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Necessary and Sufficient Conditions for the Passivicability of Linear Distributed Systems[1]

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Abstract—For a wide class of systems, as is known, hyper-minimal phase is necessary and sufficient for the strict passivicability of a system. This class contains both concentrated- and distributed-parameter systems, including parabolic equations that describe heat-exchange and diffusion processes. Our results are applicable to finite-dimensional input and output spaces, which are important for application and cover systems with different numbers of inputs and outputs for which passivity is superseded by the G-passivity of some rectangular matrix G. An example of a diffusion-type one-dimensional partial differential equation directly containing control is given. Proofs are based on the infinite-dimensional variant of the Yakubovich–Kalman lemma and Nefedov–Sholokhovich exponential stabilization theorem.

1. INTRODUCTION

Passivity-based methods of system analysis were developed in the sixties of the last century by V.M. Popov [1], G. Zames [2], and others. Conditions for the applicability of these methods for linear systems are formulated in the classical Yakubovich–Kalman–Popov (frequency) theorem [3, 4], which asserts conditions for the existence of a quadratic storage function (analog of the Lyapunov function) in the form of frequency inequalities (the SPR-property). The existence of a feedback that transforms a linear system into a strictly positive real system, i.e., passivication in modern terminology, is studied in [5, 6]. As has been found, a necessary and sufficient condition for the strict passivicability of a system is its strongly minimal phase—the so-called *hyper-minimal phase*.

The simplicity of the conditions for the solvability of the problem opens a way for designing effective methods of synthesizing systems. The passivity- and passivication-based methods of designing linear and nonlinear control systems have subsequently found wide application [7–12] and the passivicability conditions were discovered by many authors [5, 6] (see [13, 14] and the list of references in [15]). The passivication method is advantageous, because there is no need to compute the Lyapunov function explicitly for synthesizing and studying a system. It is worthwhile to extend this method to a wider class of control systems.

In [16], sufficient conditions for the solvability of operator inequalities defining the existence of a quadratic Lyapunov functional for a linear distributed system are formulated. The system is described by differential equations in a Banach space of unbounded operators that generate a continuous semigroup. The solvability conditions [16] are similar to the hyper-minimal phase conditions for the finite-dimensional case. However, unlike in the finite-dimensional case, the necessity of these conditions is still an open question.

In this paper, for a slightly more general case than that of [16], we prove that strictly minimal phase is necessary and sufficient for the existence of solutions to operator inequalities defining the

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existence of a quadratic Lyapunov functional. The results are formulated in terms of passivity and passivication in a convenient form for application to designing distributed control systems. Our results pertain to finite-dimensional input and output spaces, which are important in application. The final solution to the problem of strict passivicability is obtained for a wide class of systems.

An example on a diffusion-type one-dimensional partial differential equation containing control is given to illustrate our results.

All proofs are based on the infinite-dimensional variant of the Yakubovich–Kalman lemma and Nefedov–Sholokhovich exponential stabilization theorem and are given in the Appendix.

2. FORMULATION OF THE PROBLEM

Let us consider a class of objects described by the differential equations

$$\dot{x} = Ax + Bu, \quad y = Cx \tag{1}$$

in the Hilbert state space $X = \{x\}$. Although the results are formulated for a Hilbert state space, many of our results also hold for more general objects described by differential equations in a Banach space. Therefore, we state the formulations for Banach spaces wherever possible. We assume that control and output take finite-dimensional values, i.e., $u(t) \in R^m$ and $y(t) \in R^l$, where R^n is an n-dimensional Euclidean space. The linear operators B and C are assumed to be bounded: $B \in \mathcal{L}(R^m \to X)$ and $C^* \in \mathcal{L}(X \to R^m)$, where $\mathcal{L}(X \to Y)$ denotes the set of linear bounded operators acting from the Banach space X to the Banach space Y. The linear operator A, in general, is unbounded. We assume that the operator A generates a semigroup $U(t)$ of the class C_0. Therefore, for every $t \in (0, \infty)$, the operator $U(t) \in \mathcal{L}(X \to X)$ is defined such that

$$U(t+s) = U(t)U(s) \ \ \forall\, t \in (0,\infty) \ \ \forall\, s \in (0,\infty), \quad \lim_{t\to 0} U(t)x = x \ \ \forall\, x \in X,$$

and $\lim\limits_{\varepsilon\to 0} \varepsilon^{-1}(U(\varepsilon)x - x) = Ax$ for all x for which the limit exists (this set, denoted by $D(A)$, is dense in the Banach space X).

The solution of Eq. (1) is interpreted in the sense of the semigroup $U(t)$: a function $y(\cdot)$ is taken to be the solution if the equalities

$$x(t) = U(t)x(0) + \int_0^t U(t-s)Bu(s)\, ds \quad \text{and} \quad y(t) = Cx(t)$$

hold for all t for some function $x(\cdot)$ with values in the Banach space X.

Definition 1. A system (1) is said to be G-passive if there exists a self-adjoint positive operator $H \in \mathcal{L}(X \to X)$ such that

$$V(x(t)) \leq V(x(0)) + \int_0^t \left[u(s)^* G y(s) - \rho |x(s)|^2\right] ds \tag{2}$$

for $\rho = 0$ for the quadratic form $V(x) = x^* H x$ on the solutions of Eq. (1).

If inequality (2) holds for a positive ρ, then system (1) is said to be strictly G-passive.

Inequality (2), which contains the solution of Eq. (1) with an arbitrary locally summable function $u(\cdot)$ in the right side, is equivalent to the inequality for quadratic forms

$$2x^* H(Ax + Bu) \leq u^* GCx - \rho |x|^2 \quad \forall\, x \in D(A) \quad \forall\, u \in \mathbb{C}^m \tag{3}$$

or, which is the same thing, two operator relations

$$HA + A^*H < 0 \quad \text{and} \quad HB = (GC)^*. \tag{4}$$

(Operator inequalities are interpreted in the sense of quadratic forms.) Let us add the feedback

$$u = Ky + v \tag{5}$$

to system (1), where $v \in \mathbb{C}^m$ is a new input and K is an $m \times l$ matrix.

Definition 2. A system (1) is said to be G-passivicable (strictly G-passivicable) if there exists an $m \times l$ matrix K such that system (1), (5) is G-passive (strictly G-passive).

This paper is primarily concerned with necessary and sufficient conditions for the strict G-passivicability for some class of systems (1). The main condition for a system to belong to this class is the stabilizability, which is necessary for the passivicability of a system. In this sense, the class of infinite-dimensional system is almost not extendable. The class is almost not extendable, because additional conditions are imposed on the rate of decrement of the frequency characteristic of the system when the frequency tends to infinity. For a finite-dimensional state space of the system, these conditions automatically hold. For an infinite-dimensional space X, these conditions are satisfied, for example, by parabolic-type equations describing heat-exchange processes as well as chemical and nuclear reactions.

The passivicability problem is solved with the use of the degenerate frequency theorem, which states conditions for the solvability of the operator relations (4) in terms of frequency characteristics of system (1). For infinite-dimensional systems, this theorem was first demonstrated only as a sufficient condition. In this paper, we shall show that the frequency conditions for solvability are also necessary (4). Since the degenerate frequency theorem holds only for stable systems (1), we must also investigate the stabilizability of system (1) using dynamic output-to-control feedback. We shall formulate the solution of this problem in the form of necessary and sufficient conditions.

We now describe the class of systems (1). Let system (1), (5) be strictly G-passive for relations (4) of the form

$$H(A + KC) + (A + KC)^*H < 0, \quad HB = (GC)^*, \tag{6}$$

where $H = H^* > 0$. By the first relation, system (1), (5) is exponentially stable for $v(t) \equiv 0$. Consequently, the strict G-passivicable system (1) is exponentially stabilizable by the feedback $u = KCx$ from the state x to the finite-dimensional input u. Hence, without loss of generality, we can require that the system be exponentially stabilizable by the feedback

$$u = Dx, \tag{7}$$

where $D \in \mathcal{L}(X \to R^m)$.

An effective criterion for the exponential stabilizability of system (1) by feedback (7) from the state x to the finite-dimensional control u is given in [17]. To formulate this criterion, we require the following notation: $\sigma(A)$ is the spectrum of the operator A, $\mathrm{sp}(Q)$ is the linear span of the set Q, and $\mathbb{C}_\gamma = \{\lambda : \mathrm{Re}\lambda \geq \gamma\}$ is the extended right half-plane (for $\gamma < 0$).

Theorem (Nefedov–Sholokhovich [17]). *Let an operator A generate a semigroup of the class C^0 in a Banach space $\mathbb{X}$ and let $B \in \mathcal{L}(\mathbb{C}^m \to \mathbb{X})$. Then the following two assertions are equivalent:*

I. *There exists an operator $D \in \mathcal{L}(\mathbb{X} \to \mathbb{C}^m)$ such that $\sigma(A + BD) \subset \mathbb{C}\backslash\mathbb{C}_\gamma$, $\gamma < 0$.*

II. *The spectrum of the operator A in the closed right half-plane C_0 consists of a finite number of eigenvalues of total multiplicity $n < \infty$. The spectral projector $\mathcal{P}$ defined by the integral of the resolvent $(\lambda I - A)^{-1}$ along the contour containing the whole spectrum of A in the closed right half-plane decomposes system* (1) *into an exponentially stable subsystem and a finite-dimensional* (*not exponentially stable*) *completely controllable subsystem:*

$$\frac{dx'}{dt} = A'x' + B'u, \quad x' = \mathcal{P}x \in X', \quad y' = C'x', \tag{8}$$

$$\frac{dx''}{dt} = A''x'' + B''u, \tag{9}$$
$$x'' = (I - \mathcal{P})x \in X'', \; y'' = C''x'',$$
$$y = y' + y'',$$

where $\mathbb{X}$ is the direct sum of the subspaces $X' = \mathcal{P}\mathbb{X} = \mathbb{C}^n$ and $X'' = (I - \mathcal{P})\mathbb{X}$, $A' = A\mathcal{P}$, $A'' = A(I - \mathcal{P})$, $B' = \mathcal{P}B \in \mathcal{L}(\mathbb{C}^m \to \mathbb{C}^n)$, $B'' = (I - \mathcal{P})B$, $C' = C\mathcal{P}$, and $C'' = C(I - \mathcal{P})$,

$$\sigma(A) = \sigma(A') \cup \sigma(A''), \quad \sigma(A') \subset C_0, \quad \sigma(A'') \cap C_\gamma = \varnothing, \tag{10}$$

$$\operatorname{sp}\{B'\mathbb{C}^m, A'B'\mathbb{C}^m, \ldots, (A')^{n-1}B'\mathbb{C}^m\} = X'. \tag{11}$$

By the Nefedov–Sholokhovich theorem, every stabilizable system (1) has a finite-dimensional subsystem (8), which is uniquely defined if its "antistability" condition (10) holds. Hence we can assume that the operator $A' \in \mathcal{L}(\mathbb{C}^n \to \mathbb{C}^n)$ is defined by a matrix in some base of an n-dimensional space X'. Hence the polynomial

$$\delta(\lambda) = \det(\lambda I_n - A') \tag{12}$$

is also uniquely defined and is called the characteristic polynomial of system (1).

Let us consider the class Ξ of systems (1) satisfying the following conditions.

1. System (1) is exponentially stabilizable by some feedback $u = Dx$.
2. The resolvent $(\lambda I - A)^{-1}$ tends to zero as $|\lambda| \to \infty$ and $\lambda \in \mathbb{C}_\gamma$ for some $\gamma < 0$.
3. The operators $CA : \mathbb{X} \to \mathbb{C}^l$ and $AB : \mathbb{C}^m \to \mathbb{X}$ are bounded.

The first condition, as has been stated, is necessary for system (1) to be strictly passivicable. The other two conditions are imposed for technological reasons, since the criterion of strict G-passivicability formulated in the next section thus far does not yield to proof without them. For a finite-dimensional phase space of a system, these conditions automatically hold. For an infinite-dimensional state space, condition 2 holds, for example, for parabolic-type systems. Condition 3 in this case obviously implies that the weight functions corresponding to the operators B and C are sufficiently smooth. The example given in Section 4 pertains precisely to this class.

3. FORMULATION OF THE RESULTS

Let us take an $m \times l$ matrix G and the functions

$$\chi'(\lambda) = C'(\lambda I - A')^{-1}B', \quad \chi''(\lambda) = C''(\lambda I - A'')^{-1}B'',$$
$$\chi(\lambda) = \chi'(\lambda) + \chi''(\lambda) = C(\lambda I - A)^{-1}B, \quad \varphi(\lambda) = \delta(\lambda)\det[G\chi(\lambda)].$$

Definition 3. A system of the class Ξ is said to be of minimal phase if the function $\varphi(\lambda)$ has no zeros in $\mathbb{C}_\gamma$ for some $\gamma < 0$. A minimal phase system is said to be of hyper-minimal phase if $GCB = (GCB)^* > 0$.

By analogy with the finite-dimensional case, an operator $B \in \mathcal{L}(\mathbb{C}^m \to \mathbb{X})$ is said to be of *complete rank* if $Bu = 0$ only for a zero vector u.

Theorem 1. *A system of the class Ξ is strongly G-passivicable if and only if it is of hyper-minimal phase.*

To prove this theorem, we require two assertions, which are of independent interest and known to hold for the finite-dimensional case. The first assertion is the degenerate frequency theorem (the Yakubovich–Kalman lemma).

Theorem 2. *Let A be an exponentially stable operator generating a semigroup in the Hilbert space $\mathbb{X}$, $B \in \mathcal{L}(\mathbb{C}^m \to \mathbb{X})$, and $C \in \mathcal{L}(\mathbb{X} \to \mathbb{C}^m)$. Then for the existence of a self-adjoint positive operator $H : \mathbb{X} \to \mathbb{X}$ and a positive number δ obeying the relations*

$$\operatorname{Re} x^* H A x \leq -\delta |x|^2 \quad \forall\, x \in \mathcal{D}(A), \quad HB = C^*, \tag{13}$$

it is sufficient that

$$\operatorname{Re} \chi(\lambda) > 0 \quad \forall\, \lambda = i\omega, \quad \omega \in (-\infty, \infty), \tag{14}$$

$$\varliminf_{\omega \to \infty} g(\omega)^{-2} \operatorname{Re} \chi(i\omega) > 0, \tag{15}$$

where

$$g(\omega) = \max \left\{ \|(i\omega I - A)^{-1} B\|, \|C(i\omega I - A)^{-1}\| \right\}.$$

But if system (1) *with operator coefficients A, B, and C belongs to the class Ξ and the operator B is of complete rank, then it is necessary that inequalities* (14) *and* (15) *also hold for relations* (13) *to hold. Furthermore, relations* (13) *imply that inequality* (14) *holds not only on the imaginary axis, but also for all $\lambda \in \mathbb{C}_\Gamma$, where $\Gamma < 0$.*

The sufficiency of this theorem is proved in [16], whereas its necessity is demonstrated in the Appendix of this paper.

To demonstrate Theorem 1, we require one more auxiliary assertion concerning the stabilizability of system (1) by output-to-control feedback. Let us consider the dynamic feedback defined by the following differential equations in the Banach space $\mathbb{Z} = \{z\}$ (Hilbert space for our case):

$$\dot{z} = \mathcal{A} z + \mathcal{B} y, \ u = \mathcal{C} z + \mathcal{D} y, \tag{16}$$

where $\mathcal{A}$ is the generating operator of a semigroup of the class C^0, $\mathcal{B} \in \mathcal{L}(\mathbb{C}^l \to \mathbb{Z})$, $\mathcal{C} \in \mathcal{L}(\mathbb{Z} \to \mathbb{C}^m)$, and $\mathcal{D} \in \mathcal{L}(\mathbb{C}^l \to \mathbb{C}^m)$.

Definition 4. A system (1) (of the class Ξ) is dynamically output-stabilizable if there exist operators $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ such that system (1), (16) is exponentially stable.

Definition 5. A system (1) (of the class Ξ) is nondegenerate in the closed right half-plane if there exists a minor $\mu(\lambda)$ of the transfer function $\chi(\lambda)$ such that

$$\lim_{\lambda \to \lambda_0} \delta(\lambda) \mu(\lambda) \neq 0 \tag{17}$$

for every root $\lambda_0 \in \mathbb{C}_0$ of polynomial (12).

Theorem 3. *A system* (1) (*of the class* Ξ) *is dynamically output-stabilizable if and only if it is nondegenerate in the closed right half-plane.*

Corollary. *A dynamically output-stabilizable system is exponentially stable if its transfer function has no poles in the extended right half-plane.*

A particular case of Theorem 3 for a system with scalar input and output is given in [18]. If $l = m = 1$, then (17) takes the form

$$\delta(\lambda) = 0 \ \Rightarrow \beta(\lambda) \neq 0, \tag{18}$$

where $\chi(\lambda) = \beta(\lambda)/\delta(\lambda)$. For a finite-dimensional system, we can take $\delta(\lambda) = \det(\lambda I - A)$. If, additionally, condition (18) is satisfied for any $\lambda \in \mathbb{C}$, then the polynomials $\delta(\lambda)$ and $\beta(\lambda)$ are coprime. If (18) holds for $\mathbb{C}_0$, the greatest common divisor of the polynomials $\delta(\lambda)$ and $\beta(\lambda)$ is a Hurwitz polynomial. In either case, system (1) is output-stabilizable. Thus, Theorem 3 extends the well-known result for a finite-dimensional system to an infinite-dimensional system.

4. AN EXAMPLE

Let us examine the temperature-control problem for a homogeneous rod of unit length described by the parabolic partial differential equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial s^2} + \beta T + b(s)u(t), \quad y(t) = \int_0^1 c(s)T(s,t)\,ds, \tag{19}$$

$$\frac{\partial}{\partial s}T(0,t) = \frac{\partial}{\partial s}T(1,t) = 0, \tag{20}$$

where $\alpha > 0$, the state $T(s,t)$ for every $t \geq 0$ is a function in the space $X = L_2(0,1)$, and the distributed parameters $b(\cdot)$ and $c(\cdot)$ are also functions in the space X. Let us determine the conditions for the system to be passivicable. To apply Theorem 1 in determining the passivicability, we must verify whether system (19), (20) belongs to the class Ξ. The operator A for our case is a second-order differential operator $\alpha\partial^2/\partial s^2 + \beta I$ with domain of definition defined by the boundary conditions (20). To find the resolvent $(\lambda I - A)^{-1}$, we must solve the equation $\lambda z = Az + w$ for the function $z(\cdot)$ for an arbitrary function $w(\cdot) \in L_2(0,1)$. For our operator A, the resolvent is defined by an ordinary differential equation

$$\lambda z(s) = \alpha d^2 z(s)/ds^2 + \beta z(s) + w(s), \quad dz(0)/ds = dz(1)/ds = 0.$$

Solving this equation, we find that the relation $z(\cdot) = (\lambda I - A)^{-1}w(\cdot)$ is equivalent to the equalities

$$z(s) = \frac{[e^{-s\mu} + e^{s\mu}]}{2\alpha[\mu^2][(e^{\mu} - e^{-\mu})/\mu]} \int_0^1 [e^{-p\mu} + e^{p\mu}]w(1-p)\,dp + \int_0^s \left[\frac{e^{-p\mu} - e^{p\mu}}{2\alpha\mu}\right] w(s-p)\,dp,$$

$$\mu^2 = \frac{\lambda - \beta}{\alpha}.$$

Here all functions within square brackets are analytical functions of μ, whose expansions contain only even powers. Therefore, the resolvent degenerates only at the zeros of the function

$$\alpha[\mu^2][(e^{\mu} - e^{-\mu})/\mu] = 2(\lambda - \beta)\frac{\sin(i\sqrt{(\lambda-\beta)/\alpha})}{i\sqrt{(\lambda-\beta)/\alpha}},$$

i.e., at the points $\lambda_k = \beta - \alpha(\pi k)^2$, $k = 0, 1, 2, \ldots$, which form the spectrum of the operator A.

Let us find the conditions under which system (19), (20) belongs to the class Ξ. For this, we must verify three conditions. First, the operators CA and AB must be bounded. For our example, this implies that the functions $b(\cdot)$ and $c(\cdot)$ are twice differentiable and their first derivatives at the boundary points $s = 0$ and $s = 1$ are zero. Second, the resolvent $(\lambda I - A)^{-1}$ must tend to zero as λ tends to infinity inside the right half-plane. This is always true of parabolic equations. This is also implied by the expression for the resolvent. What now remains is to verify the last condition, namely, the stabilizability of the system by a finite-dimensional feedback. Let us apply the Nefedov–Sholokhovich theorem for this purpose.

First let us take $\beta < 0$. Then the spectral expansion (8), (9) for our system is trivial: $\mathcal{P} = 0$, $A = A''$, $\delta(\lambda) = 1$, and we obtain, instead of stabilizability, stability. This completes the proof.

Now let $0 \le \beta < \alpha\pi^2$. Then the spectrum of the operator A contains only one point λ_0 in the right half-plane $\mathrm{Re}\lambda \ge 0$. The operator $\mathcal{P}$ is the residue of the resolvent at the point λ_0. The expression for the resolvent implies that $\mathcal{P}$ is simply an operator of averaging over the rod length. The antistable finite-dimensional subsystem (8) is described by the equation

$$\dot{x}_0 = \lambda_0 x_0 + \overline{b}u,$$

where $\overline{b} = \int_0^1 b(s)\,ds$. Its controllability is defined by the inequality $\overline{b} \ne 0$ and therefore system (19), (20), by the Nefedov–Sholokhovich theorem, is finite-dimensionally stabilizable. Similarly, we can also find the antistable subsystem for large β. But for the time being let us confine ourselves to the case $\beta < \alpha\pi^2$. Therefore, $\delta(\lambda) = 1$ for $\beta < 0$ and $\delta(\lambda) = (\lambda - \lambda_0)$ for $0 \le \beta < \alpha\pi^2$.

Let us compute the transfer function $\chi(\lambda) = C(\lambda I - A)^{-1}B$. For our case,

$$w = Bu \;\Leftrightarrow w(s) = b(s)u, \quad y = Cx \;\Leftrightarrow y = \int_0^1 c(s)x(s)\,ds.$$

Let, for instance, $c(s) = b(s) \equiv c$. Then the expression for the resolvent implies that

$$CB = \int_0^1 c(s)b(s)\,ds = c^2, \quad \chi(\lambda) = c^2/(\lambda - \lambda_0).$$

Therefore, for $\beta < \alpha\pi^2$ and $b(s) = c(s) \equiv c \ne 0$, system (19), (20) not only belongs to the class Ξ, but is also of hyper-minimal phase. By Theorem 1, this is necessary and sufficient for its passivicability.

If the conditions for the passivicability of the system are satisfied, we can design a controller, using the passivication method. For example, Theorem 1 implies that the linear controller $u = Ky$ stabilizes the system for a sufficiently large negative K. But applying the results of [16], we find that the adaptive controller

$$u = Ky, \; dK/dt = -y^2$$

guarantees that $\|T(s,\cdot)\| \in L_2[0,\infty)$ in time under any initial conditions, where

$$\|T(s,t)\|^2 = \int_0^1 |T(s,t)|^2 ds.$$

5. CONCLUSIONS

A class Ξ of infinite-dimensional, in general, control systems with finite-dimensional input and output is determined. These systems satisfy the well-known result for finite-dimensional systems, namely, hyper-minimal phase is necessary and sufficient for the passivicability of a system. This class includes all finite-dimensional linear stationary systems. The class of infinite-dimensional systems also includes parabolic-type systems describing heat-exchange and diffusion processes. The main conditions for a system to belong to the class Ξ are close to the necessary and sufficient conditions for the stabilizability of a system by linear feedback. An example is given to illustrate the verification of hyper-minimal phase conditions for a diffusion-type one-dimensional partial differential equation directly containing control.

APPENDIX

Theorem 1 is demonstrated with the use of Theorems 2 and 3, which, in turn, are demonstrated with the use of certain auxiliary assertions. First, let us write an obvious equality

$$\lambda(\lambda I - A)^{-1} - I = A(\lambda I - A)^{-1} = (\lambda I - A)^{-1}A, \tag{A.1}$$

from which we obtain

$$\lambda\chi(\lambda) - CB = C(\lambda I - A)^{-1}AB = CA(\lambda I - A)^{-1}B, \tag{A.2}$$

$$\lambda C(\lambda I - A)^{-1}AB - CAB = CA(\lambda I - A)^{-1}AB, \tag{A.3}$$

$$\lambda^2\chi(\lambda) - \lambda CB - CAB = CA(\lambda I - A)^{-1}AB, \tag{A.4}$$

if $\lambda \notin \sigma(A)$ and CA and AB are bounded operators.

Proof of Theorem 2. The sufficiency of the frequency inequalities (14) and (15) for the existence of a self-adjoint positive operator H satisfying relations (13) is proved in [16]. We now show the converse is also true of systems (1) belonging to the class Ξ, i.e., relations (13) imply inequalities (14) and (15).

Let the continuous operator S be the square root of H^{-1}. Then $H^{-1} = SS$, $S = S^* > 0$. Let $z = Sx$, $\mathbf{A} = S^{-1}AS$, $\mathbf{B} = S^{-1}B$, and $\mathbf{C} = CS$. If the triple $\{u(\cdot), x(\cdot), y(\cdot)\}$ defines the process of system (1), then the triple $\{u(\cdot), z(\cdot), y(\cdot)\}$ defines the process of an equivalent system

$$\dot{z} = \mathbf{A}z + \mathbf{B}u, \quad y = \mathbf{C}z, \tag{A.5}$$

which has the same transfer function

$$\chi(\lambda) = C(\lambda I - A)^{-1}B = CSS^{-1}(\lambda I - SS^{-1}ASS^{-1})^{-1}SS^{-1}B = \mathbf{C}(\lambda I - \mathbf{A})^{-1}\mathbf{B}.$$

Expressing relations (13) in terms of system (A.5), we obtain

$$\mathbf{A} + \mathbf{A}^* \le -\varepsilon I, \quad \mathbf{B} = \mathbf{C}^*, \tag{A.6}$$

where $\varepsilon = \delta/\|H\| > 0$. Let u be a nonzero m-dimensional vector and let $\lambda = \nu + i\omega$, $\nu > -\varepsilon$. Then

$$\begin{aligned}
\mathrm{Re}\,[u^*\chi(\lambda)u] &= \mathrm{Re}\,[u^*\mathbf{B}^*(\lambda I - \mathbf{A})^{-1}\mathbf{B}u] \\
&= \frac{1}{2}[u^*\mathbf{B}^*(i\omega I + \nu I - \mathbf{A})^{-1}\mathbf{B}u + u^*\mathbf{B}^*(-i\omega I + \nu I - \mathbf{A}^*)^{-1}\mathbf{B}u] \\
&= \frac{1}{2}u^*\mathbf{B}^*[(i\omega I + \nu I - \mathbf{A})^{-1} - (i\omega I - \nu I + \mathbf{A}^*)^{-1}]\mathbf{B}u \\
&= \frac{1}{2}u^*\mathbf{B}^*[(i\omega I - \nu I + \mathbf{A}^*)^{-1}(\mathbf{A} + \mathbf{A}^* - 2\nu I)(i\omega I + \nu I - \mathbf{A})^{-1}]\mathbf{B}u \\
&= -w^*(\mathbf{A} + \mathbf{A}^* - 2\nu I)w \ge (\nu + \varepsilon)|w|^2 \ge 0,
\end{aligned}$$

where $w = (\lambda I - \mathbf{A})^{-1}\mathbf{B}u$. The equality holds only for a zero $|w|$. But if $w = 0$, then $\mathbf{B}u = (\lambda I - \mathbf{A})w = 0$. Hence $u = 0$, contrary to our assertion. Thus, inequality (14) has been demonstrated for all λ of real part greater than $-\varepsilon$.

For inequality (15), let us note that the function $g(\omega)$ in it can be replaced by $|\omega|^{-1}$. Indeed, $(i\omega - A)^{-1} \to 0$ as $\omega \to \infty$ for systems of the class Ξ. Therefore, relation (A.1) implies that

$$\frac{\|B\|}{2|\omega|} < \|(i\omega I - A)^{-1}B\| < \frac{2\|B\|}{|\omega|}, \quad \frac{\|C\|}{2|\omega|} < C\|(i\omega I - A)^{-1}\| < \frac{2\|C\|}{|\omega|}$$

for all sufficiently large $|\omega|$. Therefore, inequality (15) is equivalent to the inequality

$$\varliminf_{\omega\to\infty} \omega^2 \operatorname{Re}\chi(i\omega) > 0, \tag{A.7}$$

which we shall verify with the help of equality (A.4). Applying it to the triple $\mathbf{A}, \mathbf{B}, \mathbf{C} = \mathbf{B}^*$ for $\lambda = i\omega$, for any nonzero m-dimensional vector u we obtain

$$\begin{aligned}\omega^2 \operatorname{Re}[u^*\chi(i\omega)u] &= -\operatorname{Re}[(i\omega^2)u^*\chi(i\omega)u] \\ &= -\operatorname{Re}[i\omega u^*\mathbf{B}^*\mathbf{B}u + u^*\mathbf{B}^*\mathbf{A}\mathbf{B}u^* - u^*\mathbf{B}^*\mathbf{A}(i\omega I - \mathbf{A})^{-1}\mathbf{A}\mathbf{B}u] \\ &= -\frac{1}{2}u^*\mathbf{B}^*(\mathbf{A} + \mathbf{A}^*)\mathbf{B}u + \operatorname{Re}[u^*\mathbf{B}^*\mathbf{A}(i\omega I - \mathbf{A})^{-1}\mathbf{A}\mathbf{B}u].\end{aligned}$$

The first term in the right side of this chain of equalities does not depend on ω and is positive due the complete rank of $\mathbf{B}$ and inequalities (A.6). The second term tends to zero, because

$$\|\mathbf{B}^*\mathbf{A}(i\omega I - \mathbf{A})^{-1}\mathbf{A}\mathbf{B}\| \le \|\mathbf{B}^*\mathbf{A}\|\,\|(i\omega I - \mathbf{A})^{-1}\|\,\|\mathbf{A}\mathbf{B}\| \xrightarrow[|\omega|\to\infty]{} 0$$

since system (A.5) belongs to the class Ξ. Hence we obtain inequality (A.7) (and even with total limit, instead of partial limit). This completes the proof the theorem.

Lemma 1. *Let Ω be a neighborhood of the point $\lambda_0 \in \mathbb{C}$, let $\delta(\lambda)$ be a scalar analytical function on Ω, let $A(\lambda) = (a_{ij}(\lambda))_{i,j=1}^N$ be a matrix analytical function on Ω, let $b(\lambda) = (b_{ij}(\lambda))$ be an $N \times N$ rational matrix, and let $\mu(\lambda)$ be a minor of the matrix $B(\lambda)$. If*

$$\lim_{\lambda\to\lambda_0} \delta(\lambda)\mu(\lambda) = 0 \tag{A.8}$$

for all minors of $B(\lambda)$, then (A.8) *also holds for all minors of the matrix $B(\lambda) + A(\lambda)$.*

Proof. Without loss of generality, let us consider a matrix $A(\lambda)$ with only one nonzero element $a_{i_0j_0}(\lambda)$. Let $\mu_B(\lambda)$ be a minor of $B(\lambda)$ and let $\mu_{A+B}(\lambda)$ be a minor of $B(\lambda) + A(\lambda)$ defined by the same set of columns and rows. If $\mu_{A+B}(\lambda)$ does not depend on $a_{i_0j_0}(\lambda)$, then $\mu_{A+B}(\lambda) = \mu_B(\lambda)$ and equality (A.8) is trivial. In the contrary case, expanding $\mu_{A+B}(\lambda)$ by the elements of the i_0th row, we obtain

$$\mu_{A+B}(\lambda) = \mu_B(\lambda) \pm a_{i_0j_0}\mu'_B,$$

where μ'_B is another minor of $B(\lambda)$. Consequently,

$$\lim_{\lambda\to\lambda_0} \delta(\lambda)\mu_{A+B}(\lambda) = \lim \delta(\lambda)\mu_B(\lambda) \pm \lim a_{i_0j_0} \lim \delta(\lambda)\mu'_B = 0.$$

This completes the proof of the lemma.

Lemma 2. *A system* (1) (*of the class* Ξ) *may be nondegenerate in the closed right half-plane if and only if its finite-dimensional subsystem* (8) *is also nondegenerate in the closed right half-plane.*

Proof. Let system (1) be degenerate, i.e.,

$$\delta(\lambda_0) = 0, \quad \lim_{\lambda \to \lambda_0} \delta(\lambda)\mu(\lambda) = 0$$

at some point $\lambda_0 \in \mathbb{C}_0$ for every minor $\mu(\lambda)$ of the transfer function $\chi(\lambda)$. Then system (8), by Lemma 1, is also degenerate, because the difference between $\chi(\lambda)$ and $\chi'(\lambda)$ is an analytical matrix function $\chi''(\lambda)$ in $\mathbb{C}_0$. For this reason, degeneracy of system (8) implies the degeneracy of system (1). This completes the proof of the lemma.

Proof of Theorem 3. First we prove that nondegeneracy of system (1) is necessary for the system to be output-stabilizable. If system (1) degenerates at some point $\lambda_0 \in \mathbb{C}_0$, then, by Lemma 2, its finite-dimensional subsystem (8) also degenerates at the same point. The nondegeneracy of system (8) by a well-known finite-dimensional theorem (see, for example, Theorem 1.2.4 in [19]) is equivalent its controllability and observability. Consequently, system (8) is nonobservable since it is controllable by definition and Nefedov–Sholokhovich theorem. Therefore, there exist an eigenvalue λ_0 and an eigenvector x_0 such that

$$\operatorname{Re}\lambda_0 \geq 0, \quad x_0 \in X', \quad Ax_0 = A'x_0 = \lambda_0 x_0, \quad Cx_0 = 0.$$

The functions $z(t) \equiv 0$, $u(t) \equiv 0$, $y(t) \equiv 0$, and $x(t) = e^{t\lambda_0}x_0$ define the process of the closed-loop system (1), (16) for any feedback (16). Therefore, system (1) is not stabilizable. This completes the proof of the necessity of nondegeneracy.

To prove the sufficiency, feedback (16) must be expressed such that the closed-loop system is exponentially stable. Let system (1) be nondegenerate in the right half-plane. Then, by Lemma 2, the finite-dimensional subsystem (8) is also nondegenerate, and Theorem 1.2.4 of [19] asserts that system (8) is controllable and observable. Consequently, there exist matrices Φ and Ψ such that the operators (matrices) $A' + \Phi C'$ and $A' + B\Psi$ are Hurwitz.

Let us consider the feedback

$$\dot{z}'' = A''z'' + B''u, \tag{A.9}$$

$$\dot{z}' = A'z' + B'u + \Phi(y - C''z'' - C'z'), \tag{A.10}$$

$$u = \Psi z'. \tag{A.11}$$

Subtracting Eq. (A.9) from (9), we obtain $\dot{\varepsilon}'' = A''\varepsilon''$, where $\varepsilon''(t) = x''(t) - z''(t)$ exponentially tends to zero as $t \to \infty$, because the operator A'' is exponentially stable. Subtracting (8) from (A.10), we find that

$$\varepsilon'(t) = x'(t) - z'(t) \tag{A.12}$$

exponentially tends to zero, because

$$\dot{\varepsilon}' = A'x' - A'z' + \Phi(C'x' + C''x'' - C''z' - C'z'') = (A' + \Phi C'^{*})\varepsilon' + \Phi C''\varepsilon''.$$

Substituting (A.11) and (A.12) into (8), we obtain

$$\frac{dx'}{dt} = A'x' + B'u = A'x' + B'\Psi(x' - \varepsilon') = (A' + B'\Psi)x' - B'\Psi\varepsilon'. \tag{A.13}$$

Consequently, $dx'(t)/dt$ and $x'(t)$ exponentially tend to zero and, therefore, $u(t)$ also exponentially tends to infinity by virtue of (8). Finally, relation (9) guarantees the exponential decay of $|x''|$, because the operator A'' in Eq. (9) is exponentially stable.

Hence the feedback (A.9)–(A.11) guarantees the exponential stability of the closed-loop system. This completes the proof of the theorem.

Proof of Theorem 1. Without loss of generality, we take $m = l$ and $G = I$. If this cannot be done, then we take GC, instead of C.

Let us show that the hyper-minimal-phase system (1) is strongly G-passivicable, i.e., there exists a matrix K such that the system

$$\dot{x} = A_K x + Bu, \quad y = Cx \tag{A.14}$$

is strictly G-passive, where $A_K = A + BKC$. We shall determine the matrix K in the form $K = -kI$, where k is a real number. Let $\chi_K(\lambda) = C(\lambda I - A_K)^{-1}B$. Left- and right-multiplying the obvious identity $(\lambda I - A)^{-1} - (\lambda I - A_K)^{-1} = (\lambda I - A)^{-1}(A - A_K)(\lambda I - A_K)^{-1}$ by C and B, respectively, we obtain $\chi(\lambda) - \chi_K(\lambda) = -\chi(\lambda)K\chi_K(\lambda)$. Hence

$$\chi_K^{-1}(\lambda) - \chi^{-1}(\lambda) = -K = kI. \tag{A.15}$$

Using this equality, we shall show that the operator triple (A_K, B, C) satisfies all conditions of Theorem 2 for any sufficiently large k.

Let us begin with the exponential stability of the operator A_K. Since system (1) is of hyper-minimal phase, the inequalities $\det \chi(\lambda) \neq 0 \ \forall \lambda \in \mathbb{C}_\gamma$ and $CB > \delta I_m > 0$ hold for some $\delta > 0$. Furthermore, the functions $Q(\lambda) = C(\lambda I - A)^{-1}AB$ and $q(\lambda) = CA(\lambda I - A)^{-1}AB$ tend to zero as $|\lambda| \to \infty, \ \lambda \in \mathbb{C}_\gamma$. Indeed, $\|q(\lambda)\| \leq \|CA\|\|(\lambda I - A)^{-1}\|\|AB\|$. Since our system belongs to the class Ξ, the first and last factors are bounded, and the middle term tends to zero. A similar picture holds for $Q(\lambda)$ as well.

The matrix function $\chi(\lambda)^{-1}$ is continuous inside the domain $\mathbb{C}_\gamma$, because $\det \chi(\lambda)$ is nonzero in this domain. Taking $R > 0$, let us partition $\mathbb{C}_\gamma$ into subsets $\overline{\mathbb{C}} = \{\lambda \in \mathbb{C}_\gamma : \|Q(\lambda)\| < \delta/2, \, |\lambda| > R\}$ and $\underline{\mathbb{C}} = \mathbb{C}_\gamma \backslash \overline{\mathbb{C}}$. Let R be sufficiently large such that $\overline{\mathbb{C}}$ does not contain the spectrum of the operator A. Then the functions $q(\lambda)$ and $Q(\lambda)$ will be continuous on $\overline{\mathbb{C}}$. The set $\underline{\mathbb{C}}$ is compact, because $Q(\lambda) \to 0$ as $|\lambda| \to \infty, \ \lambda \in \mathbb{C}_\gamma$. Consequently, $\operatorname{Re} u^*\chi(\lambda)^{-1}u \geq -\underline{k}|u|^2 \ \forall u \in \mathbb{C}^m$ and $\forall \lambda \in \underline{\mathbb{C}}$, where $\underline{k} = \max \|\chi(\lambda)^{-1}\|$ for $\lambda \in \underline{\mathbb{C}}$.

If $\lambda \in \overline{\mathbb{C}}$, then $\|\mu(\lambda)\| < 1/2$, where $\mu(\lambda) = -(CB)^{-1}Q(\lambda)$. Since $(I_m - \mu(\lambda))^{-1} = I_m + \mu(\lambda)\,(I_m - \mu(\lambda))^{-1}$, we have $\|\,(I_m - \mu(\lambda))^{-1}\,\| \leq 2$. Therefore

$$\begin{aligned}
\chi^{-1}(\lambda) &= \left[\lambda^{-1}\left(CB + Q(\lambda)\right)\right]^{-1} = \lambda(CB)^{-1}\left[I_m - \mu(\lambda)\right]^{-1} \\
&= \lambda(CB)^{-1}\left[I_m + \mu(\lambda)[I_m - \mu(\lambda)]^{-1}\right] \\
&= \lambda(CB)^{-1} - (CB)^{-2}[\lambda Q(\lambda)][I_m - \mu(\lambda)]^{-1} \\
&= \lambda(CB)^{-1} - (CB)^{-2}[CAB + q(\lambda)][I_m - \mu(\lambda)]^{-1}.
\end{aligned}$$

Consequently,

$$\begin{aligned}
\operatorname{Re} u^*\chi^{-1}(\lambda)u &= \operatorname{Re} u^*\lambda(CB)^{-1}u - \operatorname{Re} u^*(CB)^{-2}[CAB + q(\lambda)][I_m - \mu(\lambda)]^{-1}u \\
&\geq \frac{\gamma}{\delta}|u|^2 - \frac{2}{\delta^2}(\|CAB\| + \|q(\lambda)\|)|u|^2.
\end{aligned}$$

Hence $\operatorname{Re} u^*\chi(\lambda)^{-1}u \geq -\overline{k}|u|^2 \ \forall \lambda \in \overline{\mathbb{C}}$, where $\overline{k} = -\gamma/\delta - 2(\|CAB\| + k_q)/\delta^2$ and $k_q = \sup \|q(\lambda)\|$ for $\lambda \in \overline{\mathbb{C}}$.

Let $k \geq 1 + \max\{\underline{k}, \overline{k}\}$. Then, relation (A.15) implies that $\operatorname{Re} u^*\chi_K^{-1}(\lambda)u > |u|^2 \ \forall \lambda \in \mathbb{C}_\gamma$ and for all u. This inequality does not hold for a nonzero m-vector v for which $\chi_K^{-1}(\lambda)v = 0$. Therefore, $\det \chi_K^{-1}(\lambda) \neq 0 \ \forall \lambda \in C_\gamma$, i.e., the matrix function $[\chi_K^{-1}(\lambda)]^{-1} = \chi_K(\lambda)$ is continuous in the domain C_γ. This implies the exponential stability of A_K. Indeed, the hyper-minimal-phase system (1) is nondegenerate in the closed right half-plane: the minor $\mu(\lambda)$ can be taken to be $\det \chi(\lambda)$. Then inequality (17) guarantees the absence of zeros for the function $\varphi(\lambda) = \delta(\lambda) \det \chi(\lambda)$ in $\mathbb{C}_\gamma$. By Theorem 3, the nondegeneracy of system (1) implies that some feedback (16) stabilizes the system. Therefore, a similar feedback

$$\dot{z} = \mathcal{A}z + \mathcal{B}y, \quad u = \mathcal{C}z + (\mathcal{D} - K)y$$

stabilizes system (A.14), because the matrices $A = A_0$ and A_K differ precisely by BKC. Applying Theorem 3 once again, we find that system (A.14) is nondegenerate. By the corollary of Theorem 3, this implies the exponential stability of the operator A_K.

Let $w = \chi_K(i\omega)u$. Then $u = \chi_K^{-1}(i\omega)w$ and

$$\begin{aligned}\operatorname{Re} u^* \chi_K(i\omega)u &= \operatorname{Re} w^* \left(\chi_K^{-1}(i\omega)\right)^* \chi_K(i\omega)\chi_K^{-1}(i\omega)w \\ &= \operatorname{Re} w^* \left(\chi_K^{-1}(i\omega)\right)^* w = \operatorname{Re} w^* \chi_K^{-1}(i\omega)w > 0,\end{aligned}$$

i.e., the transfer function $\chi_K(\lambda)$ satisfies the frequency inequality (14).

Finally, relation (A.4) implies that

$$\begin{aligned}&\omega^2 \operatorname{Re} u^* \chi_K(i\omega)u \\ &= -\operatorname{Re} u^*[CA_KB + CA_K(i\omega I - A_K)^{-1}A_KB]u \xrightarrow[\omega\to\infty]{} -\operatorname{Re} u^*C(A - kBC)Bu \\ &= -\operatorname{Re} u^*CABu + ku(CB)^2u.\end{aligned}$$

Since $CB > 0$, for sufficiently large k, we obtain $\lim\limits_{\omega} \omega^2 \operatorname{Re} \chi_K(i\omega) > 0$, which is similar to (A.7) and which for systems of the class Ξ is equivalent to the limiting frequency inequality (15).

Thus, for any sufficiently large k, the operator triple (A_K, B, C) satisfies all conditions of Theorem 2: the operator A_K is exponentially stable and the transfer function satisfies the frequency inequalities (14) and (15). Applying Theorem 2, we find that system (A.14) is strictly passive, i.e., system (1) is strictly passivicable.

We now show that the converse is also true: if system (1) is strictly passivicable, then it is of hyper-minimal phase. The strict passivicability of system (1), by definition, implies the strict passivity of system (A.14) with some matrix K, which now need not necessarily be of the form kI. Hence the operator triple $(A_K = A + BKC, B, C)$ satisfies the relations $HB = C^*$ and $\operatorname{Re} HA_K < 0$ with some bounded operator $H = H^* > 0$. Hence it immediately follows that the triple A_K is exponentially stable and the operator $CB = B^*HB$ is positive. What now remains is to verify that the function $\varphi(\lambda) = \delta(\lambda) \det \chi(\lambda)$ has no zeros in the closed right half-plane $\mathbb{C}_0$.

Let us apply Theorem 2. For the strictly passive system (A.14), it guarantees the inequality $\operatorname{Re} u^* \chi_K(\lambda)u > 0$ for every nonzero m-vector u and every λ in the extended right half-plane $\mathbb{C}_\Gamma$, $\Gamma < 0$. Consequently, $\det \chi_K(\lambda) \neq 0 \ \forall \lambda \in \mathbb{C}_\Gamma$ and $\chi_K^{-1}(\lambda)$ is a continuous matrix function in the interior of $\mathbb{C}_\Gamma$. Applying the Schur lemma and the first equality in (A.15), we obtain

$$\begin{aligned}&\det \begin{pmatrix} \lambda I - A' & -B'K \\ -C' & I - \chi''(\lambda)K \end{pmatrix} \\ &= \det(\lambda I - A') \det \left[I - \chi''(\lambda)K - C'(\lambda I - A')^{-1}B'K\right] = \det(\lambda I - A') \det [I - \chi(\lambda)K] \\ &= \det(\lambda I - A') \det \chi(\lambda) \det \left[\chi^{-1}(\lambda) - K\right] = \varphi(\lambda) \det \chi_K^{-1}(\lambda). \qquad \text{(A.16)}\end{aligned}$$

Let $\varphi(\lambda_0) = 0$, $\operatorname{Re} \lambda_0 \geq 0$. Then, by the continuity of $\chi_K^{-1}(\lambda)$, relation (A.16) implies that

$$(\lambda_0 I - A')x_0' - B'Ky_0 = 0, \ -C'x_0' + [I - \chi''(\lambda)K]y_0 = 0 \qquad \text{(A.17)}$$

for some nonzero vectors x' and y_0. The finite-dimensional subsystem (8), by Lemma 2, is nondegenerate. By Theorem 1.2.4 of [19], nondegeneracy is equivalent to simultaneous controllability and observability. Consequently, the vector y_0 cannot vanish, because equalities (A.17) take the form $\lambda_0 x_0' = A' x_0'$, $C' x_0' = 0$, which contradicts the observability assumption.

Let $v(t) = C'' G''(t)(\lambda_0 I - A'')^{-1} B K y_0$, where $G''(t)$ is an exponentially decreasing semigroup (in norm) generated by the exponentially stable operator A'' in Eq. (9). Let us consider the system of equations

$$\begin{gathered}\dot{x}'(t) = A' x'(t) + B' K y(t), \quad x'(0) = x_0', \\ \dot{x}''(t) = A'' x''(t) + B'' K[y(t) + v(t)],\ x''(0) = 0,\ y(t) = C' x'(t) + C'' x''(t).\end{gathered} \tag{A.18}$$

By virtue of (A.17), its solutions are the functions

$$x'(t) = e^{t\lambda_0} x_0', \quad x''(t) = (\lambda_0 I - A'')^{-1} B'' K e^{t\lambda_0} y_0, \quad y = e^{t\lambda_0} y_0.$$

This contradicts the stability of the operator $A_K = A + BKC$, i.e., the stability implied by the passivity of system (A.14).

Thus, the assumption that $\varphi(\lambda_0) = 0$, $\operatorname{Re} \lambda_0 \geq 0$, leads to a contradiction. Hence a strictly passivicable system is always of hyper-minimal phase. This completes the proof of the theorem.

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