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SHUNT OUTPUT FEEDBACK ADAPTIVE CONTROLLERS FOR NONLINEAR PLANTS

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Abstract: The method of parallel feedforward compensator ("shunt") developed earlier for adaptive control of linear plants is extended to a class of nonlinear minimum phase plants. The new output feedback adaptive controllers achieve global stability of the closed loop system for regulation and tracking problems. The total order of the proposed controller is equal to the relative degree of the plant.

Keywords: adaptive control, nonlinear systems, shunt compensation.

1. INTRODUCTION

The problem of adaptive control using plant output measurements attached attention of researchers for more than two decades (see e.g. Astrom and Wittenmark, 1989; Narendra and Annaswamy, 1989; Nikiforov and Fradkov, 1994). Among different existing approaches it is necessary to mention "augmented error signal" (Monopoli, 1974; Feuer and Morse, 1978), high-gain or variable structure observers (Utkin, 1981; Khalil, 1994), parallel feedforward compensators (shunts) (Mareels, 1984; Bar-Kana, 1987; Kaufman, *et al.*, 1994; Iwai and Mizumoto, 1992; Fradkov, 1994). The latter approach provides relatively simple adaptive control laws for high order plants.

Shunt compensation scheme proposed by Mareels (1984) requires r parallel filters of total order as $r(r-1)/2$, where r is relative degree of the plant. Bar-Kana (1987) suggested to use simple 1st order shunt for implicit adaptive control scheme of Sobel, *et al.*, (1982). Bartolini and Ferrara (1992) used the 1st order shunt in adaptive scheme with explicit reference model and demonstrated its applicability for nonminimum-phase linear plants. Iwai and Mizumoto (1992, 1994) suggested design procedure for shunts formed as weighted sum of $r-1$ stable linear systems without zeros. Finally Fradkov (1994) introduced shunt of order $r-1$ with two design parameters and proved that its usage in the adaptive systems with implicit reference model (Fradkov, 1974; Fomin, *et al.*, 1981) allows to stabilize MIMO minimum-phase plants with arbitrary relative degree.

In 90s the nonlinear adaptive control problems were intensively studied (see Krstic, *et al.*, 1995; Marino and Tomei, 1992; Nikiforov, *et al.*, 1996). However in most of solutions required total order of filters exceeds $2n$ or $3n$, where n is the order of plant model. The method of shunt was extended to nonlinear plants by Bar-Kana and Guez (1990), see also (Kaufman, *et al.*, 1994), where the inverse of some stabilizing feedback was suggested to use as a shunt. It guarantees existence of output feedback rendering the augmented plant strictly passive (nonlinear systems with this property are called *feedback strictly passive* (Byrnes, *et al.*, (1991) or *almost strict passive* (Bar-Kana and Guez, 1990) or *passifiable* (Seron, *et al.*, 1994). However to apply the method of Bar-Kana and Guez (1990) it is necessary to find some stabilizing controller which itself may be difficult problem.

The present paper is devoted to extending the shunt method in version of Fradkov (1994) to nonlinear plants. It is shown in section 2 that the wide class of nonlinear minimum-phase plants having relative degree r can be passified and therefore stabilized by shunt of order $r-1$. This result gives possibility of simple adaptive controllers design for regulation and tracking problems considered in the following sections.

2. STABILIZATION OF SISO MINIMUM-PHASE PLANTS

Consider nonlinear affine in control plant model

$$\dot{x} = f(x) + g(x)u, \; y = h(x) \; , \tag{1}$$

where $x \in \mathcal{R}^n$ is state vector, $u \in \mathcal{R}^1$ is input, $y \in \mathcal{R}^1$ is output, f, g, h are smooth functions such that $f(0)=0$, $h(0)=0$, i.e. origin $x=0$ is equilibrium of free system $\dot{x} = f(x)$.

The problem is to stabilize the plant (1) by means of dynamic output feedback

$$u = U(y, x_u), \; \dot{x}_u = \chi(y, x_u) \; , \tag{2}$$

ensuring boundedness of all the trajectories of (1), (2) and achievement of the goal

$$x(t) \to 0, \;\; x_u(t) \to 0, \text{ when } t \to \infty. \tag{3}$$

where $x_u \in \mathcal{R}^{n_u}$ is controller state vector.

Recall that the plant (1) is said to have relative degree r at the open set $\mathcal{D} \subset \mathcal{R}^n$, if for all $x \in \mathcal{D}$ the following conditions are satisfied

$$\begin{aligned} &L_g L_f^k h(x) = 0, \;\; k = 0, 1, \ldots, r-2 \; , \\ &L_g L_f^{r-1} h(x) \neq 0, \end{aligned} \tag{4}$$

where

$$L_\varphi \psi(x) = \sum_{i=1}^{n} \frac{\partial \psi}{\partial x_i} \varphi_i(x)$$

stands for the derivative of function $\psi(x)$ along vector field $\varphi(x)$ (Lie derivative, see (Isidori, 1989)). If system (1) has relative degree r in the open set $\mathcal{D}$, then there exists smooth nonsingular coordinate change $z = \Phi(x)$, $x \in \mathcal{D}$, such that system (1) model has in new coordinates so called Isidori's normal form

$$\begin{aligned} &\dot{z}_i = z_{i+1} \; , \;\; i = 1, \ldots, r-1 \; , \\ &\dot{z}_r = a(z) + b(z)u \; , \\ &\dot{\overline{z}} = q(z) \; , \;\; y = z_1 \; , \end{aligned} \tag{5}$$

where $a(z) = L_f^r h(\Phi^{-1}(z))$, $b(z) = L_g L_f^{r-1} h(\Phi^{-1}(z))$, $\overline{z} = (z_{r+1}, \ldots, z_n) \in \mathcal{R}^{n-r}$. The subsystem

$$\dot{\overline{z}} = q_0(\overline{z}) \; , \tag{6}$$

where $q_0(\overline{z}) = q(0, \ldots, 0, \overline{z})$ describes the motions of (1) consistent with $y(t) \equiv 0$, it is called *zero dynamics* of (1). System (1) is called ***weakly minimum phase*** (resp. ***minimum phase, exponentially minimum phase***), if zero dynamics (6) are Lyapunov stable (resp. asymptotically stable, exponentially stable). Introduce ***shunt system*** (parallel feedforward compensator) as follows

$$(p+1)^{r-1}\eta = \kappa\epsilon(p\epsilon+1)^{r-2} b(z)u \; , \tag{7}$$

where $\kappa > 0$, $\epsilon > 0$, $p = d/dt$ and consider augmented plant model described by equations (1), (7) and output equation

$$y_a = y + \eta \; , \tag{8}$$

Now we are in position to formulate the main result of this section

<u>Theorem 1.</u> Let the system (1) have relative degree r in any bounded set and the following assumptions be valid:

A1. Function $a(z)$ is locally Lipschitz and $a(0) = 0$.

A2. Function $b(z)$ is available for measurement, i.e. $b(z) \equiv b(y)$, and $b(y) \neq 0$ for all $y \in \mathcal{R}^1$.

A3. Function $q(z)$ is locally Lipschitz and plant (1) is exponentially minimum phase.

A4. Function $q(z)$ can be represented in the form

$$q(z) = q(\tilde{z}, \overline{z}) = q_0(\overline{z}) + q_1(\tilde{z}, \overline{z})\tilde{z} \; ,$$

where $\|q_1(\tilde{z}, \overline{z})\| \leq C_\alpha(1 + \|\overline{z}\|)$ for $\|\tilde{z}\| \leq \alpha$, $C_\alpha > 0$.

Then there exist numbers $\kappa > 0$, $\epsilon_0 > 0$ such that for any bounded set $\mathcal{D}_0$ of initial conditions there exists positive nonincreasing functions $K_0(\epsilon)$, $L_0(\epsilon)$ such that for any $\epsilon : 0 < \epsilon < \epsilon_0$ system (1), (7), (8) closed by feedback

$$u = -\frac{1}{b(y)}[K y_a + L \cdot \text{sign } y_a] \tag{9}$$

is asymptotically stable for $K > K_0(\epsilon)$, $L > L_0(\epsilon)$ and the goal (3) is achieved.

It follows from Theorem 1 that the plant (1) with relative degree r satisfying A1–A4 can be asymptotically stabilized by dynamic output feedback (7)–(9) of order $r-1$.

<u>Proof</u>. Suppose with no loss of generality that system (1) already has normal form (5). Fix open bounded set $\mathcal{D}_0 \subset \mathcal{R}^{n+r-1}$ of initial conditions of system (5),

(7)–(9). After change of control variable $\bar{u} = b(y)u$ equations of augmented plant may be rewritten in form

$$\begin{aligned} p^r y &= a(z) + \bar{u} , \\ \dot{\bar{z}} &= q(z) , \\ (p+1)^{r-1} w &= \kappa\epsilon(\epsilon p + 1)^{r-2}\bar{u} . \end{aligned} \quad (10)$$

Therefore the augmented output y_a satisfies equation

$$p^r (p+1)^{r-1} y_a = (p+1)^{r-1} a(z) + G(p)\bar{u} ,$$

where

$$G(\lambda) = (\lambda+1)^{r-1} + \kappa\epsilon(\epsilon\lambda+1)^{r-2}\lambda^r \quad (11)$$

is polynomial of degree $2r-2$ with leading coefficient $g_0 = \kappa\epsilon^{2r-1}$. Since $(\lambda+1)^{r-1}$ and $(\epsilon\lambda+1)^{r-1}$ are Hurwitz polynomials, it can be shown, see (Fradkov, (1994), corollary from Lemma 1) or (Andrievsky, *et al.*, 1996), that there exist number $\kappa > 0$ and function $\epsilon_0(\kappa) > 0$ such that $G(\lambda)$ is Hurwitz polynomial for $\kappa > \kappa_0$, $0 < \epsilon < \epsilon_0(\kappa)$. Pick up such κ and ϵ and introduce function $\bar{a}(t)$ satisfying differential equation

$$G(p)\bar{a}(t) = (p+1)^{r-1} a(z(t)) , \quad (12)$$

where $z(t)$ is taken along solutions of (7)–(10). System (10) may be rewritten as linear system

$$p^r (p+1)^{r-1} y_a = G(p)(\bar{a} + \bar{u}) \quad (13)$$

with new input $\bar{a} + \bar{u}$, having relative degree 1, which is minimum-phase. Therefore it can be represented in special coordinate basis with state vector $\overline{w} = (y_a, \xi)$, $\xi \in \mathcal{R}^{2r-2}$ in normal form, similar to (5):

$$\begin{aligned} \dot{y}_a &= d_1 y_a + \bar{d}^T \xi + g_0(\bar{a} + \bar{u}) , \\ \dot{\xi} &= G\xi + \bar{g} y_a , \end{aligned} \quad (14)$$

where G is $(2r-2) \times (2r-2)$ matrix, $\bar{d}, \bar{g} \in \mathcal{R}^{2r-2}$ and $\det(\lambda I - G) = G(\lambda)/g_0$ (form (14) for MIMO systems was introduced in (Utkin, 1981; Sannuti, 1983)). Finally, represent the second equation of plant (10) in form

$$\dot{\bar{z}} = q_0(\bar{z}) + q_1(\tilde{z}, \bar{z})\tilde{z} , \quad (15)$$

where $\tilde{z} = (z_1, \ldots, z_r) = S\overline{w}$, S is $(2r-2) \times r$ constant matrix; $q_1(\tilde{z}, \bar{z})$ is smooth function, continuous in $\tilde{z} = 0$, $\overline{w} = (y_a, \xi)$. Introduce also state vector of the whole system $w = (\overline{w}, \bar{z})$.

Pick up initial conditions $w_0 = (y_a(0), \xi(0), \bar{z}(0))$ in system (14), (15) from the compact set $\mathcal{D}$ corresponding to above defined set $\mathcal{D}_0$. To demonstrate asymptotic stability of system (9), (14), (15) use Lyapunov function of form

$$V_1(w) = \mu \ln(1 + V_0(\bar{z})) + \xi^T P \xi + y_a^2 , \quad (16)$$

where $\mu > 0$, $V_0(\bar{z})$ is Lyapunov function establishing exponential stability of zero dynamics (6); $P = P^T > 0$ is $(2r-2) \times (2r-2)$ positive definite matrix satisfying $PG + G^T P = -2I_{2r-2}$, I_{2r-2} is identity matrix. Function $V_0(\bar{z})$ satisfies quadratic type inequalities (Krasovskii, 1963; Hahn, 1967)

$$\begin{aligned} \rho_1 \|\bar{z}\|^2 \le V_0(\bar{z}) \le \rho_2 \|\bar{z}\|^2 , \quad \|\nabla V_0(\bar{z})\| \le \rho_3 \|\bar{z}\| , \\ \nabla V_0(\bar{z})^T q_0(\bar{z}) \le -\rho_0 \|\bar{z}\|^2 \end{aligned} \quad (17)$$

with some positive $\rho_0, \ldots, \rho_3$.

Apparently function (16) is positive definite and proper, i.e. set $\overline{\mathcal{D}} = \{w : V_1(w) \le V_0\}$ is compact for all $V_0 \ge 0$. Choose V_0 such that $\overline{\mathcal{D}} \supset \mathcal{D}$ and calculate derivative of function (16)

$$\begin{aligned} \dot{V}_1(w) \le \mu \frac{-\rho_0 \|\bar{z}\|^2 + \rho_3 \|\bar{z}\| \cdot \|q_1(\tilde{z}, \bar{z})\| \cdot \|S\| \cdot \|\overline{w}\|}{1 + V_0(\bar{z})} - \\ -2\|\xi\|^2 + 2\|\xi\| \cdot \|P\bar{g}\| \cdot |y_a| + \\ +2y_a \left[d_1 y_a + \bar{d}\xi + g_0 \bar{a} - g_0 K y_a - g_0 L \operatorname{sign} y_a \right] . \end{aligned}$$

using assumption A4 written in form $\|q_1(\tilde{z}, \bar{z})\|^2 \le C_0(1 + V_0(\bar{z}))$ for some $C_0 > 0$ the inequality (18) can be represented as $\dot{V}_1(w) \le A + B + C$, where

$$\begin{aligned} A &= \mu \frac{-\rho_0 \|\bar{z}\|^2}{1 + V_0(\bar{z})} + \mu \frac{\rho_3 C_0 \|S\| \cdot \|\bar{z}\| \cdot \|\overline{w}\|}{\sqrt{1 + V_0(\bar{z})}} - \\ &\quad -\|\xi\|^2 - |y_a|^2 , \\ B &= -\|\xi\|^2 + 2\|\xi\| \cdot |y_a| (\|P\bar{g}\| + \|\bar{d}\|) - \\ &\quad -(\|P\bar{g}\| + \|\bar{d}\|)^2 |y_a|^2 , \\ C &= y_a^2 [2d_1 + 1 + (\|P\bar{g}\| + \|\bar{d}\|)^2 - g_0 K] + \\ &\quad +2g_0 |y_a| (\bar{a}(t) - L) . \end{aligned}$$

The quantity A is a quadratic form of variables $\|\bar{z}\|/\sqrt{(1 + V_0(\bar{z}))}$ and $\|\overline{w}\|$. Therefore it is negative definite if $4\mu\rho_0 > (\mu\rho_3 C_0 \|S\|)^2$, or if

$$\mu < 4\rho_0 / (\rho_3 C_0 \|S\|)^2 . \quad (18)$$

The quantity B is already nonnegative, and C becomes nonnegative for $K > K_0$, $L > L_0$, where

$$\begin{aligned} K_0 &= [2d_1 + 1 + (\|P\bar{g}\| + \|\bar{d}\|)^2] / g_0 , \\ L_0 &= \sup_t |\bar{a}(t)| . \end{aligned} \quad (19)$$

To ensure that L_0 is finite initial conditions of filter (12) should be taken bounded (e.g. zeros), and boundedness of $a(z)$ for bounded z should be taken into account. Obviously, K_0, L_0 depend on ϵ and functions $K_0(\epsilon)$, $L_0(\epsilon)$ can be made nonincreasing.

We have proved that $\dot{V}_1(w) \le 0$ for $w \in \overline{\mathcal{D}}_0$ and, therefore all the trajectories of system are bounded. Decreasing μ, if necessary, one may ensure that $\dot{V}_1(w) \le$

$-\delta||w||^2$ for some $\delta > 0$. Therefore function $||w(t)||^2$ is integrable and in view of boundedness of trajectories we have $y_a(t) \to 0$, $\xi(t) \to 0$, $\overline{z}(t) \to 0$ when $t \to \infty$. This in turn yields $y(t) \to 0$, $y^{(k)}(t) \to 0$, $k = 0, 1, \ldots, r-1$, that proves theorem.

Remark 1. Assumption A4 overbounds quadratically the rate of growth in $\overline{z}$ of right hand side of the last equation (5). However the knowledge of this bound is not required for control algorithm. Note also that stabilization is global in case when functions $q_1(z)$ and $a(z)$ are bounded since in this case parameter μ of Lyapunov function (16) can be chosen from (18) independently of initial conditions.

Remark 2. The proposed controller is not adaptive and contains four design parameters: gains K, L and parameters of shunt κ, ϵ. The number of parameters may be reduced, since $K = L > \max\{K_0, L_0\}$ may be taken. Moreover for $r = 2$, 3 one may take $\kappa = 1$ as it seen from Hurwitz criterion for $G(\lambda)$.

Remark 3. Extension of Theorem 1 to MIMO plants is straightforward when the plant (1) has uniform relative degree $(r, r, \ldots, r)$.

3. ADAPTIVE STABILIZATION OF MINIMUM-PHASE PLANTS

Although the proposed controller does not need much apriori information about controlled plant, the required information may be further reduced by means of tuning gains K, L. Adaptation algorithm can be derived by speed-gradient method (Fradkov, 1986; Seron, *et al.*, 1994), taking Lyapunov function (16) of nonadaptive system as the goal function:

$$\dot{K} = \gamma_0 y_a^2 \, , \quad \dot{L} = \gamma_1 |y_a| \, , \qquad (20)$$

where $\gamma_0 > 0$, $\gamma_1 > 0$. Standard arguments based on Lyapunov function

$$\begin{aligned} V_2(w, K, L) &= V_1(w) + \frac{1}{2\gamma_0}(K - K_0)^2 + \\ &\quad + \frac{1}{2\gamma_1}(L - L_0)^2 \end{aligned} \qquad (21)$$

show that all the trajectories of system (1), (7)-(9), (12), (20) are bounded and the control objective (3) is achieved.

If additional structural information about plant nonlinearity $a(z)$ is available then the adequate structure of controller is worth using. Suppose for example $a(z)$ has that the following form

$$a(z) = \sum_{i=1}^{l} \theta_i a_i(y) \, , \qquad (22)$$

where $a_i(y)$ are known (measured) functions, θ_i are unknown constant coefficients. Then the structure of adaptive controller can be taken as follows:

$$u = -\frac{1}{b(y)}\left[Ky + \sum_{i=1}^{l} \hat{\theta}_i \overline{a}_i(t)\right] \, , \qquad (23)$$

where $\hat{\theta}_i$ are estimates of unknown parameters θ_i and functions $\overline{a}_i(t)$ are the outputs of l identical filters:

$$G(p)\overline{a}_i(t) = (p+1)^{r-1} a(y(t)) \, . \qquad (24)$$

After applying speed-gradient method with objective function (16) the following adaptation algorithms are obtained

$$\begin{aligned} \dot{K} &= \gamma_0 y_a^2 \, , \; \gamma_0 > 0 \, , \\ \dot{\hat{\theta}}_i &= \gamma_i y_a \overline{a}_i(t) \, , \; \gamma_i > 0, \; i = 1, \ldots, l \, . \end{aligned} \qquad (25)$$

Theorem 2. Let the conditions of Theorem 1 be valid and plant model (5) have structure (22). Then there exist values of parameters κ, ϵ such that algorithm (7), (8), (23)-(25) ensures boundedness of the trajectories and achievement of the goal (3).

Proof of the theorem is standard. It is based on Lyapunov function

$$\begin{aligned} V_3(w, K, \hat{\theta}) &= V_1(w) + \frac{1}{2\gamma_0}(K - K_0)^2 + \\ &\quad + \sum_{i=1}^{l} \frac{1}{2\gamma_i}(\hat{\theta}_i - \theta_i)^2 \end{aligned} \qquad (26)$$

and on the Theorem 1.

Remark 1. The total order of filters in the controller is $(r-1)\cdot(2l+1)$ while dynamic order of the controller is $(r-1)\cdot(2l+1)+l+1$. Using adaptive controller (23)-(25) instead of (9), (20) allows to avoid discontinuity of controller and to decrease its gain at the price of increasing its complexity. Note also that the values κ, ϵ do not depend on the plant parameters.

Remark 2. Functions $a_i(\cdot)$ in (22) may depend also on time and include not only directly measured quantities but also their derivatives up to the order of relative degree of filters (24), i.e. up to $r-1$. Therefore any linear function of $y^{(i)}$, $i = 1, \ldots, r-1$ may be added to (22).

Remark 3. More general finite-differential form of adaptation algorithms also can be used:

$$\begin{aligned} d[K + \Psi_0(y_a)]/dt &= \gamma_0 y_a^2 \, , \\ d[\hat{\theta}_i + \Psi_i(y_a)]/dt &= \gamma_i y_a \overline{a}_i(t) \, , \end{aligned} \qquad (27)$$

where $\Psi_i(y_a)$, $i = 1, \ldots, l$ are arbitrary piecevise continuous functions satisfying $\Psi_i(y_a) y_a \geq 0$. The same

Lyapunov function (26) can be used for stability proof, see (Fradkov, 1986). Finite terms $\Psi_i(y_a)$ help to improve transient performance of the system. Special cases of algorithm (27) are proportional-integral algorithm ($\Psi_i(y_a) = \beta_i y_a$, see (Kaufman, *et al.*, 1994)) and relay-integral algorithm ($\Psi_i(y_a) = \beta_i \cdot \text{sign}\, y_a$.) Solutions to the arising discontinuous differential equations may be understood in Filippov's sense.

4. TIME-VARIANT PLANTS AND TRACKING

All the above designs apply to the time-variant plant if the plant model can be transformed to the normal form, similar to (5):

$$\begin{aligned} \dot{z}_i &= z_{i+1} \ , \quad i = 1, \dots, r-1 \ , \\ \dot{z}_r &= a(z,t) + b(z,t)u \ , \\ \dot{\bar{z}} &= q(z,t) \ . \end{aligned} \qquad (28)$$

Assumptions A1–A4 should be replaced by the following ones:

A1'. Function $a(z,t)$ is smooth and locally bounded together with first partial derivatives uniformly in

$$t \geq 0; \; a(0, \dots, 0, \bar{z}, t) \equiv 0 \ .$$

A2'. $b(z,t) \equiv b(y,t)$ and there exists $\delta > 0$ such that $|b(y,t)| \geq \delta$ for all y, t .

A3'. Function $q(z,t)$ is smooth, $q(0,0) \equiv 0$ and system $\dot{\bar{z}} = q(0, \dots, 0, \bar{z}, t)$ is exponentially stable, i.e. there exists Lyapunov function $V_0(\bar{z}, t)$, satisfying standard quadratic-type inequalities, see (Krasovskii, 1963; Hahn, 1967).

A4'. Function $q(z,t)$ can be represented in the form

$$q(\tilde{z}, \bar{z}, t) = q_0(\bar{z}, t) + q_1(\tilde{z}, \bar{z}, t)\tilde{z} \ ,$$

where $||q_1(\tilde{z}, \bar{z}, t)|| \leq C_\alpha(1 + ||\bar{z}||)$

for $||\tilde{z}|| \leq \alpha$, $t \geq 0$.

Theorems 1,2 hold true for plant (28) under assumptions A1'–A4'. It allows to solve problem of tracking where the goal (3) is replaced by the goal

$$e(t) \to 0 \quad \text{as} \quad t \to \infty \ , \qquad (29)$$

where $e(t) = y(t) - y_d(t)$, $y_d(t)$ is desired trajectory of the plant output. To reduce this problem to the previous one just take $z_1 = e(t)$ in (28). Then the highest derivative of command signal $y_d^r(t)$ will appear in the second equation of (28), while augmented error will be $e_a(t) = y(t) - y_d(t) + \eta(t)$.

All proposed algorithms apply with obvious changes and Theorems 1,2 hold true (i.e. the goal (29) is achieved) if the command signal $y_d(t)$ is bounded for $t \geq 0$ together with its derivatives $y_d^{(k)}(t)$, $k = 1, \dots, r$.

5. CONCLUSIONS

The above results introduce and justify a new class of shunt-based adaptive controllers solving output feedback regulation and tracking problems for a wide class of nonlinear minimum-phase plants. The proposed controllers can be used in various application problems (control of robots, oscillatory and chaotic systems, *etc.*), see, *e.g.* (Fradkov, *et al.*, 1995). The results of the paper extend passification approach (Fradkov and Hill, 1993; Seron *et al.*, 1994) to nonlinear systems with arbitrary relative degree.

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