SHUNT COMPENSATION FOR INDIRECT SLIDING-MODE
ADAPTIVE CONTROL

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Abstract: In this paper the method for designing parallel compensator for unstable or nonminimum phase plants and new identification algorithm on sliding modes are proposed.

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1. INTRODUCTION

In recent years an interest raised to the development of adaptive schemes for plants with unknown relative degree by using output measurements only. The most interesting approach is in introduction of parallel feedback compensator. The main idea of the method is to ensure the hyper minimum phase property (HMP) of the augmented plant (plant and compensator), see (Kaufman, et al., 1994; Fradkov, 1994). This procedure simplifies the design of the adaptive controller.

In (Bartolini and Ferrara, 1992a; 1992b; Bartolini et al., 1995) a simplified adaptive control scheme has been presented, which performs the regulation of uncertain plants via pole assignment without requiring the perfect knowledge of the relative degree of the controlled plant and independently of the magnitude of the unmodelled dynamics. However, the problem of choosing compensator for unstable plants and nonminimum phase plants remains still open in the field.

In this paper the method for designing parallel compensator for unstable or nonminimum phase time-invariant linear SISO plants is considered.

2. PROBLEM STATEMENT

Consider linear time-invariant SISO plant presented in the following form

\[ \dot{x}_p(t) = A_p x_p(t) + B_p u(t), \quad y_p(t) = C_p x_p(t) \]

where \( x_p(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R} \), \( y_p(t) \in \mathbb{R} \). The plant transfer function is

\[ W_p(s) = C_p (s I_n - A_p)^{-1} B_p = \frac{B(s)}{A(s)} \]

where \( s \in \mathbb{C} \) denotes Laplace transform variable, \( \deg A(s) - n \), \( \deg B(s) = m \); \( k = n - m \) is plant relative degree. It is assumed that \( W_p(0) > 0 \), \( k > 1 \).

Let plant be time-invariant with uncertain parameters and measurable output signal \( y(t) \). The control aim is to achieve desired closed-loop system performance described by following reference-model equation (see also Bartolini and Ferrara, 1992a)

\[ A_m(p) y_p(t) = K \cdot B(p) r(t) \]

where \( r(t) \) is reference input signal, \( p \) denotes the time derivative operator \( (p = d/dt) \). \( A_m(p) \) is arbitrary chosen Hurwitz polynomial degree \( n \), \( K = A_m(0)/B(0) \).
This equation corresponds to so-called "implicit reference model" (Pradkov, 1974; Andrievsky and Pradkov, 1994) and imposes less restrictions on system performance than for explicit reference model. Gain $K$ is introduced to achieve asymptotic of system.

To achieve the aim (3) let us provide an accurate tracking of transformed reference signal $y_f(t)$ which is generated by adjustable prefiler described below. This tracking problem can be solved by means of optimizing sliding-mode (Utkin, 1981). As it can be shown, so-called hyper minimum phase (HMP) condition (Pradkov, 1974; Fomin et al., 1981) is sufficient for existence of the stable sliding-modes as well as for direct adaptive control problem solution. For SISO plants the HMP condition means that plant transfer function has all zeros in the left half-plane and $\kappa = 1$. These conditions are not assumed to be valid in the considered problem. One of the possible ways to use parallel feedforward compensator (or "shunt"), see (Mareels, 1984; Barnawa, 1987; Kaufman et al., 1994;) it makes possible to ensure requirement mentioned above for augmented plant (AP), consisting of controlled plant and shunt and allows to design adaptive control schemes without requiring plant output derivatives.

Denote the shunt transfer function as $W_c(s) = B'(s)/A'(s)$, deg $A'(s) = n'$. AP output is $y(t) = y_p(t) + \psi(t)$ and transfer function

$$W(s) = W_p(s) + W_c(s) = \frac{F(s)}{A_p(s)A'(s)}, \quad \text{(4)}$$

where $F(s) = A_p(s)B'(s) + A'(s)B_p(s)$. For tracking for $r(t)$ with desirable dynamics one has to notice that AP output $y(t)$ does not coincide with plant output $y_p(t)$ and the ideal tracking of $y(t)$ to $y_f(t)$ does not involve those one for $y_p(t)$. Hence, prefiler equations must be chosen properly. For this purpose let us find transfer function $W_f(s)$ from $r(t)$ to $y_f(t)$ under assumption that $y(t) = y_f(t)$. Taking into account (4) and shunt equation one can obtain that

$$W_f(s) = W_p(s) \frac{B'(s)A'(s)}{F(s)}, \quad \text{(5)}$$

where $W_f(s)$ is prefiler transfer function. From (3), (5) follows that control aim will be achieved if $y(t) = y_f(t)$ and $W_f(s)$ is taken as

$$W_f(s) = \frac{K \cdot F(s)}{A_m(s)A'(s)}, \quad \text{(6)}$$

where $K = A_m(0)/B(0)$.

Notice that (6) describes time-invariant filter for non-adaptive case. In presence of plant parameters uncertainty instead of (6) should be used following tuned prefiler

$$\dot{x}_f(t) = A_fx_f(t) + B_f\psi(t), \quad y_f(t) = \Theta^T(t)x_f(t), \quad \text{(7)}$$

where $x_f(t) \in \mathbb{R}^N$; $\Theta(t) \in \mathbb{R}^N$ is vector of adjustable parameters, $\Theta(t) = [\Theta_1(t), \Theta_2(t), \ldots, \Theta_N(t)]^T$, $N = n + n'$, matrices $A_f, B_f$ have regular canonical form. Nominal value of $\Theta(t) \equiv \Theta$ depends on plant parameters and should be chosen to ensure (6) for transfer function $W_f(s) = \Theta^T(sI - A_f)^{-1}B_f$. In chosen canonical form $F(s) = \sum_{i=1}^{N} \theta_i^s s^{N-i}$ is valid. Therefore one gets following linear equations for nominal values $\theta_i^s$, $i = 1, \ldots, N$

$$\sum_{i=1}^{N} \theta_i^s s^{N-i} = K(A_p(s)B'(s) + A'(s)B_p(s)) \quad \text{(8)}$$

These values depend on unknown plant parameters. The latter ones will be estimated by means of on-line identification algorithm described in section 3.

To find shunt consider the following transfer function

$$W_c(s) = \frac{n\lambda \kappa (cs + 1)^{k-2}}{(s + \lambda)^{k-1}}, \quad \lambda > 0 \quad \text{(9)}$$

The following Theorems 1,2 give the necessary property of AP (4) with shunt (9).

**Theorem 1.** Let $W_p(s)$ (2) be minimum-phase ($B(s)$ is a Hurwitz polynomial) with the relative degree $k > 1$ and $W_p(0) > 0$. Then there exist $\kappa > 0$ and function $\epsilon_0(\kappa) > 0$ such that transfer function $W(s) = W_p(s) + W_c(s)$ is HMP for all $\kappa > \kappa_0$ and $0 < \epsilon < \epsilon_0(\kappa_0)$.

**Theorem 2.** Let $W_p(s)$ be stable ($A(s)$ is a Hurwitz polynomial) with the relative degree $k > 1$ and $W_p(0) > 0$. Then for every $\epsilon > 0$ there exists sufficiently large $\kappa_0$, such that $W(s) - W_p(s) + W_c(s)$ is HMP for all $\kappa \geq \kappa_0$.

Proofs of the Theorems are given in Appendix. Note that Theorem 1 follows also from more general statement for MIMO plants in (Pradkov, 1994). However more simple than in (Pradkov, 1994) proof for SISO case is given here.

**Corollary.** Theorem 1 shows, that one can introduce shunt (9) with order deg($A_m(s)$) = $k - 1 = n - m - 1$ providing for sufficiently large $\kappa$ and small $\epsilon$ augmented plant (4) satisfying HMP condition for arbitrary given minimum-phase plant parameters domain. As it is follows from the Theorem 2, another way of shunt (9) parameters choosing provides HMP condition for stable (and, possible, nonminimum-phase) plants. For this case, the shunt equation can be simplified; namely, $W_c(s) = \kappa/(s + \lambda)$ may be taken instead of (9).

Assume that shunt (9) is chosen properly and AP (4) is HMP. Rewrite its equations in the following regular form (Utkin, 1981; Sannutti, 1983)

$$\dot{x}_1(t) = A_1x_1(t) + A_2x_2(t), \quad \text{(10)}$$

where $x_1(t) \in \mathbb{R}^n$; $\Theta(t) \in \mathbb{R}^N$ is vector of adjustable parameters, $\Theta(t) = [\Theta_1(t), \Theta_2(t), \ldots, \Theta_N(t)]^T$, $N = n + n'$, matrices $A_1, A_2$ have regular canonical form. Nominal value of $\Theta(t) \equiv \Theta$, depends on plant parameters and should be chosen to ensure (6) for transfer function $W_f(s) = \Theta^T(sI - A_f)^{-1}B_f$. In chosen canonical form $F(s) = \sum_{i=1}^{N} \theta_i^s s^{N-i}$ is valid. Therefore one gets following linear equations for nominal values $\theta_i^s$, $i = 1, \ldots, N$

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Assume that shunt (9) is chosen properly and AP (4) is HMP. Rewrite its equations in the following regular form (Utkin, 1981; Sannutti, 1983)

$$\dot{x}_1(t) = A_1x_1(t) + A_2x_2(t), \quad \text{(10)}$$
\[ \dot{x}_2(t) = A_2 x_1(t) + A_22 x_2(t) + bu(t), \quad (10) \]
\[ y(t) = c \cdot x(t), \]
where \( x_1(t) \in \mathbb{R}^{N-1} \), \( x_2(t) \in \mathbb{R} \) and \( y(t) = c_1 x_1(t) + c_2 x_2(t) \) is a measurable output, \( c_2 \neq 0 \); \( A_{11}, A_{12}, A_{21}, A_{22} \), \( b \) are unknown parameters, \( c = [c_1, c_2] \).

Now formal problem statement can be presented. One has to find a control action \( u(t) \) and adjustment law \( \Theta(t) \) in (7) such that for any given value of the plant relative degree \( k \) the plant output asymptotically satisfies (3).

The problem can be solved in two steps. The first step is to design the adjustment law for parameter estimates and to ensure their convergence to true values. The second step is to choose the control \( u(t) \) to ensure the convergence of \( s(t) = y(t) - y_f(t) \) to zero in a finite time.

### 3. THE ADJUSTMENT LAW DESIGN

In this section the least-squares-like estimator for plant parameters by using input/output measurement only is considered. The first step is designing filters to avoid the measurements of the derivatives of the plant output.

Plant equations (2) can be written as
\[ y^{(n)}(t) + a_1 y^{(n-1)}(t) + \ldots + a_n y(t) = b_0 u^{(m)}(t) + b_1 u^{(m-1)}(t) + \ldots + b_m u(t), \quad (11) \]
where \( a_1, \ldots, a_n, b_0, \ldots, b_m \) are unknown plant parameters (index \( n \) means the \( n \)th time derivative of the signal). Rewrite plant equations as follows
\[ y^{(n)}(t) = \varphi^T(t) \theta^*, \quad (12) \]
where
\[ \varphi(t) = [y^{(n-1)}(t), \ldots, y(t), u^{(m)}(t), \ldots, u(t)]^T, \]
\[ \theta^* = [-a_1, -a_2, \ldots, -a_n, b_0, b_1, \ldots, b_m]^T, \]
\( \varphi(t), \theta^* \in \mathbb{R}^{n+m+1} \). Introducing filtered signals \( \tilde{y}(t), \tilde{\varphi}(t) \) satisfying equations
\[ D(p) \tilde{y}^{(n)}(t) - y^{(n)}(t), \quad D(p) \tilde{\varphi}(t) = \varphi(t), \]
where \( D(p) = p^n + a_1 p^{n-1} + \ldots + a_n \) is arbitrary Hurwitz polynomial of \( \equiv d/dt \) one obtains from (12)
\[ \tilde{y}^{(n)}(t) = \tilde{\varphi}^T(t) \theta^*. \quad (13) \]

Signals \( \tilde{y}(t), \tilde{\varphi}(t) \) could be obtained by means of following filters
\[ \xi(t) = A_2 \xi(t) + b_2 y(t), \]
\[ \psi(t) = A_2 \psi(t) + b_2 u(t), \]
where \( \xi(t), \psi(t) \in \mathbb{R}^n \); \( A_2 \) has regular canonical form, \( \det(sI - A_2) = D(s) \). Notice, that both \( \xi \) and \( \psi \) can be implemented by using input/output measurements only. It is straightforward to see that
\[ \tilde{\varphi}(t) = [\xi_1(t), \ldots, \xi_2(t), \xi_1(t), \psi_{m+1}(t), \ldots, \psi_1(t)]^T, \]
\[ \tilde{\psi}^{(n)}(t) = y(t) - \sum_{i=1}^n \alpha_{n-i+1} \xi_i(t). \]

Let us present now the adjustment law in the form
\[ \dot{\theta}(t) = -\Gamma(t) \varphi^T(t) \tilde{\varphi}(t) \tilde{\varphi}^T(t) \theta^* - \theta^*(t) \]
\[ = -\Gamma(t) \varphi^T(t) \tilde{\varphi}^T(t) \theta^* + \Gamma(t) \varphi^T(t) \xi(t), \quad (14) \]
\[ \dot{\xi}(t) = -\Gamma(t) \varphi^T(t) \tilde{\varphi}(t) \Gamma(t) + (\Gamma(t) - \Gamma^T(t)/k_0), \quad (15) \]
where \( k_0 > \Gamma(0) \equiv \Gamma(0)^T > 0 \), \( \xi(t) \) denotes \( \tilde{\psi}^{(n)}(t) \).

To prove the parameter convergence assume that \( \psi(t) \) is persistently exciting that is guaranteed by the excitation of control action \( u(t) \). Conventional conditions of persistent excitation can be found e.g. in (Narendra and Annaswamy, 1989).

Choosing the following Lyapunov function candidate
\[ V = \frac{1}{2} ||\tilde{\varphi}(t)||^2_{\Gamma^{-1}}, \]
and evaluating its derivative along the solutions of (14), (15) one gets
\[ \dot{V}(t) = -\tilde{\varphi}^T(t) \tilde{\varphi}(t) \tilde{\varphi}^T(t) \theta^* \]
\[ + \tilde{\varphi}^T(t) \tilde{\varphi}(t) \tilde{\varphi}^T(t) \theta^* - ||\tilde{\varphi}(t)||^2_{\Gamma^{-1}(t) - I/k_0}. \]

As was shown by Bartolini, et al. (1995), under the condition of persistent excitation there exists \( \lambda \) such that \((\Gamma^{-1}(t) - I/k_0) \geq \lambda \Gamma^{-1}(t) \) and \( \dot{V}(t) \leq -2\lambda V(t) \). This in turn implies that
\[ \frac{1}{k_0} ||\tilde{\varphi}(t)||^2 \leq ||\tilde{\varphi}(t)||^2_{\Gamma^{-1}(t)} e^{-2\lambda t}. \quad (16) \]

Hence the adjustable parameters converge exponentially to their true values provided control \( u(t) \) is persistently exciting. By using estimates \( \hat{\theta}(t) \) determined by (8), (14) one can easily calculate \( \Theta(t) \) in (7) in order that aim (3) be achieved. One next step is controller design.

### 4. THE CONTROLLER DESIGN

Design control law the sliding mode on the surface \( s = y - y_f = 0 \) is organized.

At first present the error model by using (10)
\[ \dot{s}(t) = c_1 \dot{x}_1(t) + c_2 \dot{x}_2(t) - \dot{y}_f(t) = \]
\[ = c_1 A_{11} \dot{x}_1(t) + c_1 A_{12} \dot{x}_2(t) + c_2 A_{21} \dot{x}_1(t) + c_2 A_{22} \dot{x}_2(t) + c_2 b u(t) - \dot{y}_f(t), \quad (17) \]
Taking into account that
\[ x_2(t) = \frac{1}{c_2} \left( s(t) + y_f(t) - c_1 x_1(t) \right) \] (18)

after substituting it in (17) follows
\[ (c_2 b)^{-1} \dot{s}(t) = L x_1(t) + a_1 s(t) + a_1 y_f(t) - \]
\[ - (c_2 b)^{-1} y_f(t) + u(t) \] ,
(19)

where \( L \) is a \( (N-1) \times (N-1) \) vector,
\[ L = (c_2 b)^{-1} \left( c_1 A_{11} + c_2 A_{21} - \frac{c_1 A_{12} + c_2 A_{22}}{c_2} \right) \] ,
\[ a_1 = \frac{1}{c_2 (c_2 b)} (c_1 A_{12} + c_2 A_{22}) \] . Now present the error model for \( x_1(t) \). Substituting (18) in (10) yields
\[ \dot{x_1}(t) = A_s x_1(t) - \frac{A_{12}}{c_2} s(t) + \frac{A_{12}}{c_2} y_f(t) \] ,
(20)

where \( A_s = A_{11} - A_{12} c_1 / c_2 \). Equations (7), (19), (20) describe the error model. Its description is completed by using HMF property of the system. This implies that \( A_s \) is a Hurwitz matrix and \( c_2 b > 0 \). It is important to remark that \( y_f(t) \) is bounded \( |y_f(t)| \leq \bar{y}_f \) since \( A_f \) is a Hurwitz matrix and \( r(t) \), \( \theta_f(t) \) are bounded.

Choose now the control action as
\[ u(t) = -k, s(t) - \gamma \cdot \text{sign}(s(t)) \] ,

where positive parameters \( k, \gamma \) are specified below.

The stability of the system can be studied by means of the Lyapunov functions (Stotsky, 1994) of two functions
\[ V_1 = \frac{1}{2} (c b)^{-1} \dot{s}^2 + \frac{1}{2} x_1^T P x_1 \]

and
\[ V_2 = \frac{1}{2} (c b)^{-1} s^2 \] .

Evaluating \( \dot{V}_1(t) \) one has
\[ \dot{V}_1(t) = x_1^T(t) P \left( A_s x_1(t) + \frac{A_{12}}{c_2} (s(t) + y_f(t)) \right) + \]
\[ s(t) \left( L x_1(t) + a_1 s(t) + a_1 y_f(t) - \right) - \]
\[ (c_2 b)^{-1} y_f(t) - k, s(t) - \gamma \cdot \text{sign}(s(t)) \] ,

where
\[ PA_s + A_s^T P = -Q, \quad Q = Q^T > 0 \] .

It gives
\[ \dot{V}_1(t) \leq -\frac{1}{2} x_1^T(t) Q x_1(t) + \]
\[ + \frac{|P|}{|c_2|} \frac{|A_{12}|}{|c_2|} |x_1(t)||s(t)| + \]
\[ + \frac{|P|}{|c_2|} \frac{|A_{12}|}{|c_2|} |x_1(t)||\bar{y}_f(t)| + \]
\[ + |L| \frac{|c_1|}{c_2} |s(t)||s(t)| + s(t)^2 \left( \bar{a}_1 - k, \right) + \]
\[ + |s(t)| \left( \bar{a}_1 \bar{y}_f(t) + (c_2 b)^{-1} |\bar{y}_f(t)| - \gamma \right) \]
\[ \leq -\frac{\lambda_{\min}(Q)}{2} |x_1(t)|^2 \right) + \left( \bar{b} + \frac{i}{2} \right) \frac{\lambda}{2} |x_1|^2 + \]
\[ + \frac{\bar{b}}{2 \lambda} s(t)^2 + \]
\[ + \frac{\lambda}{2} |x_1(t)|^2 + \frac{\bar{b}}{2 \lambda} \bar{y}_f^2 + s(t)^2 \left( \bar{a}_1 - k, \right) + \]
\[ + |s(t)| \left( \bar{a}_1 \bar{y}_f + (c_2 b)^{-1} |\bar{y}_f(t)| - \gamma \right) \] ,

where \( \bar{b} \geq |P| \frac{|A_{12}|}{|c_2|}, \gamma \geq |L|, \bar{a}_1 > |a_1| \) and \( \lambda \) is sufficiently small number. Notice, that for any \( \lambda_{\min}(Q) \) there always exists \( \lambda > 0 \) such that \( \lambda_{\min}(Q) \geq \left( \bar{b} + \frac{i}{2} \right) \cdot \lambda \). Then
\[ \dot{V}_1(t) \leq -\frac{\lambda_{\min}(Q)}{4} |x_1(t)|^2 + s(t)^2 \left( \bar{a}_1 + \frac{\bar{b} + i}{2 \lambda} - k, \right) + \]
\[ + |s(t)| \left( \bar{a}_1 \bar{y}_f + c |\bar{y}_f(t)| - \gamma \right) + \frac{\bar{b}}{2 \lambda} \bar{y}_f^2 \] ,

where \( c \geq (c_2 b)^{-1} \). Choose \( k, \gamma \) Let
\[ k, \gamma \geq a_1 + \frac{\bar{b} + i}{2 \lambda} + \frac{\kappa}{2} \] ,
where \( \kappa > 0 \), \( \gamma_0 > 0 \) to be specified below. Then
\[ \dot{V}_1(t) \leq -\frac{1}{2} x_1^T(t) P x_1(t) \cdot \frac{\lambda_{\min}(Q)}{2 \lambda_{\max}(P)} - \]
\[ - \kappa \frac{1}{2} (c b)^{-1} s(t)^2 + \frac{\bar{b}}{2 \lambda} \bar{y}_f^2 \] ,

and
\[ \dot{V}_1(t) \leq -\kappa; V_1(t) + \frac{\bar{b}}{2 \lambda} \bar{y}_f^2 \] ,
\[ \kappa = \min \left( \frac{\lambda_{\min}(Q)}{2 \lambda_{\max}(P)}, \kappa \right) \] ,

where
\[ V_1(0) = \frac{1}{2} (c b)^{-1} s(0)^2 + \frac{1}{2} x_1^T(0) P x_1(0) \] .
Hence the following bound is true \( ||x_1(t)||^2 \leq \bar{x}_1 \), where

\[
\bar{x}_1 = \frac{1}{\lambda_{\min}(\hat{P})} \left( (cb)^{-1}(s(0)^2 + \bar{x}_1^2 (0)P \bar{x}_1(0)) + \frac{\hat{p}}{2\lambda_{\min}(\hat{P})\lambda_{\bar{v}}^2} \right).
\]

(21)

It remains to prove the convergence of \( s(t) \) in a finite time.

Taking

\[
\dot{V}_2(t) = \frac{1}{2} (cb)^{-1} s(t)^2
\]

and evaluating \( \dot{V}_2(t) \) along (19) one obtains

\[
\dot{V}_2(t) \leq |s(t)| \cdot (|L||\bar{x}_1|| + \bar{x}_1 \bar{v}_1(t) + \bar{c}\bar{v}_1(t) - \bar{a}_1 \bar{v}_1(t) - \gamma_0).
\]

Choosing \( \gamma_0 \geq \bar{x}_1 + \bar{c}/2 \), where \( \bar{x}_1 \) is upper bound of \( \bar{x}_1 \) presented by (21) \( \bar{x}_1 \leq x_1 \), \( \beta > 0 \), one gets

\[
\dot{V}_2(t) \leq -\beta \sqrt{V_2(t)}.
\]

This, in turn, implies that for all \( t > t^* \) sliding mode appears on the surface \( s = 0 \), \( t^* = 2\sqrt{V_2(0)/\beta} \).

Now let us prove the achievement of the control aim. Since \( s(t) \equiv 0 \) one has \( y(t) \equiv y_1(t) \) and one can use plant parameter estimates \( \check{\theta}(t) \) for prefilter (7) adjustment. Define \( \check{\Theta}(t) \) in (7) in accordance with (8), where are used estimates instead of true plant parameters. Finally one obtains following equation for \( \check{\Theta}(t) = [\check{\theta}_1(t), \ldots, \check{\theta}_N(t)] \)

\[
\sum_{i=1}^{N} \check{\theta}_i s^{N-i} = \check{K}(t)(\check{A}_p(s, t)B(s) + A'(s) \check{B}_p(s, t)),
\]

(22)

where \( \check{K}(t) = A_m(0)/\hat{B}(0, t) \equiv A_m(0)/\check{\theta}_{n+m+1}(t) \), \( \check{A}(s, t) = s^n - \check{\theta}_1(t) s^{n-1} - \ldots - \check{\theta}_n(t) \), \( \check{B}(s, t) = \check{\theta}_{n+1}(t) s^n + \ldots + \check{\theta}_{n+m+1}(t) \), \( A'(s) \), \( B'(s) \) are given in (7). Collecting similar terms in (22) one gets \( \check{\Theta}(t) \) in the explicit form. From parameter estimations convergence to their true values \( \theta^* \) follows that prefilter adjustable parameters error \( \check{\Theta}(t) = \check{\Theta}(t) - \theta^* \to 0 \) when \( t \to 0 \). Hence one can represent \( y_1(t) \) as \( y_1(t) = y_1^*(t) + \check{y}_1(t) \), where \( y_1^*(t) = \theta^* x_1(t) \) is an ideal prefilter output (taken from (6)) and the bounded vanishing error signal \( \check{y}_1(t) \) may be considered as addition to \( y_1(t) \) disturbance. Since the sliding-mode system was proved to be stable, \( y_1(t) \) will coincide with its ideal trajectory. Therefore finite-time sliding mode realization and exponential convergence of parameter estimates provide the control aim (3) achievement.

5. CONCLUSION

In this paper a simplified adaptive control scheme is presented for regulation of unstable and nonminimum-phase uncertain plants by means of parallel feedforward compensator (shunt) which ensures hyper-minimum phase property of augmented plant.

The proposed controller ensures finite time convergence of augmented error omitting to relay term in control law and exponential convergence of the parameter error under the condition of persistent excitation. This, in turn allows to achieve the desired dynamics to the true plant output.

REFERENCES


APPENDIX

Proof of the Theorem 1. To prove this theorem the following lemma is needed

Lemma. Let \( a(s), b(s), \gamma(s), \delta(s) \) be polynomials with positive leading coefficients \( a_0, b_0, \gamma_0, \delta_0 \), respectively, \( Q(s) = b(s)\gamma(s) + c(s)\delta(\epsilon) \). Suppose \( b(s), \gamma(s) \) and \( a_0\delta(\epsilon) + b_0\gamma_0 \) are stable polynomials. Then there exists \( \epsilon_0 > 0 \) such that \( Q(s) \) is stable for all positive \( \epsilon < \epsilon_0 \).

Proof. Note that \( \deg(b\gamma) = n - 1 \), \( \deg(Q(s)) = \deg(a\delta) = n + k - 2 \). Hence \( n - 1 \) zeros of \( Q(s) \) tend to zeros of \( b(s)\gamma(s) \) and the rest \( k - 1 \) zeros tend to infinity as \( \epsilon \to 0 \).

To analyze the behavior of \( s_i(\epsilon) \) make a change \( \epsilon = \epsilon_0 + \epsilon \) and denote \( R_\epsilon(\mu) = \epsilon^{-1}Q(\mu/\epsilon) \). Then \( R_\epsilon(\mu) = \epsilon^{-1}b(\mu/\epsilon)\gamma(\mu/\epsilon) + (a_0\mu^n + \epsilon a_1\mu^{n-1} + \ldots + \epsilon^n a_n)\delta(\mu) \) and \( R_\epsilon(\mu) \to b_0\gamma_0\mu^{n-1} + a_0\mu^n(\mu/\epsilon)\delta(\mu) \). So, \( n - 1 \) zeros of \( R_\epsilon(\mu) \) tend to zero while the rest \( k - 1 \) ones tend to zeros of \( b_0\gamma_0 + a_0\mu\delta(\mu) \). Lemma is proved.

Proof of the Theorem 2. Express \( W(s) \) as

\[
W(s) = \frac{B_p(s)(s+\lambda)^{k-1} + A_p(s)\gamma(\epsilon s + 1)\lambda^{k-2}}{A_p(s)(s+\lambda)^{k-1}}.
\]

To apply lemma one needs the polynomial \( b_0 + \kappa \mu(1 + \mu)^{k-2} \) to be stable, where \( b_0 \) denotes the leading coefficient of \( B_p(s) \). Obviously, it will be so at least for sufficiently large \( \kappa \). Theorem is proved.

Proof of the Theorem 2. Rearrange transfer function \( W(s) \) as follows

\[
W(s) = W_p(s) + W_c(s) =
\]

\[
= \frac{B_p(s)(s+\lambda)^{k-1} + \kappa\epsilon(\epsilon s + 1)\lambda^{k-2}}{A_p(s)(s+\lambda)^{k-1}}\frac{A_p(s)}{A_p(s)(s+\lambda)^{k-1}}
\]

\[
= \frac{B_p'(s) + \kappa\epsilon(\epsilon s + 1)\lambda^{k-2}(s^n + a_1s^{n-1} + \ldots + a_n)}{A_p(s)(s+\lambda)^{k-1}}
\]

\[
= \frac{\mu b_0\epsilon s^{n-1} + \mu(b_1 s^{n-2} + \ldots + b_{n-1}) + \overline{A}(s)}{\mu A_p(s)(s+\lambda)^{k-1}}
\]

where \( \mu = 1/\kappa \), \( \overline{A}(s) \) is a Hurwitz polynomial,

\[
\overline{A}(s) = a_0 s^n + a_1 s^{n-1} + \ldots a_n;
\]

\[
B'(s) = [b_0 s^n + \mu b_1 s^{n-1} + \ldots + b_{n-1}] + \lambda b_n
\]

\[
= b_1 s^{n-k-1} + \ldots + b_{n-1} + b_0, b_1 = b_1 + \lambda b_0, \ldots, b_{n-1} = b_{n-1} + \lambda b_{n-2}.
\]

Thus

\[
W(s) = \frac{\mu A_p(s)(s+\lambda)^{k-1}}{A_p(s)(s+\lambda)^{k-1}}
\]

where \( F(s) = \overline{A}_0 s^{n+k-2} + \ldots + \overline{A}_{n-2} s^n + (\overline{A}_{n-1} + \mu b_{n-1}) s^{n-1} + \ldots + \overline{A}_{n+k-2} + \mu b_{n-k} \). Therefore the numerator \( F(s) \) of \( W(s) \) will be Hurwitz for sufficiently small \( \mu > 0 \). Theorem is proved.