Adaptive control of passifiable linear systems with quantized measurements and bounded disturbances☆

Anton Selivanov[a], Alexander Fradkov[b,c], Daniel Liberzon[d]

[a]School of Electrical Engineering, Tel Aviv University, Israel
[b]Saint Petersburg State University, St. Petersburg, Russia
[c]Institute for Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg, Russia
[d]University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA

Abstract

We consider a linear uncertain system with an unknown bounded disturbance under a passification-based adaptive controller with quantized measurements. First, we derive conditions ensuring ultimate boundedness of the system. Then we develop a switching procedure for an adaptive controller with a dynamic quantizer that ensures convergence to a smaller set. The size of the limit set is defined by the disturbance bound. Finally, we demonstrate applicability of the proposed controller to polytopic-type uncertain systems and its efficiency by the example of a yaw angle control of a flying vehicle.

Keywords: Adaptive control; Quantization; Disturbance; Passification method

1. Introduction

Adaptive control plays an important role in the real world problems, where exact system parameters are often unknown. One of the possible methods for adaptive control synthesis is the *passification method* [2]. Starting from the works [3, 4] this method proved to be very efficient and useful. Nevertheless, while implementing passification-based adaptive control, several issues may arise. First of all, disturbances inherent in most systems can cause infinite growth of the control gain. This issue may be overcome by introducing the so-called "σ-modification" [5, 6]. Secondly, the measurements can experience time-varying unknown delay. This problem has been recently studied in [7]. In this paper we consider passification-based adaptive control in the presence of measurement quantization and propose a switching procedure for the controller

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Email addresses: `antonselivanov@gmail.com` (Anton Selivanov), `fradkov@mail.ru` (Alexander Fradkov), `liberzon@uiuc.edu` (Daniel Liberzon)

parameters that ensures the convergence of the system state to an ellipsoid whose size depends on the upper bound of the disturbance.

Control with limited information has attracted growing interest in the control research community lately [8, 9, 10, 11]. Due to limited sensing capabilities, defects of sensors and limited communication channel capacities it is reasonable to assume that only approximate value of the output is available to a controller. These sensor and communication imposed constraints can be modeled by quantization [12].

Although adaptive control of uncertain systems received considerable interest and has been widely investigated, there are few works devoted to adaptive control with quantized measurements. In [13] the performance of an adaptive observer-based chaotic synchronization system under information constrains has been analyzed. A binary coder-decoder scheme has been proposed and studied in [14] for synchronization of passifiable Lurie systems via limited-capacity communication channel. In [15] a direct adaptive control framework for systems with *input* quantizers has been developed. In [16] a supervisory control scheme for uncertain systems with quantized measurements has been proposed. In supervisory control schemes usually a finite family of candidate controllers is employed together with an estimator-based switching logic to select the active controller at every time.

Differently from these works, the control scheme proposed here does not require any estimator or observer. Unlike [16] we consider adaptive tuning of the controller gain, rather than switching between several known controllers. At the same time, to ensure convergence to a smaller set, our controller switches parameters of the adaptation law.

Notations. By $\|\cdot\|$ we denote Euclidean norm for vectors and spectral norm for matrices. For $P \in \mathbb{R}^{n\times n}$ notation $P > 0$ means that P is symmetric and positive-definite, $\lambda_{\max}(P)$, $\lambda_{\min}(P)$ are the maximum and minimum eigenvalues, respectively, P^T denotes transposed matrix P.

2. System description

Consider an uncertain linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t), \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

with state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}$, output $y \in \mathbb{R}^l$, and constant uncertain matrices A, B, C of appropriate dimensions. Unknown disturbance $w(t) \in \mathbb{R}^n$ has a bounded norm:

$$\|w(t)\| \le \Delta_w, \quad t \ge 0.$$

Following [2] we introduce the notion of *hyper-minimum-phase* (HMP) systems.

Definition 1. *For a given $g \in \mathbb{R}^l$ the transfer function $g^T W(s) = g^T C(sI - A)^{-1}B$ is called hyper-minimum-phase (HMP) if $g^T W(s)\det(sI - A)$ is a Hurwitz polynomial with a positive leading coefficient $g^T CB > 0$.*

Assumption 1. *There exists $g \in \mathbb{R}^l$ such that $\|g\| = 1$ and the transfer function $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP.*

The condition $\|g\| = 1$ is imposed only to simplify calculations and is not restrictive since if $g^T W(s)$ is HMP then $\|g\|^{-1} g^T W(s)$ is also HMP.

Remark 1. *The search of the vector g satisfying Assumption 1 in general is a difficult problem. It is equivalent to the search of a Hurwitz polynomial in an affine family of polynomials which is probably NP-hard (cannot be solved in a polynomial time, see [17]). One approach based on Monte-Carlo method can be found in [18].*

2.1. Passification lemma

Our results are based on the following lemma [4, 19].

Lemma 1 (Passification lemma). *The rational function $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP if and only if there exist a matrix P, a vector $\theta_* \in \mathbb{R}^l$, and a scalar $\varepsilon > 0$ such that*

$$P > 0, \quad P\bar{A} + \bar{A}^T P < -\varepsilon P, \quad PB = C^T g, \tag{2}$$

where $\bar{A} = A - B\theta_*^T C$.

Remark 2. *If $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP then there exists θ such that the input $u = -\theta^T y + v$ makes the system*

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned}$$

strictly passive with respect to a new input v, i.e. there exist functions $V(x) = x^T P x$, with $P > 0$, and $\varphi(x) \geq 0$, where $\varphi(x) > 0$ for $x \neq 0$, such that

$$V(x(t)) \leq V(x(0)) + \int_0^t \left[y^T(s) g v(s) - \varphi(x(s)) \right] ds.$$

Remark 3. *Passification lemma is also contained in [20] (implicitly) and in [21] (explicitly). This lemma provides conditions for existence of an output static feedback $u = -\theta^T y$ that renders the closed-loop system strictly positive real (SPR). If no such constant output feedback exists, then no dynamic output feedback with a proper transfer matrix exists to make the closed-loop system SPR [22]. More subtle results for the case of non-strict passivity can be found in [23].*

2.2. Quantizer model

Further we will assume that the controller receives quantized measurements. Following [8] we introduce a *quantizer with a quantization range M and a quantization error bound Δ_e* as a mapping $q\colon y \mapsto q(y)$ from $\mathbb{R}^l$ to a finite subset of $\mathbb{R}^l$ such that

$$\|y\| \le M \Rightarrow \|q(y) - y\| \le \Delta_e.$$

We will refer to the quantity $e = q(y) - y$ as the *quantization error*. The concrete codomain of q is not important for our further analysis, therefore, can be chosen arbitrary. The value of M is usually dictated by the effective range of a sensor.

By *dynamic quantizer* we will mean the mapping

$$q_\mu(y) = \mu q\left(\frac{y}{\mu}\right), \tag{3}$$

where $\mu > 0$. For each positive μ one obtains a quantizer with the quantization range μM and the quantization error bound $\mu\Delta_e$. We can think of μ as the "zoom" variable: increasing μ corresponds to zooming out and essentially obtaining a new quantizer with larger quantization range and quantization error bound, whereas decreasing μ corresponds to zooming in and obtaining a quantizer with a smaller quantization range but also a smaller quantization error bound. A useful example to keep in mind is a camera with optical zooming capability: one can zoom in and out while the number of photodiodes in the image sensor is fixed. Another example is the system with digital communication channel that can transmit a finite number of bytes. In this case one needs to encode all possible values of the output signal to transmit it through a communication channel. Obviously, in such case one can reduce the quantization error by reducing the range.

3. Ultimate boundedness

Together with the system (1) that satisfies Assumption 1 with some g we consider the adaptive controller

$$\begin{aligned} u(t) &= -\theta^T(t) q(y(t)), \\ \dot\theta(t) &= \gamma q(y(t)) q^T(y(t)) g - a\theta(t), \end{aligned} \tag{4}$$

where $\gamma > 0$ is a controller gain parameter and $a > 0$ is a regularizing parameter. Since $q(y(t))$ is piece-wise continuous we consider right-hand side derivative. As it has been previously shown [24] adaptive controllers similar to (4) without quantization ($q(y) = y$) can ensure ultimate boundedness of the system (1). Here we analyze this controller in the case of quantized measurements.

We will derive our results using the following Lyapunov function

$$V(x,\theta) = x^T P x + \gamma^{-1}\|\theta - \theta_*\|^2, \tag{5}$$

where P, θ_* satisfy (2). For convenience define the following quantities:

$$\Lambda_C = \|C\|, \quad \lambda_P = \lambda_{\min}(P), \quad \Lambda_P = \lambda_{\max}(P). \tag{6}$$

Remark 4. *Since chattering on the boundaries between the quantization regions is possible, solutions to differential equation* (1), (4) *are to be interpreted in the sense of Filippov. However, this issue will not play a significant role in the subsequent stability analysis. Indeed, all upper bounds on* $\dot{V}$ *that we will establish remain valid (almost everywhere) along Filippov's solutions (cf. [25]).*

First we prove the following lemma.

Lemma 2. *Under Assumption 1 consider the system* (1), (4) *with a quantization range* $M > 0$*. Denote*

$$\begin{aligned} \alpha &= \varepsilon - \nu - 2\sigma^{-1}\lambda_P^{-1}\Lambda_C^2, \\ a &= \alpha + \gamma(\sigma + \|\theta_*\|^{-1})\Delta_e^2, \\ \beta &= \nu^{-1}\Lambda_P\Delta_w^2 + a\gamma^{-1}\|\theta_*\|^2 + (\sigma\|\theta_*\|^2 + \|\theta_*\|)\Delta_e^2, \end{aligned} \tag{7}$$

where ε *is from* (2) *and* $\nu > 0$, $\sigma > 0$ *are such that* $\alpha > 0$*. If* Δ_e *and* Δ_w *are such that*

$$\frac{\beta}{\alpha} < \frac{M^2\lambda_P}{\Lambda_C^2} \tag{8}$$

and

$$V(x(t_*), \theta(t_*)) < \frac{M^2\lambda_P}{\Lambda_C^2} \tag{9}$$

then for $t \geq t_*$

$$V(x(t), \theta(t)) \leq \left(V(x(t_*), \theta(t_*)) - \frac{\beta}{\alpha} \right) e^{-\alpha(t-t_*)} + \frac{\beta}{\alpha}, \tag{10}$$

where $t_* \geq 0$ *is arbitrary time instant.*

Proof. See Appendix A.

The following remark will be useful later.

Remark 5. *One can easily see that*

$$\frac{\beta}{\alpha} = c_\gamma + c_w\Delta_w^2 + c_e\Delta_e^2,$$

where

$$\begin{aligned} c_\gamma &= \gamma^{-1}\|\theta_*\|^2, \\ c_w &= \alpha^{-1}\nu^{-1}\Lambda_P, \\ c_e &= 2\alpha^{-1}(\|\theta_*\| + \|\theta_*\|^2\sigma). \end{aligned} \tag{11}$$

Remark 6. *Lemma 2 asserts that the state of the system* (1), (4) *converges from the ellipsoid* $V(x,\theta) < M^2\lambda_P\Lambda_C^{-2}$ *to a smaller ellipsoid* $V(x,\theta) \leq c_\gamma + c_w\Delta_w^2 + c_e\Delta_e^2$*. The size of the initial ellipsoid is such that* $y(t_*)$ *is in the quantization range. The condition* (8) *guarantees that the values* Δ_w, Δ_e *are small enough so that the limit ellipsoid is smaller than the initial one and, therefore,* $y(t)$ *is in the quantization range for* $t \geq t_*$.

The next theorem follows directly from Lemma 2, Remark 5, and the fact that c_γ can be made arbitrary small by increasing the controller gain parameter γ.

Theorem 1. *Consider the system* (1), (4) *under Assumption 1 with a quantization range M and a controller parameter a given by* (7)*. If Δ_e and Δ_w are such that*

$$c_w \Delta_w^2 + c_e \Delta_e^2 < \frac{M^2 \lambda_P}{\Lambda_C^2},$$

where c_w, c_e are given by (11) *with positive ν, σ such that $\alpha > 0$, then for $\gamma > 0$ such that $c_\gamma + c_w \Delta_w^2 + c_e \Delta_e^2 < M^2 \lambda_P \Lambda_C^{-2}$, the trajectories of the system are ultimately bounded for any initial conditions satisfying*

$$\Lambda_P \|x(0)\|^2 + \gamma^{-1} \|\theta(0) - \theta_*\|^2 < \frac{M^2 \lambda_P}{\Lambda_C^2}.$$

Corollary 1. *The system* (1), (4) *under Assumption 1 is ultimately bounded for any controller parameters $\gamma > 0$ and $a > 0$ if the quantization error bound $\Delta_e > 0$ and $\|x(0)\|$ are sufficiently small.*

4. Switching control

Under conditions of Lemma 2 the state of the system (1), (4) converges from the ellipsoid (9) to a smaller ellipsoid $V(x, \theta) \leq c_\gamma + c_w \Delta_w^2 + c_e \Delta_e^2$. Consequently, the output converges to a smaller set and if the controller "zooms in" onto this smaller set it will reduce the maximum quantization error Δ_e. This, in turn, will decrease the value $c_\gamma + c_w \Delta_w^2 + c_e \Delta_e^2$ and ensure convergence to an even smaller set. By repeating this zooming procedure one will obtain a sequence of converging ellipsoids. Below we give a mathematical description of this idea.

Consider the following controller

$$\begin{aligned} u(t) &= -\theta^T(t) q_{\mu(t)}(y(t)), \\ \dot{\theta}(t) &= \gamma q_{\mu(t)}(y(t)) q_{\mu(t)}^T(y(t)) g - a(t)\theta(t), \end{aligned} \tag{12}$$

where $q_{\mu(t)}$ is a dynamic quantizer, $\mu(t)$, $a(t)$ are piecewise constant (switching) parameters to be determined later.

Suppose there is a known V_0 such that

$$V(x(0), \theta(0)) < V_0.$$

Let us choose a zooming parameter $\mu_0 > 0$ such that

$$V_0 \leq \frac{\mu_0^2 M^2 \lambda_P}{\Lambda_C^2}.$$

This will ensure that $\|y(0)\| < \mu_0 M$, that is $y(0)$ is in the quantization range. Assume that Δ_w and Δ_e are such that $c_w \Delta_w^2 + c_e \mu_0^2 \Delta_e^2 < V_0$. From (11) one can see that c_γ can be made arbitrary small by choosing a large enough controller gain parameter $\gamma > 0$. Let us fix some $\gamma > 0$, $\epsilon > 0$ such that

$$c_\gamma + c_w \Delta_w^2 + c_e \mu_0^2 \Delta_e^2 + \epsilon < V_0.$$

Following (7) we choose

$$a_0 = \alpha + \gamma \mu_0^2 \Delta_e^2 (\sigma + \|\theta_*\|^{-1}).$$

Let us require the quantizer to change its zoom when $V(x(t), \theta(t)) < V_1 = c_\gamma + c_w \Delta_w^2 + c_e \mu_0^2 \Delta_e^2 + \epsilon$. Then (10) suggests that the first switching instance should have the form

$$t_1 = t_0 + \frac{1}{\alpha} \ln \frac{V_0 - c_\gamma - c_w \Delta_w^2 - c_e \mu_0^2 \Delta_e^2}{\epsilon},$$

where $t_0 = 0$ and α is defined in (7). Inequality $V(x(t_1), \theta(t_1)) < V_1$ implies

$$\|y(t_1)\| < \Lambda_C \sqrt{V_1 \lambda_P^{-1}} = \mu_1 M,$$

where $\mu_1 = \mu_0 \sqrt{V_1 V_0^{-1}}$. Then one should recalculate the regularizing parameter

$$a_1 = \alpha + \gamma \mu_1^2 \Delta_e^2 (\sigma + \|\theta_*\|^{-1}).$$

Since the maximum quantization error $\mu_0 \Delta_e$ has changed to a smaller quantity $\mu_1 \Delta_e$, the limit value for $V(x(t), \theta(t))$ is now given by

$$c_\gamma + c_w \Delta_w^2 + c_e \mu_1^2 \Delta_e^2.$$

By repeating the procedure described above one obtains the following sequence of parameters for $i = 1, 2, \ldots$

$$\begin{aligned} V_i &= c_\gamma + c_w \Delta_w^2 + c_e \mu_{i-1}^2 \Delta_e^2 + \epsilon, \\ \mu_i &= \mu_0 \sqrt{V_i V_0^{-1}}, \\ a_i &= \alpha + \gamma \mu_i^2 \Delta_e^2 (\sigma + \|\theta_*\|^{-1}), \\ t_i &= t_{i-1} + \frac{1}{\alpha} \ln \frac{V_{i-1} - c_\gamma - c_w \Delta_w^2 - c_e \mu_{i-1}^2 \Delta_e^2}{\epsilon}. \end{aligned} \tag{13}$$

Note that the parameters of switching are predefined. To switch the zooming variable μ one needs to guarantee that the output y doesn't leave some compact set. This can be done in terms of the state $x(t)$ using Lyapunov function (5). Since $x(t)$ is not known, the value of V cannot be calculated. Therefore, we use known upper bounds V_i for V on $[t_i, t_{i+1})$ that can be calculated "a priori". The next lemma gives the limit value for V_i.

Lemma 3. *For any positive scalars c_γ, c_w, c_e, Δ_w, Δ_e, ϵ, V_0, μ_0 if*

$$c_\gamma + c_w \Delta_w^2 + c_e \mu_0^2 \Delta_e^2 + \epsilon < V_0$$

then the sequence

$$V_{i+1} = c_\gamma + c_w \Delta_w^2 + c_e \frac{V_i}{V_0} \mu_0^2 \Delta_e^2 + \epsilon$$

monotonically decreases to the value

$$V_\infty = \frac{c_\gamma + c_w \Delta_w^2 + \epsilon}{1 - c_e \mu_0^2 \Delta_e^2 V_0^{-1}}.$$

Proof. See Appendix B.

Now we minimize the quantity V_∞ by choosing appropriate σ, ν. The values c_γ and ϵ can be chosen arbitrary small. By minimizing the quantity $c_w/(1 - c_e \mu_0^2 \Delta_e^2 V_0^{-1})$ with respect to σ, ν one finds that

$$\begin{aligned} \sigma &= \frac{\Lambda_C}{\mu_0 \Delta_e \|\theta_*\|} \sqrt{V_0 \lambda_P^{-1}}, \\ \nu &= \frac{\varepsilon}{2} - \|\theta_*\| \mu_0^2 \Delta_e^2 V_0^{-1} - 2 \frac{\mu_0 \Delta_e \|\theta_*\| \Lambda_C}{\sqrt{\lambda_P V_0}}. \end{aligned} \tag{14}$$

Then

$$V_\infty = \frac{c_\gamma + \epsilon}{1 - c_e \mu_0^2 \Delta_e^2 V_0^{-1}} + \frac{\Lambda_P \Delta_w^2}{\nu^2}. \tag{15}$$

Remark 7. *By substituting σ, ν given by* (14) *into* (7) *we obtain*

$$\alpha = \frac{\varepsilon}{2} + \|\theta_*\| \mu_0^2 \Delta_e^2 V_0^{-1} > 0.$$

Relation $c_e \mu_0^2 \Delta_e < V_0$ is equivalent to $(\|\theta_*\| + \|\theta_*\|^2 \sigma) \mu_0^2 \Delta_e^2 V_0^{-1} < \alpha/2$, therefore,

$$\nu = \frac{\varepsilon}{2} - \sigma^{-1} \lambda_P^{-1} \Lambda_C^2 - (\|\theta_*\| + \|\theta_*\|^2 \sigma) \mu_0^2 \Delta_e^2 V_0^{-1} > \frac{\varepsilon}{2} - \sigma^{-1} \lambda_P^{-1} \Lambda_C^2 - \frac{\alpha}{2} = \frac{\nu}{2}.$$

That is ν given in (14) *is positive.*

Remark 8. *In [1] for a linear system without disturbances it has been shown that adaptive controller* (12) *can ensure convergence of V given by* (5) *to any vicinity of the origin. The quantity $\Lambda_P \Delta_w^2 \nu^{-2}$ that appears in* (15) *is the one that cannot be improved due to unknown disturbance inherent in the system.*

One could note that according to (13) there may exist such finite t_∞ that $t_i \to t_\infty$. That is the controller should be able to switch infinitely often. To avoid this issue we choose some value $\zeta > 0$ and stop switching when $V_i < V_\infty + \zeta$.

The next theorem summarizes the aforementioned ideas.

Theorem 2. *Under Assumption 1 consider the system* (1), (12) *with quantizer range M. If Δ_e, Δ_w are such that*

$$c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 < V_0, \tag{16}$$

where c_w, c_e are given by (11) *with σ, ν given by* (14) *and α given by* (7)*, then for any δ there exists a positive integer l such that adaptive controller* (12) *with positive γ and ϵ satisfying*

$$\frac{c_\gamma + \epsilon}{1 - c_e\mu_0^2\Delta_e^2V_0^{-1}} < \delta\lambda_P, \quad c_\gamma + c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 + \epsilon < V_0$$

and switching parameters

$$a(t) = \begin{cases} a_i, & t \in [t_i, t_{i+1}), \quad 0 \le i < l, \\ a_l, & t \ge t_l, \end{cases}$$
$$\mu(t) = \begin{cases} \mu_i, & t \in [t_i, t_{i+1}), \quad 0 \le i < l, \\ \mu_l, & t \ge t_l, \end{cases}$$

where a_i, μ_i, t_i are given in (13)*, ensures that*

$$\|x(t)\|^2 < \frac{\Lambda_P\Delta_w^2}{\lambda_P\nu^2} + \delta, \quad t \ge t_l \tag{17}$$

for initial conditions that satisfy

$$\Lambda_P\|x(0)\|^2 + \gamma^{-1}\|\theta(0) - \theta_*\|^2 < V_0 \le \frac{\mu_0^2M^2\lambda_P}{\Lambda_C^2}. \tag{18}$$

Moreover, $\|\theta(t)\|$ is a bounded function.

Proof. See Appendix C.

Remark 9. *To obtain convergence conditions for the system* (1), (12) *without quantization one can use Theorem 2 with $\Delta_e \to 0$, $M \to \infty$. Then* (16), (18) *are always true, switching procedure* (13) *vanishes and* (17) *in view of* (14) *transforms to*

$$\|x(t)\|^2 < \frac{4\Lambda_P}{\varepsilon^2\lambda_P}\Delta_w^2 + \delta. \tag{19}$$

This estimate coincides with [26, Theorem 2.13].

Remark 10. *The value of ε from* (2) *is the stability level that can be achieved by using the control law $u(t) = -\theta_*y(t)$. Larger ε leads to smaller c_e and, therefore,* (16) *is satisfied with a larger maximum quantization error Δ_e.*

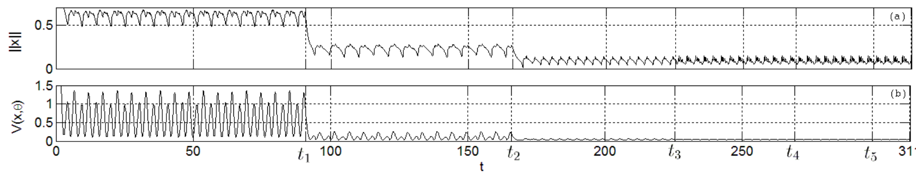

Figure 1: (a): norm of the state; (b): Lyapunov function (5).

Remark 11. *Our results are applicable to the system* (1) *with uncertain* A *that resides in the polytope*

$$A = A_\xi = \sum_{i=1}^{N} \xi_i A_i, \quad 0 \le \xi_i, \quad \sum_{i=1}^{N} \xi_i = 1. \tag{20}$$

If $g^T W_\xi(s) = g^T C(sI - A_\xi)^{-1} B$ *is HMP for all* ξ *from* (20)*, then* (2) *are feasible for each* ξ *with some* θ_ξ *and* P_ξ*. To apply the results of this paper one should take*

$$\begin{aligned} \varepsilon &= \min_{\xi \in \Xi} \varepsilon_\xi, \quad \theta_* = \underset{\theta_\xi, \xi \in \Xi}{\operatorname{argmax}} \|\theta_\xi\|, \\ \lambda_P &= \min_{\xi \in \Xi} \lambda_{\min}(P_\xi), \quad \Lambda_P = \max_{\xi \in \Xi} \lambda_{\max}(P_\xi). \end{aligned} \tag{21}$$

The existence of these quantities follows from Lemma 1, compactness of a set of ξ*, and continuity of the matrix* A_ξ *in* ξ*.*

The relations (2) *are feasible for* $\theta_\xi = k_* g$ *with large enough* k_* *[2]. Since* (2) *are affine in* A_ξ*, to obtain the values from* (21) *one can solve linear matrix inequalities*

$$P > 0, \;\; P(A_i - Bk_* g^T C) + (A_i - Bk_* g^T C)^T P < -\varepsilon P, \;\; PB = C^T g, \;\; i = 1, \dots, N,$$

with a decision variable P *and tuning parameters* ε*,* k_**. To find appropriate tuning parameters one should first set* $\varepsilon = 0$ *and find the minimum* k_* *such that LMIs are feasible. Then by increasing* k_* *one will obtain larger allowable values for* ε*.*

5. Example: yaw angle control

We demonstrate applicability of our results by an example of a yaw angle control. Under several simplifying assumptions [27] dynamics of the lateral motion of an aircraft can be described by (1) with

$$A = \begin{bmatrix} a_{11} & 1 & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Table 1: Parameters of switching: t_i — instants of switching, V_i — upper bound for $V(x(t), k(t))$ on $[t_i, t_{i+1})$, μ_i — zooming parameter, a_i — regularizing parameter.

i	t_i	V_i	μ_i	a_i
0	0	1000	1	311.67
1	90.85	144.66	0.38	45.19
2	165.98	24.89	0.158	7.88
3	225.38	8.12	0.09	2.66
4	269.08	5.78	0.076	1.92
5	297.27	5.45	0.074	1.82

where x_1 is a sideslip angle, x_3 and x_2 are the yaw angle and its rate, respectively, $u(t)$ is the rudder angle. Following [27] we take $a_{22} = 1.3$, $b_1 = 19/15$, $b_2 = 19$ and suppose that a_{11}, a_{21} are uncertain parameters:

$$a_{11} \in [0.1, 1.5], \quad a_{21} \in [25, 40]. \tag{22}$$

For $g = \frac{\sqrt{2}}{2}(1,1)^T$ the transfer function

$$g^T W(s) = \frac{b_2 s^2 + (b_1 a_{21} - b_2 a_{11} + b_2)s + b_1 a_{21} - b_2 a_{11}}{s\sqrt{2}(s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{21})}$$

is HMP for all a_{11}, a_{21} from (22). Using Remark 11 we find that (2) are satisfied with $\varepsilon = 0.25$, $\theta_* = 5.3g$,

$$P \approx \begin{bmatrix} 2.3 & -0.15 & -2 \\ -0.15 & 0.05 & 0.17 \\ -2 & 0.17 & 5.15 \end{bmatrix},$$

where P is given up to hundredth. We take

$$V_0 = 10^3, \quad \mu_0 = 1, \quad \Delta_w = 0.1, \quad \Delta_e = 0.01.$$

For these parameters (16) is satisfied and, therefore, Theorem 2 can be applied. For $\delta = 2$ it is sufficient to take $\gamma = 10^3$ and $\epsilon = 10^{-2}$.

The results of numerical simulations for $a_{11} = 0.75$, $a_{21} = 33$ are presented in Fig. 1. Initial conditions were chosen randomly such that $\theta(0) = (0,0)^T$, $V(x(0), \theta(0)) \le V_0$. The values of all switching parameters are presented in Table 1. The switching procedure stops after 5 switches. As one can see μ_i is decreasing, this corresponds to “zooming in”.

6. Conclusions

We considered hyper-minimum-phase uncertain linear system with bounded disturbance. First we proved that if the disturbance and quantization error bounds are small enough the standard passification-based adaptive controller ensures ultimate boundedness of the closed-loop system. Then we showed that by using a dynamic quantizer with switching "zoom" variable one can ensure convergence to a smaller ellipsoid. The size of this ellipsoid is defined by the disturbance bound. Finally, we demonstrated applicability of the proposed controller to polytopic-type uncertain systems and its efficiency by the example of a yaw angle control of a flying vehicle.

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Appendix A. Proof of Lemma 2

Under Assumption 1 it follows from Lemma 1 that relations (2) are valid for some matrix P and vector θ_*, therefore, Lyapunov function (5) can be constructed. Its derivative along the trajectories of the system (1), (4) has the form

$$\begin{aligned}\dot{V} &= 2x^T P[Ax - B\theta^T q(y)] + 2x^T Pw \\ &+ 2(\theta-\theta_*)^T q(y) q^T(y) g - 2a\gamma^{-1}(\theta-\theta_*)^T\theta \\ &= 2x^T P[Ax - B\theta_*^T Cx] + 2q^T(y) g(\theta_*-\theta)^T q(y) \\ &- 2e^T(t) g(\theta_*-\theta)^T q(y) - 2y^T g\theta_*^T e + 2x^T Pw \\ &+ 2(\theta-\theta_*)^T q(y) q^T(y) g - 2a\gamma^{-1}(\theta-\theta_*)^T\theta.\end{aligned}$$

Here we used the relation $PB = C^T g$ from (2) and notation $e = q(y) - y$. Condition (9) implies $\|y(t_*)\| < M$. Since $y(t)$ is continuous in t, $\|y(t)\| < M$ on $[t_*, T)$ for some $T > t_*$. Thus $\|e(t)\| \le \Delta_e$ for $t \in [t_*, T)$. Since $\|g\| = 1$ and $2a^T b \le a^T Qa + b^T Q^{-1} b$ for any vectors a, b and a matrix $Q > 0$, for $t \in [t_*, T)$ we obtain

$$\begin{aligned}&- 2e^T(y) g(\theta_*-\theta)^T q(y) \le 2\Delta_e |(\theta_*-\theta)^T q(y)| \\ &\le 2\Delta_e |(\theta_*-\theta)^T y| + 2\Delta_e |(\theta_*-\theta)^T e| \\ &\le (\sigma + \|\theta_*\|^{-1})\Delta_e^2 \|\theta_*-\theta\|^2 + \sigma^{-1}\|y\|^2 + \|\theta_*\|\Delta_e^2,\end{aligned}$$

$$-2y^T g\theta_*^T e \le \sigma^{-1} x^T C^T g g^T Cx + \sigma\Delta_e^2 \|\theta_*\|^2,$$

$$
\begin{aligned}
2x^T P w &\le \nu x^T P x + \nu^{-1}\Lambda_P \Delta_w^2,\\
-2a\gamma^{-1}(\theta-\theta_*)^T\theta &= -2a\gamma^{-1}\|\theta-\theta_*\|^2 - 2a\gamma^{-1}(\theta-\theta_*)^T\theta_*\\
&\le -a\gamma^{-1}\|\theta-\theta_*\|^2 + a\gamma^{-1}\|\theta_*\|^2.
\end{aligned}
$$

Then

$$
\begin{aligned}
\dot V + \alpha V - \beta \le\; & -(\varepsilon - \nu - 2\sigma^{-1}\lambda_P^{-1}\Lambda_C^2 - \alpha)x^T P x\\
& - (a - \gamma\sigma\Delta_e^2 - \gamma\|\theta_*\|^{-1}\Delta_e^2 - \alpha)\gamma^{-1}\|\theta_* - \theta\|^2\\
& + \nu^{-1}\Lambda_P\Delta_w^2 + a\gamma^{-1}\|\theta_*\|^2 + \sigma\Delta_e^2\|\theta_*\|^2 + \|\theta_*\|\Delta_e^2 - \beta.
\end{aligned}
$$

By substituting values from (7) we find that $\dot V \le -\alpha V + \beta$. It follows from the comparison principle [28] that for $t \in [t_*, T)$

$$
V(x(t),\theta(t)) \le \left(V(x(t_*),\theta(t_*)) - \frac{\beta}{\alpha}\right) e^{-\alpha(t-t_*)} + \frac{\beta}{\alpha}.
$$

The latter together with (8), (9) implies $T = \infty$.

Appendix B. Proof of Lemma 3

For $i = 0$ we have

$$
V_1 = c_\gamma + c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 + \epsilon < V_0.
$$

Suppose that $i > 0$ and for $j < i$ it has been proved that $V_j < V_{j-1}$. Then

$$
\begin{aligned}
V_i &= c_\gamma + c_w\Delta_w^2 + c_e\frac{V_{i-1}}{V_{i-2}}\frac{V_{i-2}}{V_0}\mu_0^2\Delta_e^2 + \epsilon\\
&< c_\gamma + c_w\Delta_w^2 + c_e\frac{V_{i-2}}{V_0}\mu_0^2\Delta_e^2 + \epsilon = V_{i-1}.
\end{aligned}
$$

Therefore V_i is a monotonically decreasing sequence of positive numbers, and, therefore, it has a limit value, which is a solution of the equation

$$
V = c_\gamma + c_w\Delta_w^2 + c_e\frac{V}{V_0}\mu_0^2\Delta_e^2 + \epsilon,
$$

i.e. $V = V_\infty$.

Appendix C. Proof of Theorem 2

Let us choose $\zeta > 0$ such that

$$
\frac{c_\gamma + \epsilon}{1 - c_e\mu_0^2\Delta_e^2 V_0^{-1}} + \zeta \le \delta\lambda_P.
$$

Under conditions of Theorem 2, Lemma 2 implies (10) for $t \in [t_0, t_1]$, $t_* = t_0$, therefore,

$$
V(x(t),\theta(t)) < V_0, \quad \forall t \in [t_0, t_1].
$$

Consider $t \in [t_i, t_{i+1}]$ and assume that for $j < i$ it has been proved that

$$V(x(t), \theta(t)) < V_j, \quad \forall t \in [t_j, t_{j+1}].$$

By applying Lemma 2 on $[t_{i-1}, t_i]$ with $t_* = t_{i-1}$ and substituting $t = t_i$ into (10) one arrives at

$$V(x(t_i), \theta(t_i)) < c_\gamma + c_w \Delta_w^2 + c_e \mu_{i-1}^2 \Delta_e^2 + \epsilon = V_i.$$

Moreover,

$$V_i = \mu_i^2 V_0 = \frac{\mu_i^2 M^2 \lambda_P}{\Lambda_C^2} = \frac{M_i^2 \lambda_P}{\Lambda_C^2},$$

where $M_i = \mu_i M$. Thus, (9) is satisfied with $M = M_i$, $t_* = t_i$. Relation (13) implies

$$c_\gamma + c_w \Delta_w^2 + c_e \mu_i^2 \Delta_e^2 < V_i = \frac{M_i^2 \lambda_P}{\Lambda_C^2}.$$

That is (8) is true with $\beta = \nu^{-1} \Lambda_P \Delta_w^2 + a_i \gamma^{-1} \|\theta_*\|^2 + (\sigma \|\theta_*\|^2 + \|\theta_*\|) \mu_i^2 \Delta_e^2$, $M = M_i$, $t_* = t_i$. Therefore, Lemma 2 can be applied on $[t_i, t_{i+1}]$. By induction we conclude that

$$V(t) < V_i, \quad \forall t \in [t_i, t_{i+1}).$$

Since $V_i \to V_\infty$ there exists l such that

$$V_l \leq V_\infty + \zeta \leq \frac{\Lambda_P \Delta_w^2}{\nu^2} + \delta \lambda_P.$$

Thus, if switching stops after t_l, one obtains that for $t \geq t_l$

$$V(x(t), \theta(t)) < \frac{\Lambda_P \Delta_w^2}{\nu^2} + \delta \lambda_P,$$

therefore, for $t \geq t_l$

$$\|x(t)\|^2 < \frac{\Lambda_P \Delta_w^2}{\lambda_P \nu^2} + \delta.$$

Function $\|\theta(t)\|$ is bounded since $V(x(t), \theta(t))$ is bounded.