t\|\xi(s)\|^2\,ds. \end{aligned} \tag{15}$$

Since $V(h)<\infty$, (15) implies $V(t)<\infty$. But if $k(t)\to\infty$ then $V(t)\to\infty$, therefore, $k(t)$ is bounded. Since $V(h)$ and $V(t)$ are bounded, $\int_h^t\|\xi(s)\|^2\,ds<\infty$. It follows from Barbalat's lemma [16] that $\xi(t)\to 0$ when $t\to\infty$, that is $x(t)\to 0$ when $t\to\infty$ and the system (2), (3) is asymptotically stable with respect to $x(t)$. Finally,

$$k(t)=k(h)-\gamma^{-1}\int_h^t\left(g^Ty(s)\right)^2\,ds.$$

Since $V(h)$ and $V(t)$ are bounded, $\int_h^\infty\|\xi(s)\|^2\,ds<\infty$, thus $\int_h^\infty\left(g^Ty(s)\right)^2\,ds<\infty$. Hence, there exists $\lim_{t\to\infty}k(t)=k(h)-\gamma^{-1}\int_h^\infty\left(g^Ty(s)\right)^2\,ds$, i. e. $k(t)$ tends to a constant value. ■

Remark 5: Note that (3) contains $\gamma>0$. As it can be seen from (10) increase of γ leads to increase of the domain of stability. At the same time the larger γ is the slower the speed of convergence is. For $\gamma^{-1}=0$ the system may be unstable.

Remark 6: If $h=0$, i. e. (2) does not contain time delay, one can choose $R=S=G=0$. Then the existence of $P>0$ such that $\Phi_1<0$ is guaranteed by Passification lemma. As far as Φ_1 does not contain μ, in the delay-free case the system (2), (3) is globally asymptotically stable with respect to $x(t)$. Theorem 1 gives an upper bound for $r(t)$ such that (2), (3) remains asymptotically stable within the domain of stability (10).

Remark 7: The smaller h is, the larger M_k can be taken such that (8), (9) are feasible. That is smaller delay allows for larger domain of stability (10).

Remark 8: LMIs of Theorem 1 are affine in A. Therefore, if A resides in the uncertain polytope

$$A = \sum_{j=1}^{M} \mu_j(t) A^{(j)}, \quad 0 \le \mu_j(t) \le 1, \quad \sum_{j=1}^{M} \mu_j(t) = 1,$$

one have to solve these LMIs simultaneously for all the M vertices $A^{(j)}$, applying the same decision matrices.

IV. EXAMPLE: FLYING VEHICLE PITCH CONTROL

Let us illustrate the proposed adaptive control method by example of a flying vehicle pitch control. Under several simplifying assumptions, dynamics of the pitch angle can be described [9] by (2) with

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & a_0 \\ 0 & 1 & a_1 \end{bmatrix}, B = \begin{bmatrix} b_0 \\ b_1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & c_1 \end{bmatrix}. \quad (16)$$

In [9] for the system (2), (16) without delay the adaptive algorithm (3) was synthesized and analyzed. Here we study the system (2), (16), (3) in presence of unknown fast varying time delay in the measurements.

Let the matrix A contain uncertain parameters: $a_0 \in [\alpha_0, \beta_0]$, $a_1 \in [\alpha_1, \beta_1]$. For $g = (1,1)^T$ the transfer function

$$W(\lambda) = g^T C(\lambda I - A)^{-1} B = \frac{(b_1\lambda + b_0)(\lambda + 1 + c_1 - a_1)}{\lambda(\lambda^2 - a_1\lambda - a_0)}$$

is hyper-minimum-phase if

$$b_0 > 0, \quad b_1 > 0, \quad a_1 < c_1 + 1.$$

Since (8) is affine in A to ensure stability conditions one have to solve (8), (9) simultaneously for four vertices given by $A^{(1)} = A|_{\substack{a_0=\alpha_0 \\ a_1=\alpha_1}}$, $A^{(2)} = A|_{\substack{a_0=\alpha_0 \\ a_1=\beta_1}}$, $A^{(3)} = A|_{\substack{a_0=\beta_0 \\ a_1=\alpha_1}}$, $A^{(4)} = A|_{\substack{a_0=\beta_0 \\ a_1=\beta_1}}$ with the same R, tuning parameter k_*, and P such that $PB = C^T g$.

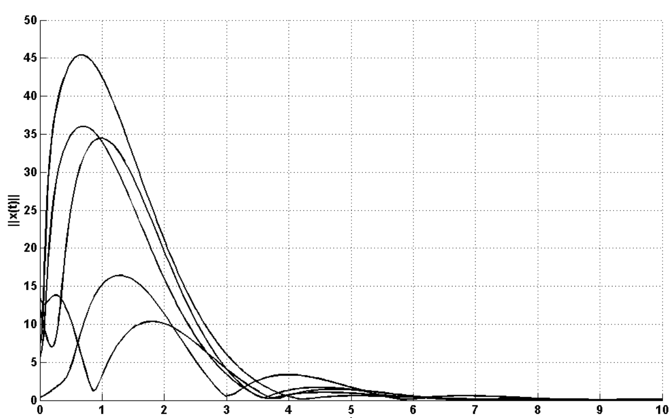

Fig. 1. $\|x(t)\|$ for $a_0 = 13.15$, $a_1 = 0.8$, $\gamma = 10$, $k(h) = 0$, $h = 0.001$ and different initial conditions such that $\|x(0)\|$ satisfies (10).

For simulations we take $b_0 = 19.76$, $b_1 = 15.2$, $c_1 = 0.8$ and suppose that $a_0 \in [0.5, 20]$, $a_1 \in [0.1, 1.6]$. Then (8), (9) are feasible in vertices $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, and $A^{(4)}$ for $h = 0.001$, $M_k = 6.5$, $k_* = -4$. For $k(h) = 0$, $\gamma = 10$ the domain of stability (10) is $\|x(0)\| < 15.33$.

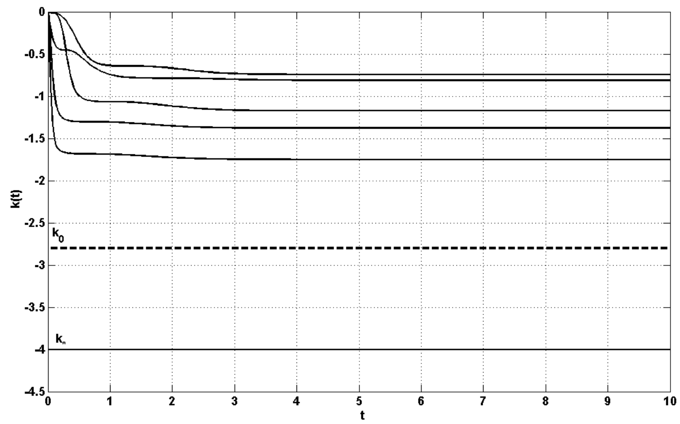

Fig. 2. Tuning gains $k(t)$, k_*, and the static gain k_0. All parameters values are same as in Fig. 1.

In Fig. 1 one can see norms of five different solutions of (2), (3) with $k(h) = 0$ and different randomly chosen initial conditions $\|x(0)\| < 15.33$. Since (2), (3) is asymptotically stable with respect to $x(t)$ all norms tend to zero. In Fig. 2 the corresponding gains $k(t)$ are depicted. As it was proved all gains tend to constant values.

On basis of the proof for Theorem 1 one can easily prove that if the conditions of Theorem 1 are satisfied, the controller $u(t) = k_* g^T y(t)$ insures global asymptotic stability of the closed-loop system (2), (3). Note that the limiting values of $|k(t)|$ are much smaller than $|k_*|$. Moreover, in order to compare the adaptive output feedback (3) and the static output feedback $u(t) = k_0 g^T y(t)$ we found the minimum value $k_0 = -2.8$ such that (8), (9) with P such that $PB = C^T g$ are feasible for $\mu = 0$ in all vertices $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, and $A^{(4)}$. Obviously, $|k_0|$ should be not more than $|k_*|$ since k_* insures feasibility of LMIs for all $\mu \in [-M_k, M_k]$. The value of k_0 is depicted with a dashed line. As one may notice all $|k(t)|$ tend to values that are smaller than $|k_0|$. The reason is that the static controller allows one to stabilize any system with $a_0 \in [\alpha_0, \beta_0]$, $a_1 \in [\alpha_1, \beta_1]$. On the other hand the adaptive controller allows one to use a smaller controller gain that depends on the system parameters and initial values $x(0)$.

V. CONCLUSIONS

We have analyzed an adaptive output feedback controller for a linear (probably uncertain) systems in the presence of time-varying measurements delay. Under the assumption that the delay-free system is hyper-minimum-phase we have derived LMIs for the local stability of the closed-loop system with respect to its state, whereas the tuning gain tends to a constant value. The domain of stability can be broadened by increasing the value of γ from (3) but this leads to the reduction of the speed of convergence. For $h = 0$ our results recover results for a delay free case. Theorem 1 gives an upper bound on the time-varying delay that preserves the stability of the states of the closed-loop system. An application of Theorem 1 to the case of polytopic-type uncertainties and advantage of the adaptive output control over the static output control are demonstrated on the example of a pitch angle control of a flying vehicle. It is shown that an adaptive gain tends to a constant value that is smaller than the static gain that ensures the stability of the entire class of uncertain systems. Moreover, smaller values of initial conditions yield smaller limiting value of a controller gain.

REFERENCES

[1] M. Krstic, "Nonlinear and delay-adaptive systems," *IEEE Control System Magazine*, vol. 30, no. 1, pp. 22–41, 2010.

[2] M. Balas, S. Gajendar, and L. Robertson, "Adaptive Tracking Control of Linear Systems with Unknown Delays and Persistent Disturbances (or Who You Callin' Retarded?)," in *AIAA Guidance, Navigation, and Control Conference*, Reston, Virigina, Aug. 2009, pp. 1–15.

[3] R. Ben Yamin, I. Yaesh, and U. Shaked, "Robust simple adaptive model following for linear time-delay systems with guaranteed H-infinity performance," *IMA Journal of Mathematical Control and Information*, vol. 27, no. 4, pp. 475–491, 2010.

[4] V. Klinshov and V. Nekorkin, "Synchronization in networks of pulse oscillators with time-delay coupling," *Cybernatics and physics*, vol. 1, no. 2, pp. 106–112, 2012.

[5] A. Proskurnikov, "Average consensus in symmetric nonlinear multi-agent networks with non-homogeneous delays," *Cybernatics and physics*, vol. 1, no. 2, pp. 128–133, 2012.

[6] A. Tsykunov, *Adaptive control of systems with aftereffect*. Moscow: Nauka, 1984. [In Russian]

[7] R. Ortega and R. Lozano, "Globally stable adaptive controller for systems with delay," *International Journal of Control*, vol. 47, no. 1, pp. 17–23, 1988.

[8] M. J. Balas and J. P. Nelson, "New robustness theorem with application to adaptive control of nonlinear systems with input/output delays," in *IEEE International Conference on Control Applications (CCA)*, Sep. 2011, pp. 1437–1442.

[9] A. L. Fradkov, "Synthesis of adaptive system of stabilization of linear dynamic plants," *Automation and Remote Control*, no. 12, pp. 96–103, 1974.

[10] B. R. Andrievskii and A. L. Fradkov, "Method of passification in adaptive control, estimation, and synchronization," *Automation and Remote Control*, vol. 67, no. 11, pp. 1699–1731, 2006.

[11] A. L. Fradkov, "Speed-gradient scheme and its application in adaptive control problems," *Automation and Remote Control*, vol. 40, no. 9, pp. 1333–1342, 1980.

[12] ——, "Passification of Non-square Linear Systems and Feedback Yakubovich-Kalman-Popov Lemma," *European journal of control*, no. 6, pp. 573–582, 2003.

[13] K. Liu and E. Fridman, "Delay-dependent methods : when does the first delay interval need a special analysis?" in *IFAC Workshop on Time Delay Systems*, Grenoble, France, 2013.

[14] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston: Birkhauser, 2003.

[15] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[16] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.

[17] E. Fridman and U. Shaked, "Delay-dependent stability and H-infinity control: Constant and time-varying delays," *International Journal of Control*, vol. 76, no. 1, pp. 48–60, 2003.

[18] E. Fridman and Y. Orlov, "Exponential stability of linear distributed parameter systems with time-varying delays," *Automatica*, vol. 45, no. 1, pp. 194–201, 2009.