Information Transmission by Adaptive Synchronization with Chaotic Carrier and Noisy Channel [1]

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Abstract

The problem of signal transmission by means of a pair of synchronized chaotic systems (transmitter and receiver) is considered. The approach of adaptive observations is used for design of the receiver. Two kinds of adaptive algorithms are investigated in presence of additive noise in the channel. The results are illustrated by a number of numerical examples of signal transmission based on the pairs of synchronizing Chua, Duffing and Metzler systems.

1 Introduction

In recent years a growing interest was observed in the problem of synchronizing chaotic systems [1, 2, 14]. It was motivated not only by scientific interest in the problem, but also by practical applications in different fields particularly in telecommunications [2, 3, 4, 10]. Of practical interest is the problem of synchronizing two or more systems when the designer of the receiver does not know not only initial state but also some parameters. This more complicated problem is referred to as *adaptive synchronization* [7, 8, 9, 13, 16]. Its solution may be used in communications in the case when parameter modulation is used for message transmission, see, e.g. [9] where the solution based on adaptive observers in an idealized setting with neglected noise was proposed.

In this paper the scheme related to that of [9] is investigated in the case of noisy channel. Two kinds of adaptive observers are considered. Numerical examples for carrier signals generated by different chaotic systems (namely, Chua, Duffing and Metzler equations) are used given.

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2 Adaptive synchronization algorithms

Consider the transmitter described by state space equations in Lur'e form:

$$\dot{x}_d = Ax_d + \varphi_0(y_d) + B\sum_{i=1}^{m}\theta_i\varphi_i(y_d), \quad y_d = Cx_d \tag{1}$$

where $x_d \in \mathbb{R}^n$ is the transmitter state vector, $y_d \in \mathbb{R}^l$ is the vector of outputs (transmitted signals), $\theta = \mathrm{col}\,(\theta_1, \ldots, \theta_m)$ is the vector of transmitter parameters (possibly representing a message). It is assumed that the nonlinearities $\varphi_i(\cdot)$, $i = 0, 1, \ldots, m$, matrices A, C and vector B are known.

The receiver is designed as another dynamical system that provides estimates $\hat{\theta}_i$, $i = 1, \ldots, m$ of the transmitter parameters based on the (noisy) observations of the transmitted signal $y_d(t)$. The problem is to design receiver equations

$$\dot{z} = F(z, y_r), \tag{2}$$

$$\hat{\theta} = h(z, y_r) \tag{3}$$

ensuring convergence

$$\overline{\lim_{t\to\infty}} \left\|\hat{\theta}(t) - \theta\right\| \le \Delta, \tag{4}$$

where $y_r(t) = y_d(t) + \xi(t)$ is the received signal, $\xi(t)$ is channel noise, $\hat{\theta}(t) = \mathrm{col}\left(\hat{\theta}_1(t), \ldots, \hat{\theta}_m(t)\right)$ is the vector of parameter estimates, $\Delta \ge 0$ is given accuracy bound.

One of receivers considered below was proposed in [9]. This receiver is a kind of adaptive observer. Its simplest version for the case when A, B, C are known is as follows:

$$\dot{x} = Ax + \varphi_0(y_r) + B\left(\sum_{i=1}^{m}\hat{\theta}_i\varphi_i(y_r) + \hat{\theta}_0 G(y_r - y)\right), \quad y = Cx, \tag{5}$$

$$\dot{\hat{\theta}}_i = \psi_i(y_r, y), \quad i = 0, 1, 2, \ldots, m, \tag{6}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^l$, $\theta_0 \in \mathbb{R}$ and $G \in \mathbb{R}^l$ is the vector of weights. The adaptation algorithm (6) is provided by standard gradient/speed-gradient techniques as follows:

$$\dot{\hat{\theta}}_i = -\gamma_i (y - y_r)\varphi_i(y_r), \quad i = 1, \dots, m, \tag{7}$$

$$\dot{\hat{\theta}}_0 = -\gamma_0 (y - y_r)^2, \tag{8}$$

where γ_i, $(i = 0, 1, \dots m)$ are positive adaptation gains. In presence of noise in the channel algorithm (7), (8) has to be regularized (robustified) [5, 6, 15]. One way of regularization is introducing the *parametric feedback*. It provides the following form of adaptive algorithm

$$\dot{\hat{\theta}}_i = -\gamma_i (y - y_r)\varphi_i(y_r) - \alpha_i \hat{\theta}_i, \quad i = 1, \dots, m, \tag{9}$$

$$\dot{\hat{\theta}}_0 = -\gamma_0 (y - y_r)^2 - \alpha_0 \hat{\theta}_0, \tag{10}$$

where $\alpha_i > 0$ $(i = 0, 1, \dots m)$ are regularization gains.

To formulate properties of the proposed algorithms introduce two definitions.

Definition 1 ([6]). The system $\dot{x} = \bar{A}x + \bar{B}u$, $y = \bar{C}x$ with transfer matrix $W(\lambda) = \bar{C}(\lambda I - \bar{A})^{-1}\bar{B}$, where $u,\ y \in \mathbb{R}^l$ and $\lambda \in \mathcal{C}$ is called *hyper-minimum-phase* if it is *minimum-phase* (i.e. the polynomial $\varphi(\lambda) = \det(\lambda I - \bar{A}) \det W(\lambda)$ is Hurwitz), and the matrix $\bar{C}\bar{B} = \lim_{\lambda \to \infty} \lambda W(\lambda)$ is symmetric and positive definite.

Remark: For $l = 1$ the system of order n is hyper-minimum-phase if the numerator of its transfer function is a Hurwitz polynomial of degree $n - 1$ with positive coefficients.

Definition 2.A vector-function $f : [0, \infty) \to \mathbb{R}^m$ is called *persistently exciting (PE) on* $[0, \infty)$, if it is measurable and bounded on $[0, \infty)$ and there exist $\alpha > 0, T > 0$ such that

$$\int_t^{t+T} f(s) f(s)^T \, ds \geq \alpha I \tag{11}$$

for all $t \geq 0$.

Properties of the algorithm (7), (8) are described in the following Theorem proved in [9].

Theorem 1. Let $\xi \equiv 0$ and all the trajectories of the transmitter (1) be bounded and linear system with the transfer function $W(\lambda) = GC(\lambda I - A)^{-1}B$ be hyper-minimum-phase. Then all the trajectories of the receiver (5), (7), (8) are bounded and the auxiliary goal

$$\overline{\lim_{t \to \infty}} \|x(t) - x_d(t)\| \leq \Delta_x \tag{12}$$

holds for $\Delta_x = 0$. If, in addition, the vector-function $\mathrm{col}\,(\varphi_1(y_r), \dots, \varphi_m(y_r))$ satisfies the PE condition, then also (4) holds for $\Delta = 0$.

The following Theorem describes properties of the receiver with regularized adaptation algorithm (9), (10).

Theorem 2. Let the noise function $\xi(t)$ be bounded: $|\xi(t)| \leq \Delta_\xi$; all the trajectories of the transmitter 1 be bounded and linear system with transfer function $W(\lambda) = GC(\lambda I - A)^{-1}B$ be hyper-minimum-phase. Then all the trajectories of the system (1), (5), (9), (10) are bounded and the goals (4), (12) hold for some $\Delta > 0$, $\Delta_x > 0$. If, in addition, $\Delta_\xi > 0$ is sufficiently small, and gains $\alpha_i > 0$, $i = 0, 1, \dots, m$ are chosen sufficiently small, then the values Δ in (4) and Δ_x in (12) can be chosen arbitrary small.

Theorem 2 follows from the results of [9] and [5].

Another variant of the adaptive observer based on the well known Lion's method [12], consists of augmented signals implementation. These signals are generated by the row of filters, therefore the parameter estimates can be based on output measurements only.

Let us describe in detail. The transfer function $W(\lambda)$ of the linear part in Lur'e form (1) can be written as

$$W(\lambda) = \frac{b_0 \lambda^k + b_1 \lambda^{k-1} + \dots + b_k}{\lambda^n + a_1 \lambda^{n-1} + \dots + a_n},$$

so that for output and input signals of the linear part is valid

$$y_r^{(n)} + a_1 y_r^{(n-1)} + \dots + a_n y_r = b_0 u^{(k)} + \dots + b_{k-1} u, \tag{13}$$

where $u(t)$ stands for the output of nonlinear part (1), i.e. $u(t) \triangleq \varphi_0(y_r) + B \sum_{i=1}^{m} \theta_i \varphi(y_r)$. Equation (13) leads to the similar interrelation with respect to outputs $\tilde{y}_r(t)$, $\tilde{u}(t)$ of the identical filters, actuated by the signals $y_r(t), u(t)$:

$$\tilde{y}_r^{(n)} + a_1 \tilde{y}_r^{(n-1)} + \dots + a_n \tilde{y}_r = b_0 \tilde{u}^{(k)} + \dots + b_{k-1} \tilde{u}, \tag{14}$$

In contrast to (13), all derivatives in (14) can be measured without differentiation of input/output signals. In the case of unknown parameters a_i, b_j in (13), the following *implicit adjustable model* can be written

$$\begin{aligned} &\tilde{y}_r^{(n)} + \hat{a}_1(t) \tilde{y}_r^{(n-1)} + \cdots \\ &+ \hat{a}_n(t) \tilde{y}_r = \hat{b}_0(t) \tilde{u}^{(k)} + \dots + \hat{b}_{k-1}(t) \tilde{u}, \end{aligned} \tag{15}$$

where the *adjustable parameters* $\hat{a}_i(t), \hat{b}_j(t)$ stand for the estimates of *a priori* unknown parameters a_i, b_j of (13). Signal

$$\begin{aligned} \delta(t) &\triangleq \tilde{y}_r^{(n)} + \hat{a}_1(t) \tilde{y}_r^{(n-1)} + \cdots \\ &+ \hat{a}_n(t) \tilde{y}_r - \hat{b}_0(t) \tilde{u}^{(k)} - \dots - \hat{b}_{k-1}(t) \tilde{u} \end{aligned} \tag{16}$$

can be referred as an *identification error* and its magnitude has to be minimizied by the identification algorithm. Introducing the regressor ϕ and the vector of adjustable parameters $\hat{\theta}$ as

$$\phi \triangleq [\tilde{y}_r^{(n-1)}, \dots, \tilde{y}_r, -\tilde{u}^{(k)}, - \dots, -\tilde{u}]^T,$$

$$\hat{\theta}(t) \triangleq [\hat{a}_1(t), \ldots, \hat{a}_n(t), \hat{b}_0(t), \ldots, \hat{b}_{k-1}(t)]^T,$$

one gets (16) in the form

$$\delta(t) = \tilde{y}_r^{(n)} + \phi(t)^T \hat{\theta}(t). \qquad (17)$$

Appliyng the speed-gradient techniques and the *matrix square-root algorithm*, one finally obtains the identification law as follows:

$$\dot{\Gamma} = -\gamma \Gamma \phi \phi^T \Gamma + \alpha \Gamma, \qquad (18)$$

$$\dot{\theta} = -\gamma \Gamma \phi \delta, \qquad (19)$$

where $\Gamma = \Gamma(t)$ is square *gain matrix*, α, γ are algorithm parameters. The particular form of algorithm (16) – (19) is given in the next section.

As an example we consider the problem of synchronizing pairs of Chua, Metzler and Duffing systems with unknown parameters and incomplete measurements.

3 Examples: communication using Chua, Metzler and Duffing systems

Consider an example of information transmission where both transmitter and receiver system are implemented as a Chua's circuit, similarly to [3]. The transmitter model in dimensionless form is given as:

$$\begin{array}{rcl} \dot{x}_{d_1} & = & p(x_{d_2} - x_{d_1} + f(x_{d_1}) + s f_1(x_{d_1})) \\ \dot{x}_{d_2} & = & x_{d_1} - x_{d_2} + x_{d3} \\ \dot{x}_{d_3} & = & -q x_{d_2} \end{array} \qquad (20)$$

where $f(z) = M_0 z + 0.5(M_1 - M_0) f_1(z)$, $f_1(z) = |z+1| - |z-1|$, M_0, M_1, p, q are the transmitter parameters, $s = s(t)$ is the signal to be reconstructed in the receiver. Assume that the transmitted signal is $y_r(t) = x_{d_1}(t)$, and the values of the parameters p, q are known.

The parameters M_0, M_1 are assumed to be *a priori* unknown which motivates the use of an adaptation for the receiver design. The receiver designed according to the results of Section 2 is modeled as

$$\begin{array}{rcl} \dot{x}_1 & = & p(x_2 - x_1 + f(y_r) + c_1 f_1(y_r) + c_0(x_1 - y_r)), \\ \dot{x}_2 & = & x_1 - x_2 + x_3, \\ \dot{x}_3 & = & -q x_2, \end{array} \qquad (21)$$

where c_0, c_1 are the adjustable parameters. The adaptation algorithm (9), (10), takes the form

$$\begin{array}{rcl} \dot{c}_0 & = & -\gamma_0 (y_r - x_1)^2 - \alpha_0 c_0, \\ \dot{c}_1 & = & -\gamma_1 (x_1 - y_r) f_1(y_r) - \alpha_1 c_1, \end{array} \qquad (22)$$

where γ_0, γ_1 are the adaptation gains, α_0, α_1 are the regularization gains. .

First we examine the ability of the system (21), (22) to receive and to decode messages. To this end we verify the conditions of the Theorem 1 assuming that $s(t) =$ const. Clearly, if $s(t)$ is a time-varying binary signal, we can only expect that the results of Theorem 1 can be used if the parameter estimation is fast enough, at least much faster than the actual parameter modulation. Writing the error equations yields

$$\left\{ \begin{array}{rcl} \dot{e}_1 & = & p(e_2 - e_1 + (c_1 - s) f_1(y_r) + c_0 e_1) \\ \dot{e}_2 & = & e_1 - e_2 + e_3 \\ \dot{e}_3 & = & -q e_2, \end{array} \right. \qquad (23)$$

where $e_i = x_i - x_{d_i}$, $i = 1, 2, 3$. The system (23) is, obviously in Lur'e form with

$$A = \begin{bmatrix} -p & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = [1\ 0\ 0],$$

$\hat{\theta}_1 = c_1$, $\theta_1 = s$, $\theta_0 = c_0$.

The transfer function of the linear part is

$$W(\lambda) = \frac{\lambda^2 + \lambda + q}{\lambda^3 + (p+1)\lambda^2 + q\lambda + pq} \qquad (24)$$

We see that the order of the system is $n = 3$, while the numerator polynomial is Hurwitz and has degree 2 for all $q > 0$ and all real p. Therefore the hyper-minimum-phase condition holds for $q > 0$ and any p, M_0, M_1. Thus, Theorem 1 yields the boundedness of all receiver trajectories $x(t)$ and convergence of the observation error: $e(t) \to 0$. In particular, $y_r(t) - x_1(t) \to 0$. Furthermore, to be able to reconstruct the signal $s(t)$ the receiver should provide convergence $c_1(t) - s \to 0$ for constant s. According to Theorem 1, this will be the case if the PE condition (see Definition 2) holds, which reads as

$$\int_{t_0}^{t_0+T} f_1^2(y_r(t))\, dt \geq \varrho \qquad (25)$$

for some $T > 0$, $\varrho > 0$ and all $t_0 \geq 0$. To verify (25), we note that condition (25) basically means that the trajectory of the transmitter $x_d(t)$ does not converge to the plane $x_{d_1} = 0$ when $t \to \infty$. This is not the case, at least when the system (20) exhibits chaotic behavior. Indeed, in this case the value $x_{d_1}(t)$ leaves the interval $(-1, 1)$ (where $f_1(z)$ is linear). infinitely many times, say at t_k , $k = 1, 2, \ldots$. The time intervals $\Delta t_k = t_{k+1} - t_k$ between t_k can be overbounded by constant, if the trajectory does not converge to the set $x_{d_1} = 0$.

We may also evaluate a lower bound for ϱ in (25):

$$\varrho_0 = \underline{\lim}_{T \to \infty} \frac{1}{T} \int_0^T f_1^2(x_{d_1}(t))\, dt. \qquad (26)$$

The value of ϱ_0 characterizes the parameter convergence rate. It follows from the standard convergence rate results (see e.g. [15]) that if $\varrho_0 > 0$, then the convergence $c_1(t) - s \to 0$ is exponential, with rate $\gamma_1 \varrho_0$, at least for sufficiently small $\gamma_1 > 0$. Ergodicity arguments suggest that

$$\varrho_0 \geq \frac{\overline{x}_{d_1}^2}{\mu}, \qquad (27)$$

where $\overline{x}_{d_1}^2$ is the average value of $x_{d_1}^2(t)$ over the attractor Ω, and $\mu = \sup_{x \in \Omega} |x_{d_1}(t)|$.

Now let us turn to application to the posed problem the algorithm (17) – (19). In accordance with the transfer function of the linear part (24), equation (16) in the considered example can be written as

$$\delta(t) = \tilde{y}^{(3)}(t) + (1+p)\tilde{y}^{(2)}(t) + q\tilde{y}^{(1)}(t) + pq\tilde{y}(t) - \tilde{u}^{(2)}(t) - \tilde{u}^{(1)}(t) - q\tilde{u}, \qquad (28)$$

where $\tilde{y}^{(i)}$, $\tilde{u}^{(i)}$ stand for outputs, inputs and their time derivatives of two third order Butterworth low-pass filters with input signals $y_r(t), u(t)$ respectively; $y_r(t)$ is the received signal (in the case of an ideal channel, $y_r(t) \equiv x_{d_1}(t)$ in (20)); the "input of the model linear part" $u(t)$ is determined as $u = p\tilde{f}(y_d) + p\tilde{s}\tilde{f}_1(y_d)$, where $\tilde{s} = \tilde{s}(t)$ stands for the transmitter parameter estimate, the nonlinear part is described by functions $\tilde{f}_1(y_d) = |y_r + 1| - |y_r - 1|$, $\tilde{f}(y_r) = M_0 y_r + 0.5(M_1 - M_0)\tilde{f}_1(y_r)$, p, q, M_0, M_1 are given constants. Signal $\tilde{f}_1(y_r(t))$ excites the next one Butterworth filter to obtain the regressor $\phi(t)$ as $\phi = -p(\tilde{f}_{1_f}^{(2)} + p\tilde{f}_{1_f}^{(1)} + q\tilde{f}_{1_f})$, where $\tilde{f}_{1_f}^{(i)}$ stand for output filter signal and its derivatives. The adaptive algorithm (16), providing estimate $\tilde{s}(t)$ of the received signal $s(t)$ takes the form

$$\begin{aligned} \dot{\Gamma} &= -\gamma\Gamma^2\phi^2 + \alpha\Gamma, \\ \dot{\tilde{s}}(t) &= -\gamma\Gamma\phi\delta. \end{aligned} \qquad (29)$$

Analogously, Metzler or Duffing systems can be also used for communication with adaptive observer/identification algorithms.

Metzler transmitter equation can be written in the following form:

$$y_r^{(3)} + a_1 y_r^{(2)} + a_2 y_r^{(1)} - a_3(t) y_r + a_4 y_r^2 = 0, \qquad (30)$$

where $y_r(t)$ is a transmitted signal, $a_3(t)$ is a modulating signal.

Duffing transmitter is written as follows

$$y_r^{(2)} + a_1 y_r^{(1)} + a_2(t) y_r^3 + a_3 y_r = B \sin \omega_0 t, \qquad (31)$$

where $a_2(t) = a_{2_0} + \Delta a_2(t)$, $\Delta a_2(t)$ is a modulating signal.

Both schemes (9), (10) and (16) – (19) can be immediately applied to adaptive receiver development.

In practice the channel is subjected to noise, i.e. the investigation of the signal transmission in the case of noisy channel is very important. In this report is assumed that the white noise $\xi(t)$ added to the transmitter output, so that the received signal $y_r(t)$ is modeled as

$$y_r(t) = y_d(t) + \xi(t), \qquad (32)$$

where $\xi(t)$ is a Gaussian white noise with zero average value and intensity σ. [1]

4 Simulation results

We carried out simulations for the above schemes. Parameter values were selected as $p = 9; q = 14.286; M_0 = 5/7; M_1 = -6/7$. For these parameter values the system (20) possesses a chaotic attractor, resembling that of the system used in [3] (after some rescaling of space and time variables).

The initial conditions for the transmitter were taken as $x_d(0) = [0.3\ 0.3\ 0.3]$. For the receiver zero initial conditions were chosen for the state x_0 as well as for the adjustable parameters $c_0(0)$, $c_1(0)$. In order to eliminate the influence of initial conditions no message was transmitted during the first 20 time units ("tuning" or "calibration" of the receiver), i.e. $s(t) \equiv 1$ for $0 \leq t \leq 20$. The time history of observation errors and parameter estimates during tuning show that all observation errors and parameter estimation error $c_1(t) - s$ tend to zero rapidly. The value $c_0(t)$ tends to some constant value.

After the tuning period the square wave message

$$s(t) = s_0 + s_1 \operatorname{sign} \sin\left(\frac{2\pi t}{T_0}\right), \qquad (33)$$

where $s_0 = 1.005$, $s_2 = 0.005$ was sent. Received signal $y_r(t)$ is shown on the Fig. 1.

The simulation shows, that the reconstructed signal $y(t)$ coincides with the transmitted signal $y_r(t)$ with very good accuracy. However both observation errors and estimation errors do not decay completely during the interval when $s(t)$ is constant. Nevertheless, a reliable reconstruction of the signal $s(t)$ is very well possible. The accuracy of estimation can be easily improved by increasing the adaptation gain γ_1. The achievable information transmission rate depends on the highest frequencies in the carrier spectrum. Fig. 2 *a* shows the message signal $s(t)$ (with the period $T_0 = 10$), its estimate via adaptive observer algorithm (21), (22) $\hat{s}(t)$ and output of the first-order low-pass filter $s_f(t)$. This filter is used for separation the message in the case of

[1] More precisely, $\xi(t)$ is modeled as a piecewise constant random process with sample time Δ_t and $\xi(t_k) = \zeta_k\sqrt{\Delta_t}$, $(k = 0, 1, 2, \ldots,\ t_k = k\Delta_t)$, where ζ_k are Gaussian random numbers, having zero mean and the standard deviation σ.

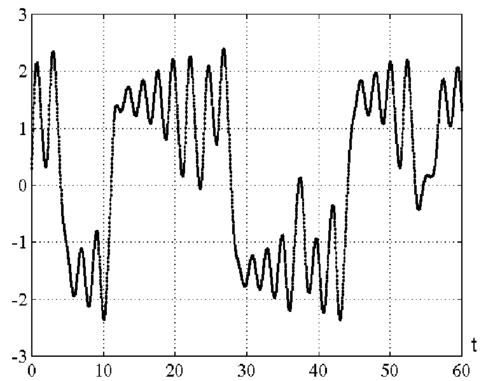

Figure 1: Received signal $y_r(t)$.

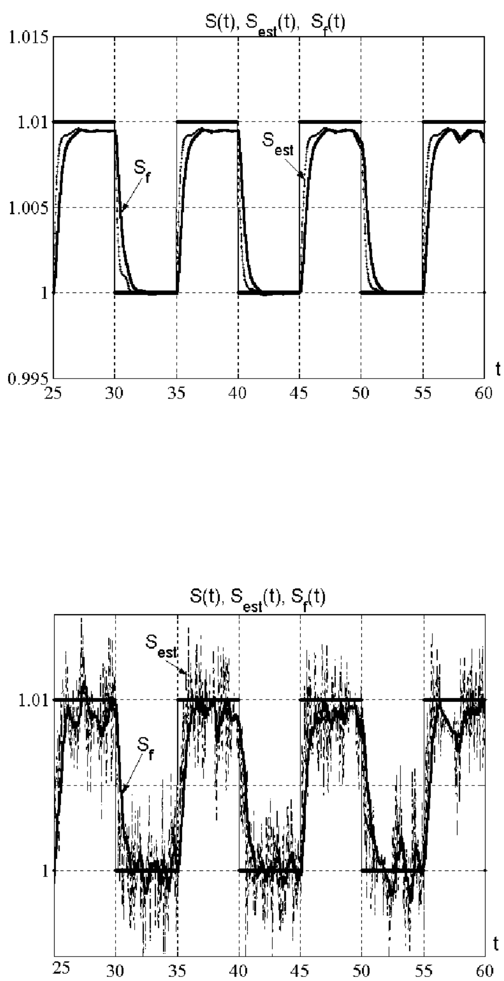

Figure 2: Parameter estimation by means of the adaptive observer (21), (22).

the noisy channel. In the simulation were taken following parameters of the algorithm: $\gamma_0 = 10$, $\alpha_0 = 1$ $\gamma_1 = 5$, $\alpha_1 = 0.2$, filter pass band is equal to 3.5. Fig. 2 *b* illustrates the influence of the noise with $\sigma = 10^{-3}$ in the channel. One can notice, that the message can be recognized in this case too.

The more complicated algorithm (28), (29) is more noise-immune, as it can be seen from the Fig. 3 *b*, where is taken $\sigma = 0.1$ and no one post-filter is used. Algorithm (29) parameters are taken as: $\gamma = \mathrm{sgn}(t - t_0)$, (where $\mathrm{sgn}(\cdot)$ denotes the unit step function, $t_0 = 5$) $\alpha = 5$, $\Gamma(t_0) = 10^{-5}$, band pass of Butterworth filters is equal to 5.

Fig. 4 shows the simulation results of implementation information transmission via Metzler system (30) and algorithm (19) – (29) in the cases of ideal and noisy channel. Following transmitter parameters where taken: $a_1 = 0.4$, $a_2 = 0.95$, $a_{3_0} = 0.78$, $a_4 = -0.6$. Modulating signal $\Delta a_3 = 0.01 a_{3_0} = 7.8 \cdot 10^{-3}$. Butterworth low-pass filters in the receiver algorithm (19) has band pass equal to 1, the initial value $\Gamma(t_0) = 1$, initial transmitter conditions are taken as $y_r(0) = 1, y_r^{(1)}(0) = y_r^{(2)}(0) = 0$.

5 Conclusion

The proposed adaptive observer-based synchronization scheme demonstrates good signal and parameter reconstruction abilities. It allows to achieve high information transmission rate. The results of the paper demonstrate that regularization of adaptation algorithms provides reasonable values of synchronization error.

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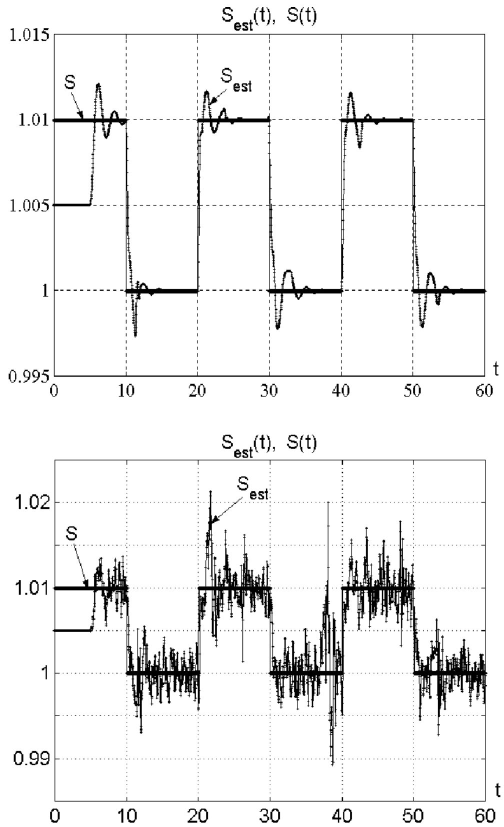

Figure 3: Parameter estimation by means of the identification algorithm (28), (29).

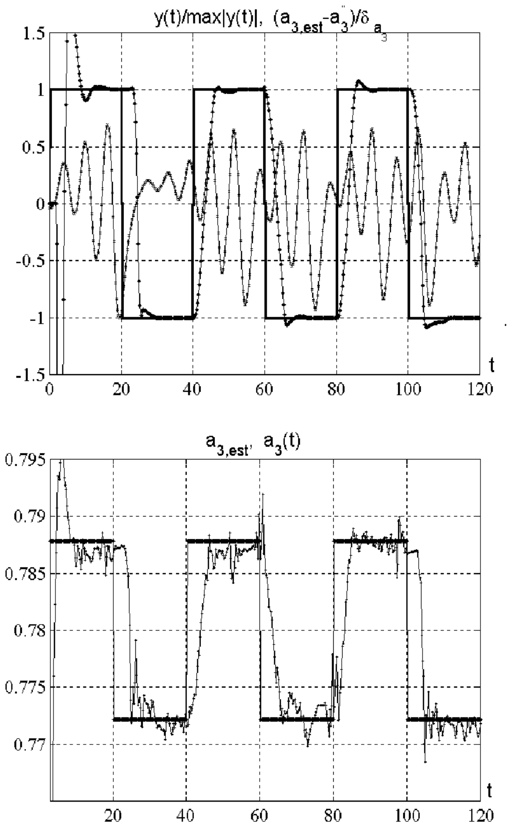

Figure 4: Signal transmission via Metzler transmitter/receiver.

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