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SHUNT COMPENSATION FOR INDIRECT SLIDING-MODE ADAPTIVE CONTROL

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Abstract: In this paper the method for designing parallel compensator for unstable or non-minimum phase plants and new identification algorithm on sliding modes are proposed.

Keywords: model reference adaptive control, sliding modes, parameter identification, shunt compensation

1. INTRODUCTION

In recent years an interest raised to the development of adaptive schemes for plants with unknown relative degree by using output measurements only. The most interesting approach is in introduction the parallel feed-forward compensator. The main idea of the method is to ensure the hyper minimum phase property (HMP) of the augmented plant (plant and compensator), see (Kaufman, *et al.*, 1994; Fradkov, 1994). This procedure simplifies the design of the adaptive controller. In (Bartolini and Ferrara, 1992a; 1992b; Bartolini *et al.*, 1995) a simplified adaptive control scheme has been presented, which performs the regulation of uncertain plants via pole assignment without requiring the perfect knowledge of the relative degree of the controlled plant and independently of the magnitude of the unmodelled dynamics. However, the problem of choosing compensator for unstable plants and nonminimum phase plants remains still open in the field.

In this paper the method for designing parallel compensator for unstable or nonminimum phase time-invariant linear SISO plants is considered.

2. PROBLEM STATEMENT

Consider linear time-invariant SISO plant presented in the following form

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t), \; y_p(t) = C_p x_p(t), \qquad (1)$$

where $x_p(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}$, $y_p(t) \in \mathcal{R}$. The plant transfer function is

$$W_p(s) = C_p \left(sI_n - A_p\right)^{-1} B_p = \frac{B(s)}{A(s)} \qquad (2)$$

where $s \in C$ denotes Laplas transform variable, $\deg A(s) = n$, $\deg B(s) = m$; $k = n - m$ is plant relative degree. It is assumed that $W_p(0) > 0$, $k > 1$.

Let plant be time-invariant with uncertain parameters and measurable output signal $y(t)$. The control aim is to achieve desired closed-loop system performance described by following reference-model equation (see also Bartolini and Ferrara, 1992a)

$$A_m(p) y_p(t) = K \cdot B(p) r(t) \, , \qquad (3)$$

where $r(t)$ is reference input signal, p denotes the time derivative operator ($p \equiv d/dt$) , $A_m(p)$ is arbitrary chosen Hurwitz polynomial degree n , $K = A_m(0)/B(0)$.

This equation corresponds to so-called "implicit reference model" (Fradkov, 1974; Andrievsky and Fradkov, 1994) and imposes less restrictions on system performance than for explicit reference model. Gain K is introduced to achieve astatism of system.

To achieve the aim (3) let us provide an accurate tracking of transformed reference signal $y_f(t)$ which is generated by adjustable prefilter described below. This tracking problem can be solved by means of organizing sliding-mode (Utkin, 1981). As it can be shown, so called hyper minimum phase (HMP) condition (Fradkov, 1974; Fomin, *et al.*, 1981) is sufficient for existence of the stable sliding-modes as well as for direct adaptive control problem solution. For SISO plants the HMP condition means that plant transfer function has all zeros in the left half-plane and $k = 1$. These conditions are not assumed to be valid in the considered problem. One of the possible way is to use parallel feedforward compensator (or "*shunt* "), see (Mareels, 1984; Bar-Kana, 1987; Kaufman, *et al.*, 1994;). It makes possible to ensure requirement mentioned above for augmented plant (AP), consisting of controlled plant and shunt and allows to design adaptive control schemes that do not require plant output derivatives.

Denote the shunt transfer function as $W_c(s) = B'(s)/A'(s)$, $\deg A'(s) = n'$. AP output is $y(t) = y_p(t) + y_c(t)$ and transfer function

$$W(s) = W_p(s) + W_c(s) = \frac{F(s)}{A_p(s)A'(s)}, \qquad (4)$$

where $F(s) = A_p(s)B'(s) + A'(s)B_p(s)$. For tracking for $r(t)$ with desirable dynamics one has to notice that AP output $y(t)$ does not coincides with plant output $y_p(t)$ and the ideal tracking of $y(t)$ to $y_f(t)$ does not involve those one for $y_p(t)$. Hence prefilter equations must be chosen properly. For this purpose let us find transfer function $W_r(s)$ from $r(t)$ to $y_p(t)$ under assumption that $y(t) \equiv y_f(t)$. Taking into account (4) and shunt equation one can obtain that

$$W_r(s) = W_f(s)\frac{B(s)A'(s)}{F(s)}, \qquad (5)$$

where $W_f(s)$ is prefilter transfer function. From (3) , (5) follows that control aim will be achieved if $y(t) \equiv y_f(t)$ and $W_f(s)$ is taken as

$$W_f(s) = \frac{K \cdot F(s)}{A_m(s)A'(s)}, \qquad (6)$$

where $K = A_m(0)/B(0)$.

Notice that (6) describes time-invariant filter for non-adaptive case. In the presence of plant parameters uncertainty instead of (6) should be used following tuned prefilter

$$\dot{x}_f(t) = A_f x_f(t) + B_f r(t),\ y_f(t) = \Theta^T(t)x_f(t), \qquad (7)$$

where $x_f(t) \in \mathcal{R}^N$; $\Theta(t) \in \mathcal{R}^N$ is vector of adjustable parameters, $\Theta(t) = |\vartheta_1(t), \vartheta_2(t), \ldots, \vartheta_N(t)|^T$, $N = n + n'$, matrices A_f , B_f have regular canonical form. Nominal value of $\Theta(t) \equiv \Theta_*$ depends on plant parameters and should be chosen to ensure (6) for transfer function $W_f(s) = \Theta_*^T(sI - A_f)^{-1}B_f$. In the chosen canonical form $F(s) = \sum_{i=1}^N \vartheta_i^* s^{N-i}$ is valid. Therefore one gets following linear equations for nominal values ϑ_i^*, $i = 1, \ldots, N$

$$\sum_{i=1}^{N} \vartheta_i^* s^{N-i} = K(A_p(s)B'(s) + A'(s)B_p(s)). \qquad (8)$$

These values depend on unknown plant parameters. The latter ones will be estimated by means of on-line identification algorithm described in section 3.

To find shunt consider the following transfer function

$$W_c(s) = \frac{\kappa\epsilon(\epsilon s + 1)^{k-2}}{(s + \lambda)^{k-1}}, \quad \lambda > 0. \qquad (9)$$

The following Theorems 1,2 give the necessary property of AP (4) with shunt (9).

<u>Theorem 1</u>. Let $W_p(s)$ (2) be minimum-phase ($B(s)$ be a Hurwitz polynomial) with the relative degree $k > 1$ and $W_p(0) > 0$. Then there exist $\kappa_0 > 0$ and function $\epsilon_0(\kappa) > 0$ such that transfer function $W(s) = W_p(s) + W_c(s)$ is HMP for all $\kappa > \kappa_0$ and $0 < \epsilon < \epsilon_0(\kappa_0)$.

<u>Theorem 2</u>. Let $W_p(s)$ be stable ($A(s)$ be a Hurwitz polynomial) with the relative degree $k > 1$ and $W_p(0) > 0$. Then for every $\epsilon > 0$ there exists sufficiently large κ_0 , such that $W(s) = W_p(s) + W_c(s)$ is HMP for all $\kappa \geq \kappa_0$.

Proofs of the Theorems are given in Appendix. Note that Theorem 1 follows also from more general statement for MIMO plants in (Fradkov, 1994). However more simple than in (Fradkov, 1994) proof for SISO case is given here.

Corollary. Theorem 1 shows, that one can introduce shunt (9) with order $\deg(A_s(s)) = k - 1 = n - m - 1$ providing for sufficiently large κ and small ϵ augmented plant (4) satisfying HMP condition for arbitrary given minimum-phase plant parameters domain. As it is follows from the Theorem 2, another way of shunt (9) parameters choosing provides HMP condition for stable (and, possible, nonminimum-phase) plants. For this case, the shunt equation can be simplified; namely $W_c(s) = \kappa/(s + \lambda)$ may be taken instead of (9).

Assume that shunt (9) is chosen properly and AP (4) is HMP. Rewrite its equations in the following regular form (Utkin, 1981; Sannuti, 1983)

$$\dot{x}_1(t) \ = \ A_{11}x_1(t) + A_{12}x_2(t),$$

$$\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + bu(t), \quad (10)$$
$$y(t) = c \cdot x(t),$$

where $x_1(t) \in \mathcal{R}^{N-1}$, $x_2(t) \in \mathcal{R}$ and $y(t) = c_1x_1(t) + c_2x_2(t)$ is a measurable output, $c_2b > 0$; A_{11}, A_{12}, A_{21}, A_{22} , b are unknown parameters, $c = [c_1, \; c_2]$.

Now formal problem statement can be presented . One have to find a control action $u(t)$ and adjustment law $\Theta(t)$ in (7) such that for any given value of the plant relative degree k the plant output asymptotically satisfies (3).

The problem can be solved in two steps. The first step is to design the adjustment law for parameter estimates and to ensure their convergence to true values. The second step is to choose the control $u(t)$ to ensure the convergence of $s(t) = y(t) - y_f(t)$ to zero in a finite time.

3. THE ADJUSTMENT LAW DESIGN

In this section the least-squares-like estimator for plant parameters by using input/output measurement only is considered. The first step is designing filters to avoid the measurements of the derivatives of the plant output.

Plant equations (2) can be written as

$$y^{(n)}(t) + a_1y^{(n-1)}(t) + \ldots + a_ny(t) =$$
$$= b_0u^{(m)}(t) + b_1u^{(m-1)}(t) + \ldots + b_mu(t), \quad (11)$$

where $a_1, \ldots a_n, b_0, \ldots, b_m$ are unknown plant parameters (index n means the nth time derivative of the signal). Rewrite plant equations as follows

$$y^{(n)}(t) = \varphi^T(t)\theta^*, \quad (12)$$

where

$$\varphi(t) = | \; y^{n-1}(t), \ldots, \dot{y}(t), y(t), u^m(t), \ldots, u(t) \; |^T ,$$
$$\theta^* = | -a_1, \; -a_2, \ldots, \; -a_n, \; b_0, b_1, \ldots, b_m \; |^T ,$$

$\varphi(t), \theta^* \in \mathcal{R}^{n+m+1}$. Introducing filtered signals $\tilde{y}(t)$, $\tilde{\varphi}(t)$ satisfying equations

$$D(p)\tilde{y}^{(n)}(t) = y^{(n)}(t) \quad , \quad D(p)\tilde{\varphi}(t) = \varphi(t) ,$$

where $D(p) = p^n + d_1p^{n-1} + \ldots + d_n$ is arbitrary Hurwitz polynomial of $p \equiv d/dt$ one obtains from (12)

$$\tilde{y}^{(n)} = \tilde{\varphi}^T(t)\theta^* \quad , \quad (13)$$

Signals $\tilde{y}(t)$, $\tilde{\varphi}(t)$ could be obtained by means of following filters

$$\dot{\xi}(t) = A_d\xi(t) + b_dy(t) ,$$
$$\dot{\psi}(t) = A_d\psi(t) + b_du(t) ,$$

where $\xi(t), \psi(t) \in \mathcal{R}^n$; A_d , b_d has regular canonical form, $\det(sI - A_d) = D(s)$. Notice, that both ξ and ψ can be implemented by using input/output measurements only. It is straightforward to see that

$$\tilde{\varphi}(t) = |\xi_n(t), \ldots, \xi_2(t), \xi_1(t), \psi_{m+1}(t), \ldots, \psi_1(t)|^T ,$$
$$\tilde{y}^{(n)}(t) = y(t) - \sum_{i=1}^{n} d_{n-i+1}\xi_i(t) .$$

Let us present now the adjustment law in the form

$$\dot{\theta}(t) = -\Gamma(t)\tilde{\varphi}(t)\tilde{\varphi}^T(t)(\theta^T(t) - \theta^*(t)) =$$
$$= -\Gamma(t)\tilde{\varphi}(t)\tilde{\varphi}^T(t)\theta^T(t) + \Gamma(t)\tilde{\varphi}(t)\tilde{\xi}(t) , (14)$$

$$\dot{\Gamma}(t) = -\Gamma(t)\tilde{\varphi}(t)\tilde{\varphi}^T(t)\Gamma(t) + (\Gamma(t) - \Gamma^2(t)/k_0) , \quad (15)$$

where $k_0I > \Gamma(0) = \Gamma(0)^T > 0$, $\tilde{\xi}(t)$ denotes $\tilde{y}^{(n)}(t)$.

To prove the parameter convergence assume that $\tilde{\varphi}(t)$ is persistently exciting that is guaranteed by the excitation of control action $u(t)$. Conventional conditions of persistent excitation can be found *e.g.* in (Narendra and Annaswamy, 1989).

Choosing the following Lyapunov function candidate

$$V = \frac{1}{2}||\tilde{\theta}||^2_{\Gamma^{-1}(t)}$$

and evaluating its derivative along the solutions of (14), (15) one gets

$$\dot{V}(t) = -\tilde{\theta}^T(t)\tilde{\varphi}(t)\tilde{\varphi}^T(t)\tilde{\theta}(t) +$$
$$+ \; \tilde{\theta}^T(t)\tilde{\varphi}(t)\tilde{\varphi}^T(t) \; \tilde{\theta}(t) - ||\tilde{\theta}(t)||^2_{(\Gamma^{-1}(t) - I/k_0)} .$$

As was shown by Bartolini, *et al.*, (1995), under the condition of persistent excitation there exists λ such that $(\Gamma^{-1}(t) - I/k_0) \geq \lambda\Gamma^{-1}(t)$ and $\dot{V}(t) \leq -2\lambda V(t)$. This in turn implies that

$$\frac{1}{k_0}||\tilde{\theta}(t)||^2 \; \leq \; ||\tilde{\theta}(t)||^2_{\Gamma^{-1}(0)} \cdot e^{-2\lambda t} . \quad (16)$$

Hence the adjustable parameters converge exponentially to their true values provided control $u(t)$ is persistently exciting. By using estimates $\theta(t)$ determined by (8), (14) one can easily calculate $\Theta(t)$ in (7) in order that aim (3) be achieved. One next step is controller design.

4. THE CONTROLLER DESIGN

Design control law the sliding mode on the surface $s = y - y_f = 0$ is organized.

At first present the error model by using (10)

$$\dot{s}(t) = c_1\dot{x}_1(t) + c_2\dot{x}_2(t) - \dot{y}_f(t) =$$
$$= c_1A_{11}x_1(t) + c_1A_{12}x_2(t) + \quad (17)$$
$$+ \; c_2A_{21}x_1(t) + c_2A_{22}x_2(t) + c_2bu(t) - \dot{y}_f(t) .$$

Taking into account that

$$x_2(t) = \frac{1}{c_2}\left(s(t) + y_f(t) - c_1 x_1(t) \right) \qquad (18)$$

after substituting it in (17) follows

$$\begin{aligned}(c_2 b)^{-1}\dot{s}(t) &= Lx_1(t) + a_1 s(t) + a_1 y_f(t) - \\ &- (c_2 b)^{-1}\dot{y}_f(t) + u(t) \ , \qquad (19)\end{aligned}$$

where L is $1 \times (N-1)$ vector,

$$L = (c_2 b)^{-1}\left(c_1 A_{11} + c_2 A_{21} - \frac{c_1 A_{12} + c_2 A_{22}}{c_2} c_1 \right),$$

$a_1 = \frac{1}{c_2(c_2 b)}(c_1 A_{12} + c_2 A_{22})$. Now present the error model for $x_1(t)$. Substituting (18) in (10) yields

$$\dot{x}_1(t) = A_* x_1(t) - \frac{A_{12}}{c_2} s(t) + \frac{A_{12}}{c_2} y_f(t) , \qquad (20)$$

where $A_* = A_{11} - A_{12}c_1/c_2$. Equations (7), (19), (20) describe the error model. Its description is completed by using HMP property of the system. This implies that A_* is a Hurwitz matrix and $c_2 b > 0$. It is important to remark that $y_f(t)$ is bounded ($|y_f(t)| \leq \overline{y}_f$) since A_f is a Hurwitz matrix and $r(t)$, $\theta_f(t)$ are bounded.

Choose now the control action as

$$u(t) = -k_s s(t) - \gamma \cdot \mathrm{sign}(s(t)) ,$$

where positive parameters k_s and γ are specified below.

The stability of system can be studied by means of the consecutive application (Stotsky, 1994) of two Lyapunov functions

$$V_1 = \frac{1}{2}(cb)^{-1}s^2 + \frac{1}{2}x_1^T P x_1$$

and

$$V_2 = \frac{1}{2}(cb)^{-1}s^2 .$$

Evaluating $\dot{V}_1(t)$ one has

$$\begin{aligned}\dot{V}_1(t) &= x_1^T(t)P\left(A_* x_1(t) + \frac{A_{12}}{c_2}(s(t) + y_f(t)) \right) + \\ &+ s(t)\Big(Lx_1(t) + a_1 s(t) + a_1 y_f(t) - \\ &- (c_2 b)^{-1}\dot{y}_f(t) - k_s s(t) - \gamma \cdot \mathrm{sign} s(t) \Big) ,\end{aligned}$$

where

$$PA_* + A_*^T P = -Q, \quad Q = Q^T > 0 .$$

It gives

$$\begin{aligned}\dot{V}_1(t) &\leq -\frac{1}{2}x_1^T(t)Qx_1(t) + \\ &+ \frac{||P|| \cdot ||A_{12}||}{|c_2|}||x_1(t)|| \cdot |s(t)| + \\ &+ \frac{||P|| \cdot ||A_{12}||}{|c_2|}||x_1(t)|| \cdot \overline{y}_f(t) + \\ &+ ||L|| \cdot ||x_1(t)|| \cdot |s(t)| + s(t)^2\left(\overline{a}_1 - k_s\right) + \\ &+ |s(t)|\left(\overline{a}_1\overline{y}_f(t) + (c_2 b)^{-1}|\dot{y}_f(t)| - \gamma\right) \\ &\leq -\frac{\lambda_{\min}(Q)}{2}||x_1(t)||^2 + \left(\overline{p} + \overline{l}\right)\frac{\lambda}{2}||x_1||^2 + \\ &+ \frac{\overline{p} + \overline{l}}{2\lambda}s(t)^2 + \\ &+ \overline{p}\frac{\lambda}{2}||x_1(t)||^2 + \frac{\overline{p}}{2\lambda}\overline{y}_f^2 + s(t)^2\left(\overline{a}_1 - k_s\right) + \\ &+ |s(t)|\left(\overline{a}_1\overline{y}_f + (c_2 b)^{-1}|\dot{y}_f(t)| - \gamma\right) ,\end{aligned}$$

where $\overline{p} \geq ||P|| \cdot ||A_k||/|c_2|$, $\overline{l} \geq ||L||$, $\overline{a}_1 \geq ||a_1||$ and λ is sufficiently small number. Notice, that for any $\lambda_{\min}(Q)$ there always exists $\lambda > 0$ such that $\frac{\lambda_{\min}(Q)}{4} \geq \left(\overline{p} + \overline{l}\right) \cdot \lambda$ Then

$$\begin{aligned}\dot{V}_1(t) \leq -\frac{\lambda_{\min}(Q)}{4}||x_1(t)||^2 + s(t)^2\left(\overline{a}_1 + \frac{\overline{p} + \overline{l}}{2\lambda} - k_s\right) + \\ + |s(t)|\left(\overline{a}_1\overline{y}_f + \overline{c}|\dot{y}_f(t)| - \gamma\right) + \frac{\overline{p}}{2\lambda}\overline{y}_f(t)^2 \ ,\end{aligned}$$

where $\overline{c} \geq (c_2 b)^{-1}$. Choose k_s and γ . Let

$$k_s \geq a_1 + \frac{\overline{p} + \overline{l}}{2\lambda} + \frac{\kappa}{2}\overline{c} ,$$

$$\gamma \geq \overline{a}_1\overline{y}_f + \overline{c}|\dot{y}_f(t)| + \gamma_0 ,$$

where $\kappa > 0$, $\gamma_0 > 0$ to be specified below. Then

$$\begin{aligned}\dot{V}_1(t) &\leq -\frac{1}{2}x_1^T(t)Px_1(t) \cdot \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} - \\ &- \kappa\frac{1}{2}(cb)^{-1}s(t)^2 + \frac{\overline{p}}{2\lambda}\overline{y}_f^2 ,\end{aligned}$$

and

$$\dot{V}_1(t) \leq -\kappa_1 V_1(t) + \frac{\overline{p}}{2\lambda}\overline{y}_f^2,$$

$$\kappa_1 = \min\left(\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \ , \ \kappa \right) ,$$

where

$$V_1(0) = \frac{1}{2}(cb)^{-1}s(0)^2 + \frac{1}{2}x_1^T(0)Px_1(0) .$$

Hence the following bound is true $||x_1(t)||^2 \le \bar{x}_1$, where

$$\bar{x}_1 = \frac{1}{\lambda_{\min}(P)}\Big((cb)^{-1}s(0)^2 + x_1^T(0)Px_1(0)\Big) + \\ + \frac{\bar{p}}{2\lambda_{\min}(P)\lambda\kappa_1}\bar{y}_f^2 \; . \qquad (21)$$

It remains to prove the convergence of $s(t)$ in a finite time.

Taking

$$V_2(t) = \frac{1}{2}(cb)^{-1}s(t)^2$$

and evaluating $\dot{V}_2(t)$ along (19) one obtains

$$\dot{V}_2(t) \le |s(t)| \cdot \Big(||L|| \cdot ||x_1|| + \bar{a}_1\bar{y}_f(t) + \\ + \bar{c}|\dot{y}_f(t)| - \bar{a}_1\bar{y}_f - \bar{c}|\dot{y}_f(t)| - \gamma_0\Big) \; .$$

Choosing $\gamma_0 \ge \bar{\epsilon}x_b + \bar{c}\beta/2$, where x_b is upper bound of $\bar{x}_1$ presented by (21) ($\bar{x}_1 \le x_b$), $\beta > 0$, one gets

$$\dot{V}_2(t) \le -\beta\sqrt{V_2(t)} \; .$$

This, in turn, implies that for all $t \ge t^*$ sliding mode appears on the surface $s = 0$, $t^* = 2\sqrt{V_2(0)}/\beta$.

Now let us prove the achievement of the control aim. Since $s(t) \equiv 0$ one has $y(t) \equiv y_f(t)$ and one can use plant parameter estimates $\theta(t)$ for prefilter (7) adjustment. Define $\Theta(t)$ in (7) in accordance with (8), where are used estimates instead of true plant parameters. Finally one obtains following equation for $\Theta(t) = |\vartheta_1(t), \ldots, \vartheta_N(t)|$

$$\sum_{i=1}^{N}\vartheta_i s^{N-i} = \widehat{K}(t)(\widehat{A}_p(s,t)B'(s) + A'(s)\widehat{B}_p(s,t)) \; , \quad (22)$$

where $\widehat{K}(t) = A_m(0)/\widehat{B}(0,t) \equiv A_m(0)/\theta_{n+m+1}(t)$, $\widehat{A}(s,t) = s^n - \theta_1(t)s^{n-1} - \ldots - \theta_n(t)$, $\widehat{B}(s,t) = \theta_{n+1}(t)s^m + \ldots + \theta_{n+m+1}(t)$, $A'(s)$, $B'(s)$ are given in (7). Collecting similar terms in (22) one gets $\Theta(t)$ in the explicit form. From parameter estimations convergence to their true values θ^* follows that prefilter adjustable parameters error $\tilde{\Theta}(t) = \Theta(t) - \Theta^* \to 0$ when $t \to 0$. Hence one can represent $y_f(t)$ as $y_f(t) = y_f^*(t) + \tilde{y}_f(t)$, where $y_f^*(t) = \Theta^* x_f(t)$ is an ideal prefilter output (taken from (6)) and the bounded vanishing error signal $\tilde{y}_f(t)$ may be considered as addition to y_f disturbance. Since the sliding-mode system was proved be stable, $y_p(t)$ will coincide with its ideal trajectory. Therefore finite-time sliding mode realization and exponential convergence of parameter estimates provide the control aim (3) achievement.

5. CONCLUSION

In this paper a simplified adaptive control scheme is presented for regulation of unstable and nonminimum-phase uncertain plants by means of parallel feedforward compensator (shunt) which ensures hyper-minimum phase property of augmented plant.

The proposed controller ensures finite time convergence of augmented error omitting to relay term in control law and exponential convergence of the parameter error under the condition of persistent excitation. This, in turn allows to achieve the desired dynamics to the true plant output.

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APPENDIX

Proof of the Theorem 1. To prove this theorem the following lemma is needed

Lemma. Let $\alpha(s), \beta(s), \gamma(s), \delta(s)$ be polynomials with positive leading coefficients α_0, β_0, γ_0, δ_0, respectively, $Q_\epsilon(s) = \beta(s)\gamma(s) + \epsilon(s)\alpha(s)\delta(\epsilon s)$. Suppose $\beta(s), \gamma(s)$ and $\alpha_0 s\delta(s) + \beta_0\gamma_0$ are stable polynomials. Then there exists $\epsilon_0 > 0$ such that $Q_\epsilon(s)$ is stable for all positive $\epsilon < \epsilon_0$.

Proof. Note that $\deg(\beta\gamma) = n - 1$, $\deg(Q_\epsilon(s)) = \deg(\alpha\beta) = n + k - 2$. Hence $n - 1$ zeros of $Q_\epsilon(s)$ tend to zeros of $\beta(s)\gamma(s)$ and the rest $k - 1$ zeros $s_i(\epsilon)$, $i = 1, \dots k - 1$ tend to infinity as $\epsilon \to 0$. To analyze the behavior of $s_i(\epsilon)$ make a change ϵs to μ and denote $R_\epsilon(\mu) = \epsilon^{n-1}Q_\epsilon(\mu/\epsilon)$. Then $R_\epsilon(\mu) = \epsilon^{n-1}\beta(\mu/\epsilon)\gamma(\mu/\epsilon) + (\alpha_0\mu^n + \epsilon\alpha_1\mu^{n-1} + \dots + \epsilon^n\alpha_n)\delta(\mu)$ and $R_\epsilon(\mu) \to \beta_0\gamma_0\mu^{n-1} + \alpha_0\mu^n\delta(\mu) = \mu^{n-1}(\beta_0\gamma_0 + \alpha_0\mu\delta(\mu))$. So, $n - 1$ zeros of $R_\epsilon(\mu)$ tend to zero while the rest $k - 1$ ones tend to zeros of $\beta_0\gamma_0 + \alpha_0\mu\delta(\mu)$. Lemma is proved.

Proof of the Theorem 1. Express $W(s)$ as

$$W(s) = \frac{B_p(s)(s+\lambda)^{k-1} + A_p(s)\kappa\epsilon(\epsilon s+1)^{k-2}}{A_p(s)(s+\lambda)^{k-1}} .$$

To apply lemma one needs the polynomial $b_0 + \kappa\mu(1+\mu)^{k-2}$ to be stable, where b_0 denotes the leading coefficient of $B_p(s)$. Obviously,it will be so at least for sufficiently large κ . Theorem is proved.

Proof of the Theorem 2. Rearrange transfer function $W(s)$ as follows

$$\begin{aligned} W(s) &= W_p(s) + W_c(s) = \\ &= \frac{B_p(s)(s+\lambda)^{k-1} + \kappa\epsilon(\epsilon s+1)^{k-2}\cdot A_p(s)}{A_p(s)(s+\lambda)^{k-1}} = \\ &= \frac{B'(s) + \kappa\epsilon(\epsilon s+1)^{k-2}(s^n + a_1 s^{n-1} + \dots + a_n)}{A_p(s)(s+\lambda)^{k-1}} = \\ &= \frac{\mu b_0' s^{n-1} + \mu(b_1' s^{n-2} + \dots + b_{n-1}') + \overline{A}(s)}{\mu \cdot A_p(s)(s+\lambda)^{k-1}} , \end{aligned}$$

where $\mu = 1/\kappa$, $\overline{A}(s)$ is a Hurwitz polynomial,

$$\overline{A}(s) = \overline{a}_0 s^n + \overline{a}_1 s^{n-1} + \dots \overline{a}_n ,$$

$$B'(s) = \left[b_0 s^{n-k} + \overline{B}(s)\right](s+\lambda)^{k-1} ,$$

$\overline{B}(s) = b_1 s^{n-k-1} + \dots + b_{n-k}$, $b_0' = b_0$, $b_1' = b_1 + \lambda b_0, \dots, b_{n-1}' = b_{n-k}\lambda^{k-1}$. Thus

$$W(s) = \frac{F(s)}{\mu A_p(s)(s+\lambda)^{k-1}} ,$$

where $F(s) = \overline{a}_0 s^{n+k-2} + \dots + \overline{a}_{k-2}s^n + +(\overline{a}_{k-1} + \mu b_{k-1}')s^{n-1} + \dots + \overline{a}_{n+k-2} + \mu b_{n-k}'$. Therefore the numerator $F(s)$ of $W(s)$ will be Hurwitz for sufficiently small $\mu > 0$. Theorem is proved.