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Exploring nonlinearity by feedback ☆

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Abstract

The possibilities of studying nonlinear behavior of physical systems by small feedback action are discussed. Analytical bounds of possible system energy change by feedback are established. It is shown that for a 1-DOF nonlinear oscillator, the change of energy by feedback can reach the limit achievable for a linear oscillator by a harmonic (non-feedback) action. The results are applied to different physical problems: evaluating the amplitude of action leading to escape from a potential well; stabilizing unstable modes of a nonlinear oscillator (pendulum); using feedback testing signals in spectroscopy. These and related studies are united by similarity of their goals (examination possibilities and limitations for changing a system behavior by feedback) and by unified methodology borrowed from cybernetics (control science). They, therefore, constitute a part of physics which can be called *cybernetical physics*.

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1. Introduction: physics and control

Before a few decades, the interest of the physics community in control theory was not substantial. However, the situation changed dramatically in the 1990s after it was discovered that even small feedback introduced into a chaotic system can change its behavior significantly, e.g. turn the chaotic motion into the periodic one [1]. The seminal paper [1] gave rise to an avalanche of publications demonstrating metamorphoses of numerous systems – both simple and complicated – under action of feedback. However, the potential of modern nonlinear control theory (e.g. [2–4]) still was not seriously exhausted although the key role of nonlinearity was definitely appreciated. On the other hand, new problems are not very traditional for control theorists: the desired point or the desired trajectory of the system is not specified whilst the 'small feedback' requirement is imposed instead. It took some time to realize

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that such kind of problems are typical for control of more general oscillatory behavior and to work out the unified approach to nonlinear control of oscillations and chaos [5].

It needs only one more effort to make the next step and to pose the problem of systematic studying of the properties of physical (as well as chemical, biological, etc.) systems by means of small feedback actions.

The present paper is aimed at demonstrating the very first consequences of such an approach for physics and other natural sciences. First, the general approach to feedback design for oscillatory systems is explained by example of 1-DOF nonlinear oscillator. Then, possible applications to nonlinear spectroscopy escape from a potential well and stabilization of unstable modes are considered. Finally, some mechanism explaining the efficiency of small feedback is discussed and avenues of further research are outlined.

2. Exciting nonlinear oscillator: feedback resonance

Consider the controlled 1-DOF oscillator, modeled after appropriate rescaling by the differential equation

$$\ddot{\varphi} + \Pi(\varphi)' = u, \tag{1}$$

where φ is the phase coordinate, $\Pi(\varphi)$ the potential energy function and u the controlling variable. The state vector of the system (1) is $x = (\varphi, \dot{\varphi})$ and its important characteristic is the total energy $H(\varphi, \dot{\varphi}) = (1/2)\dot{\varphi}^2 + \Pi(\varphi)$. The state vector of the uncontrolled (free) system moves along the energy surface (curve) $H(\varphi, \dot{\varphi}) = H_0$. The behavior of the free system depends on the shape of $\Pi(\varphi)$ and the value of H_0. For example, for a simple pendulum, we have $\Pi(\varphi) = \omega_0^2(1 - \cos\varphi) \geq 0$. Obviously, choosing $H_0 : 0 < H_0 < 2\omega_0^2$, we obtain oscillatory motion with amplitude $\varphi_0 = \arccos(1 - H_0/\omega_0^2)$. For $H_0 = 2\omega_0^2$, the motion along the separatrix including upper equilibrium is observed, while for $H_0 > 2\omega_0^2$, the energy curves become infinite and the system exhibits permanent rotation with average angular velocity $\langle\dot{\varphi}\rangle \approx \sqrt{2H_0}$.

Let us put the question: is it possible to significantly change the energy (i.e. behavior) of the system by means of an arbitrarily small controlling action?

The answer is well known when the potential is quadratic, $\Pi(\varphi) = (1/2)\omega_0^2\varphi^2$, i.e. system dynamics are linear,

$$\ddot{\varphi} + \omega_0^2\varphi = u. \tag{2}$$

In this case, we may use the harmonic external action

$$u(t) = \bar{u}\sin\omega t \tag{3}$$

and for $\omega = \omega_0$, watch the unbounded resonance solution $\varphi(t) = -(\bar{u}t/2\omega_0)\cos\omega_0 t$.

However, for nonlinear oscillators, the resonant motions are more complicated with interchange of energy absorption and emission. It is well known that even in the case of a simple pendulum, the harmonic excitation can give rise to chaotic motions. The reason, roughly speaking, is that the natural frequency of a nonlinear system depends on the amplitude of oscillations.

Therefore, the idea is to create resonance in a nonlinear oscillator by changing the frequency of external action as a function of oscillation amplitude. To implement this idea we need to make $u(t)$ depend on the current measurements $\varphi(t), \dot{\varphi}(t)$, which exactly means introducing a feedback

$$u(t) = U(\varphi(t), \dot{\varphi}(t)). \tag{4}$$

Now the problem is how to find the feedback law (4) in order to achieve the energy surface $H(\varphi, \dot{\varphi}) = H^*$. This problem falls into the field of control theory. To solve it, we suggest to use the so-called speed-gradient (SG) method (see [5–8] and Section 6 below). For the system (1) the SG-method with the choice of the goal function $Q(x) = [H(x) - H^*]^2$ produces simple feedback laws

$$u = -\gamma \left(H - H^*\right) \dot{\varphi}, \tag{5}$$

$$u = -\gamma \operatorname{sign} \left(H - H^*\right) \cdot \operatorname{sign} \dot{\varphi}, \tag{6}$$

where $\gamma > 0$, $\operatorname{sign}(H) = 1$, for $H > 0$, $\operatorname{sign}(H) = -1$ for $H < 0$ and $\operatorname{sign}(0) = 0$. It can be proved (see Section 6 for general statement) that the goal $H(x(t)) \to H^*$ in the systems (1) and (5) (or (1) and (6)) will be achieved from almost all initial conditions provided that the potential $\Pi(\varphi)$ is smooth and its stationary points are isolated. It is worth noting that since the motion of the controlled system belongs to the finite energy layer between H_0 and H^*, the right hand side of Eq. (5) is bounded. Therefore, taking a sufficiently small gain γ, we can achieve the given energy surface $H = H^*$ by means of arbitrarily small control. Of course, this seemingly surprising result holds only for conservative (lossless) systems.

Now, let the losses be taken into account, i.e. the system is modeled as

$$\ddot{\varphi} + \varrho\dot{\varphi} + \Pi(\varphi)' = u, \tag{7}$$

where $\varrho > 0$ is the damping coefficient. Then it is not possible any more to reach an arbitrary level of energy. The lower bound $\bar{H}$ of the energy value reachable by a feedback of amplitude $\bar{u}$ can be calculated as (see Section 6)

$$\bar{H} = \frac{1}{2}\left(\frac{\bar{u}}{\varrho}\right)^2. \tag{8}$$

In order to achieve the energy (8), the parameters of feedback should be chosen properly; namely, parameter values of the algorithm (5) providing energy (8) under restriction $|u(t)| \le \bar{u}$ are as follows: $H^* = 3\bar{H}$, $\gamma = \varrho/2\bar{H}$. For the algorithm (6) any value H^* exceeding $\bar{H}$ is appropriate if $\gamma = \bar{u}$ (see Section 6).

Note that H^* does not have the meaning of the desired energy level in the presence of losses. It leaves some freedom of parameter choice. Exploiting this observation, we may take H^* sufficiently large in the algorithm (6) and arrive at its simplified form

$$u = -\gamma \operatorname{sign} \dot{\varphi}, \tag{9}$$

that looks like introducing negative Coulomb friction into the system.

It is worth comparing the bound (Eq. (8)) energy with the energy level achievable for a linear oscillator

$$\ddot{\varphi} + \varrho\dot{\varphi} + \omega_0^2\varphi = u(t), \tag{10}$$

where $\varrho > 0$ is the damping coefficient, by harmonic (non-feedback) action. The response of the model to the harmonics $u(t) = \bar{u}\sin\omega t$ is also harmonics $\varphi(t) = A\sin(\omega t + \varphi_0)$ with the amplitude

$$A = \frac{\bar{u}}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \varrho^2\omega^2}}. \tag{11}$$

Let ϱ be small, $\varrho^2 < 2\omega_0^2$. Then, A reaches its maximum for resonant frequency: $\omega^2 = \omega_0^2 - \varrho^2/4$, and the system energy averaged over the period is

$$\bar{H} = \frac{1}{2}\left(\frac{\bar{u}}{\varrho}\right)^2 + \mathrm{O}(\varrho^2). \tag{12}$$

Comparison of Eqs. (8) and (12) shows that for a nonlinear oscillator affected by feedback, the change of energy can reach the limit achievable for linear oscillator by harmonic (non-feedback) action, at least in the case of small damping. Therefore, feedback allows a nonlinear oscillator to achieve as deep a resonance as can be achieved by harmonic excitation for the linear case.

Let us consider some possible applications.

3. 'Superoptimal' escape from a potential well

The study of escape from a potential well is important in many fields of physics and mechanics [9,10]. Sometimes escape is an undesirable event and it is important to find conditions preventing it (e.g. buckling of the shells, capsize of the ships, etc.). In other cases, escape is useful and the conditions guaranteeing it are needed. Escape may correspond to a phase transition in the system. In all cases, the conditions of achieving escape by means of as small an external force as possible are of interest.

In [10] such a possibility (optimal escape) has been studied for typical nonlinear oscillators (7) with a single-well potential $\Pi_{\mathrm{e}}(\varphi) = (\varphi^2/2) - (\varphi^3/3)$ (so-called 'escape equation') and a twin-well potential $\Pi_{\mathrm{D}}(\varphi) = (-\varphi^2/2) + (\varphi^4/4)$ (Duffing oscillator). The least amplitude of a harmonic external forcing $u(t) = \bar{u} \sin \omega t$ for which no stable steady state motion exists within the well was determined by intensive computer simulations. For example, for an escape equation with $\varrho = 0.1$, the optimal amplitude was evaluated as $\bar{u} \approx 0.09$, while for the Duffing twin-well equation with $\varrho = 0.25$ the value of amplitude was about $\bar{u} \approx 0.21$. We performed computer simulations for the case of the Duffing oscillator. The results agree with those of [10]. The typical time histories of input and output for $\bar{u} = 0.208$ are shown in Fig. 1. It is seen that escape does not occur.

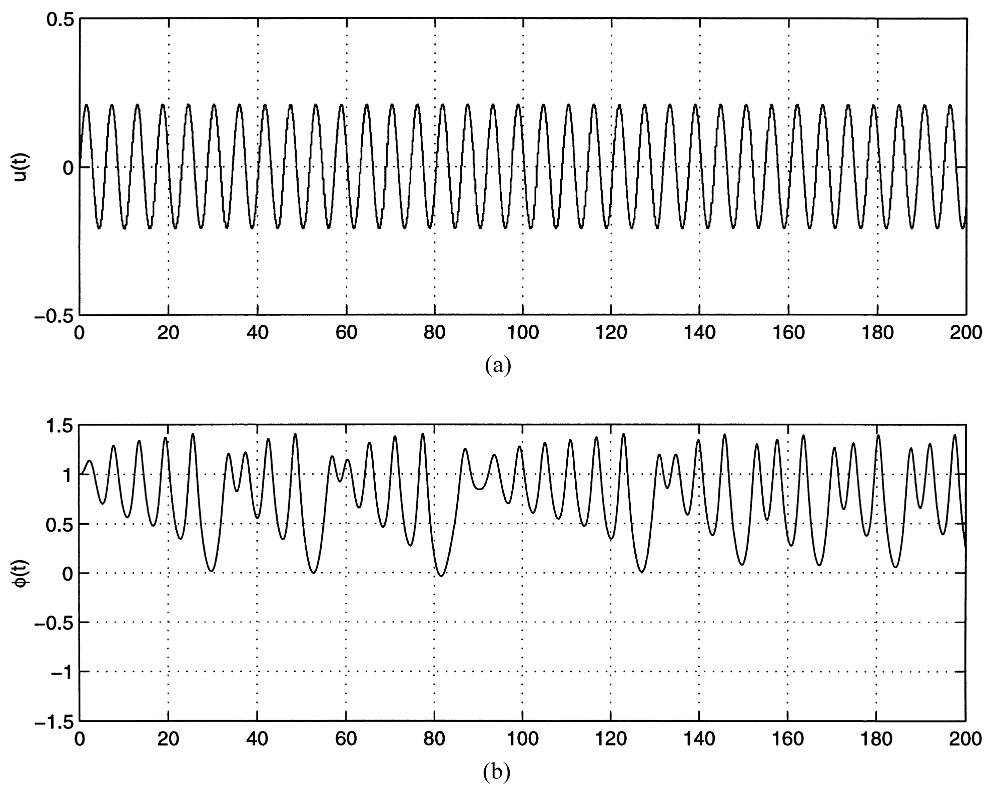

Fig. 1. Dynamics of Duffing oscillator driven by harmonics. (a) Harmonic input $u(t)$; (b) corresponding dynamics of oscillator output $\phi(t)$.

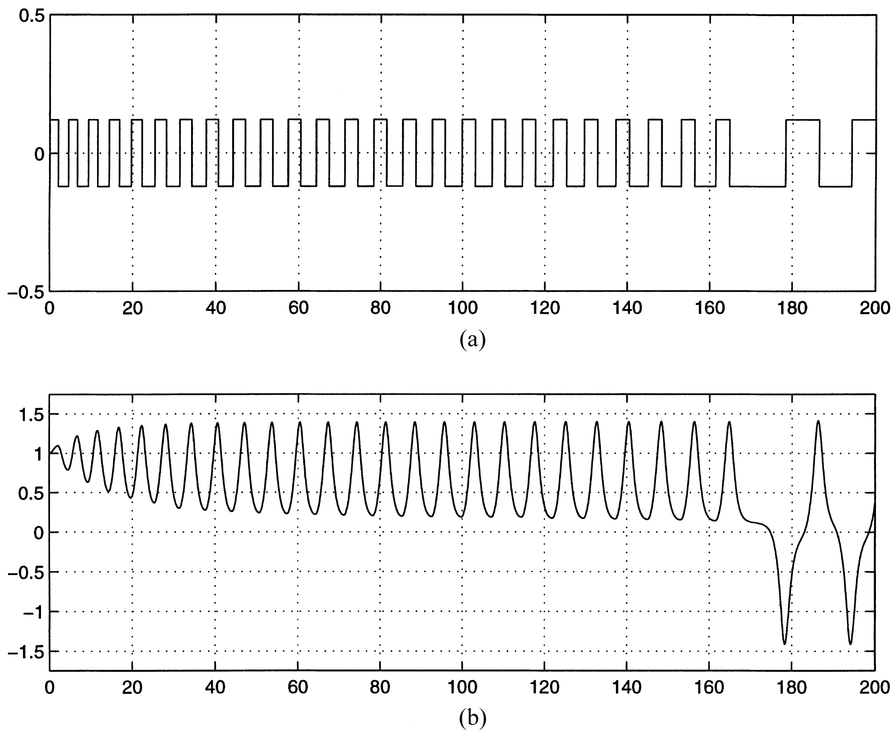

Fig. 2. Dynamics of the Duffing oscillator driven by feedback. (a) Feedback input $u(t)$; (b) corresponding dynamics of oscillator output $\phi(t)$.

Using feedback forcing, we may expect reducing the escape amplitude. In fact using the formula (8), the amplitude of feedback (5) or (6) leading to escape can be easily calculated, by just substituting the height of potential barrier

$$\max_{\Omega} \Pi(\varphi) - \min_{\Omega} \Pi(\varphi)$$

for $\bar{H}$ into Eq. (8) where Ω is the well corresponding to the initial state. For example, in the case of the escape equation $\bar{H} = 1/6$, $\varrho = 0.1$ and $\bar{u} = 0.0577$, while for the Duffing oscillator with $\bar{H} = 1/4$, $\varrho = 0.25$ the escape amplitude is estimated as $\bar{u} = 0.1767$. The obtained values are substantially smaller than those evaluated in [10]. The less the damping, the bigger the difference between the amplitudes of feedback and non-feedback signals leading to escape. Simulation exhibits still stronger difference: escape for the Duffing oscillator occurs for $\bar{u} = \gamma = 0.122$, if the feedback (6) or (9) is applied, see Fig. 2. Note that the oscillations in the feedback systems have both variable frequency and variable shape.

We also studied the dependence of escape amplitudes on the damping by means of computer simulations in the range of damping coefficient ϱ varying from 0.01 to 0.25. Simulations confirmed the theoretical conclusion that the feedback escape amplitude is proportional to the damping (Fig. 3). We may evaluate the efficiency of feedback μ as the ratio of escape amplitudes for harmonic ($\bar{u}_{\mathrm{h}}$) and feedback ($\bar{u}_{\mathrm{f}}$) forcing,

$$\mu = \frac{\bar{u}_{\mathrm{h}}}{\bar{u}_{\mathrm{f}}}. \tag{13}$$

Fig. 4 shows that the efficiency of feedback is inversely proportional to the damping for small values of damping.

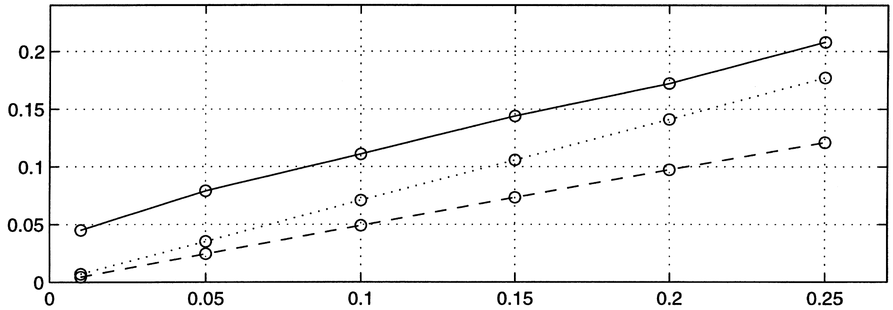

Fig. 3. Dependence of escape amplitudes on damping (solid line – harmonic forcing; dashed line – feedback forcing (simulation); dotted line – feedback forcing (theoretical estimate)).

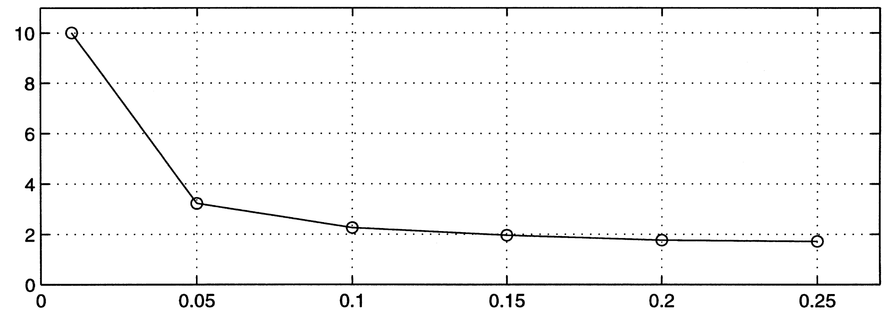

Fig. 4. Efficiency of feedback $\mu = \bar{u}_h/\bar{u}_f$ versus damping ϱ.

4. Stabilization of unstable modes

In the 1940s, Kapitsa surprised his colleagues by an experiment with a rod eccentrically mounted on a horizontal motor shaft. The demonstration showed that the upper unstable equilibrium of the swinging rod (pendulum) can be made stable by sufficiently fast vibrations of the pivot. The experimental results were explained both by Kapitsa himself and by Bogoliubov by means of the method of averaging (history and explanations see, e.g. in [11]). It triggered the development of a new field in mechanics called 'vibrational mechanics' with numerous applications in science and technology [12].

Let us revise Kapitsa's experiment from the feedback point of view. The model of Kapitsa's pendulum differs from the one mentioned in Section 2 in that the controlling action is the vertical acceleration of the pivot rather than the applied torque. The model of the system under control is as follows:

$$\ddot{\varphi} + \omega_0^2 \sin\varphi = bu \sin\varphi, \tag{14}$$

where $b > 0$ is the scaling parameter. Harmonic vibration of the pivot with acceleration $u(t) = \omega^2 \sin\omega t$ for sufficiently large ω leads to swinging it up. Can it also be done by a small feedback?

Applying the speed-gradient method to the system (14) results in feedback laws close to Eqs. (5) and (6),

$$u = -\gamma \left(H - H^*\right) \dot{\varphi} \sin\varphi, \tag{15}$$

$$u = -\gamma \,\mathrm{sign}\left[\left(H - H^*\right) \dot{\varphi} \sin\varphi\right]. \tag{16}$$

It can be shown, (see [13]) that the law (16) provides stabilization of the upper equilibrium $\varphi = \pi$ for arbitrary $\gamma > 0$, if $H^* = 2\omega_0^2$ is the energy corresponding to the upper equilibrium. The initial condition can be taken arbitrarily except the lower equilibrium $\varphi = 0$. Therefore, unlike the non-feedback forcing in Kapitsa's experiment the feedback allows one to stabilize an unstable equilibrium by an arbitrarily small forcing.

Other examples of such an advantageous property of feedback are provided in the literature on control of chaos, where highly unstable orbits are shown to be stabilizable by tiny corrections [1,14,15]. It is also worth noting that the swinging of the pendulum has become a kind of benchmark example in control literature. For example, energy control algorithms for simple (torque driven) pendulum were proposed in [7]. Algorithms related to Eqs. (15) and (16) for the case of horizontally (rather than vertically) driven pendulum were studied in [16,17]. Also, techniques inspired by those developed in [1] were applied to control the parametrically excited pendulum, (see [18] and the references therein). An advantage of the speed-gradient method used in this section is its applicability to more general, higher-dimensional systems (see Section 6 below).

5. Feedback spectroscopy

The conventional spectroscopy is based upon applying a harmonic signal to the physical system under examination. Though the energy eigenvalues in the spectroscopy theory are predicted by quantum mechanical calculations, to explain the dynamics of resonant interaction between radiation and matter, the classical harmonic oscillator model is usually used [19]. Real multi-DOF system has a variety of natural modes with different natural frequencies and different losses. The most interesting are the resonant frequencies, corresponding to small damping which produce the resonant peaks (lines) on the spectrogram. The resonant peaks can be evaluated by scanning over the frequency range of input signal. What is the role of nonlinearity?

The conventional methods treat anharmonicity as perturbation changing resonance conditions for large deviations from equilibrium. As a result, some energy is reflected instead of being absorbed by the system and the energy value (12) cannot be achieved for larger $\bar{u}$.

Let us try feedback. Applying the signal of form (5) and using the nonlinear oscillator model (7), we can achieve the energy level (8) coinciding with Eq. (12). Thus, we get an opportunity of giving full degree of excitation to the system and evaluating its energy absorbing ability at higher energy levels. Since the nonlinearity is essential only for small damping ϱ, i.e. near linear resonances, the 'feedback' spectroscopy techniques should incorporate the conventional ones in order to determine the near-resonant regions and to give initial excitation to the system.

It is important that for excitation, we may use simple feedback (6) which does not require measuring energy and looks just like introduction of a negative damping into the system. Therefore, the obtained resonant energy value does not depend on the shape of potential, i.e. the kind of anharmonicity does not matter.

Of course, the feedback excitation is not easy to implement because it should depend not only on the intensity but also on the phase of the radiation. However, the development of ultrafast controlled lasers [20], growth of the productivity of the computers and increase of speed and accuracy measurements give hope for the experimental verification of the approach. It is already quite realistic for the fields dealing with lower frequencies, e.g. for ultrasonic investigations. Another approach to nonlinear resonance spectroscopy which does not use energy considerations was suggested in [21].

6. Speed-gradient algorithms and energy control

Various algorithms for the control of nonlinear systems were proposed in the literature, see e.g. [2–4,22]. However, the overwhelming part of control theory and technology deals with stabilization of prespecified points and trajectories. For purposes of 'small control' design, the following 'speed-gradient' procedure is convenient [5–8].

Let the controlled system be modeled as

$$\dot{x} = F(x, u), \tag{17}$$

where $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the input (controlling signal). Let the goal of control be expressed as the limit relation

$$Q(x(t)) \to 0 \quad \text{when} \ \ t \to \infty. \tag{18}$$

In order to achieve the goal (18), we may apply the SG-algorithm in the *finite* form

$$u = -\Psi(\nabla_u \dot{Q}(x, u)), \tag{19}$$

where $\dot{Q} = (\partial Q/\partial x)F(x, u)$ is the speed of changing $Q(x(t))$ along the trajectories of Eq. (17), vector $\Psi(z)$ forms a sharp angle with the vector z, i.e. $\Psi(z)^{\mathrm{T}} z > 0$ when $z \neq 0$ (superscript 'T' stands for transpose). The first step of the speed-gradient procedure is to calculate the speed $\dot{Q}$. The second step is to evaluate the gradient $\nabla_u \dot{Q}(x, u)$ with respect to controlling input u. Finally, the vector-function $\Psi(z)$ should be chosen to meet the sharp angle condition. For example, the choice $\Psi(z) = \gamma z, \ \ \gamma > 0$ yields the *proportional* (with respect to speed-gradient) feedback

$$u = -\gamma \nabla_u \dot{Q}(x, u), \tag{20}$$

while the choice $\Psi(z) = \gamma \operatorname{sign} z$, where sign is understood componentwise, yields the *relay* algorithm

$$u = -\gamma \operatorname{sign}(\nabla_u \dot{Q}(x, u)). \tag{21}$$

The *integral* form of the SG-algorithm

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\gamma \nabla_u \dot{Q}(x, u), \tag{22}$$

also can be used as well as combined, e.g. proportional–integral forms.

The underlying idea of the choice (20) is that moving along the antigradient of the speed $\dot{Q}$ provides decrease of $\dot{Q}$. It may eventually lead to negativity of $\dot{Q}$ which, in turn, yields decrease of Q and, eventually, achievement of the primary goal (18). However, to prove Eq. (18) some additional assumptions are needed, see [5–8].

Let us illustrate the derivation of SG-algorithms for the Hamiltonian controlled system of the form

$$\dot{q} = \nabla_p H(q, p) + \nabla_p H_1(q, p)u, \qquad \dot{p} = -\nabla_q H(q, p) - \nabla_q H_1(q, p)u, \tag{23}$$

where $x = (q, p)$ is the $2n$-dimensional state vector, H is the Hamiltonian of the free system, H_1 is the interaction Hamiltonian. In order to control the system to the desired energy level H^*, the energy related goal function $Q(q, p) = (H(q, p) - H^*)^2$ is worth choosing. The first step of speed-gradient design yields

$$\dot{Q} = 2(H - H^*)\dot{H} = 2(H - H^*)[(\nabla_q H)^{\mathrm{T}} \nabla_p H_1 - (\nabla_p H)^{\mathrm{T}} \nabla_q H_1]u = 2(H - H^*)\{H, H_1\}u,$$

where $\{H, H_1\}$ is the Poisson bracket. Since $\dot{Q}$ is linear in u, the second step yields $\nabla_u \dot{Q} = 2(H - H^*)\{H, H_1\}$. Now different forms of SG-algorithms can be produced. For example, proportional form (20) is as follows,

$$u = -\gamma (H - H^*)\{H, H_1\}, \tag{24}$$

where $\gamma > 0$ is the gain parameter. For a special case $n = 1, \ \ H_1(q, p) = q, \ \ q = \varphi$, it turns into the algorithm (5), while for $H_1 = -\cos\varphi$, we obtain the algorithm (15). Analysis of the system containing the feedback is based on the following result (proof see in [5]).

Theorem 1. *Let functions H, H_1 and their partial derivatives be smooth and bounded in the region $\Omega_0 = \{(q, p) : |H(q, p) - H^*| \leq \Delta\}$. Let the unforced system (for $u = 0$) have only isolated equilibria in Ω_0.*

Then, any trajectory of the system with feedback either achieves the goal or tends to some equilibrium. If, additionally, Ω_0 does not contain stable equilibria, then the goal will be achieved for almost all initial conditions from Ω_0.

Similar results are also valid for the goals expressed in terms of several integrals of motions and for the general nonlinear systems with SG-algorithms (see [8]).

Now, consider the 1-DOF oscillator with losses (7) controlled by the algorithm (9) (extension to n-DOF systems (see in [23])). Let the goal be increasing the energy of the system. Evaluating the energy change and substituting $u(t)$ from Eq. (9)) with $\gamma = \bar{u}$ yields

$$\dot{H} = \frac{\partial H}{\partial p}(-\varrho p + u) = -\varrho p^2 + pu = |p|(\bar{u} - \varrho|p|).$$

Therefore, $\dot{H} \geq 0$ in the region defined by the inequality $|p| \leq \bar{u}/\varrho$, which is equivalent to the restriction on the kinetic energy $p^2/2 \leq (\bar{u}/\varrho)^2/2$. The latter inequality holds if the condition

$$H \leq \frac{1}{2}\left(\frac{\bar{u}}{\varrho}\right)^2 \tag{25}$$

is imposed on the total energy of the system. Hence, the energy increases as long as Eq. (25) remains valid. It justifies the estimate (8).

In the case when the feedback (5) is used, we obtain $\dot{H} = -\varrho p^2 - (H - H^*)\gamma p^2 = p^2(\gamma(H - H^*) - \varrho)$, and $\dot{H} \geq 0$ within the region $H \leq H^* - \varrho/\gamma$. It yields the estimate

$$\bar{H} = H^* - \frac{\varrho}{\gamma}. \tag{26}$$

However, H^* cannot be taken arbitrarily large because of control amplitude constraint $|u| \leq \bar{u}$ which is equivalent to $\gamma|H - H^*|\,|p| \leq \bar{u}$. The above inequality is valid if

$$\gamma^2(H - H^*)^2 H \leq \tfrac{1}{2}\bar{u}^2. \tag{27}$$

Since Eq. (27) should be valid in the whole range of energies $0 \leq H \leq H^*$, it is sufficient to require it for $H = H^*/3$ providing a maximum of the left hand side of Eq. (27). Therefore, the maximum γ consistent with Eq. (27) is $\gamma = \bar{u}/((2/3)H^*)^{3/2}$. Substituting the above γ into Eq. (26) and taking maximum over H^*, we obtain that the bound (8) is achieved with the choice $\gamma = \varrho(2\bar{H})$, $H^* = 3\bar{H}$.

7. Discussion

The fundamental question of physics, mechanics and other natural sciences is: what it is possible and why? In this paper, we attempted to investigate what it is possible to do with a physical system by feedback. It was shown that if the system is close to conservative, its energy can be changed in a broad range by a small feedback, creating the phenomenon of *feedback resonance*. The 1-DOF nonlinear oscillator was taken as an example but the results hold for much more general systems.

The nature of such an efficiency of feedback is very similar to the case of control of chaos [1,14]. The method of [1] and other related methods apply a small control on the cross-sections where the trajectory passes near an unstable periodic orbit (the trace of this trajectory on the section is a fixed point). By virtue of the recurrence property of chaotic motion, the trajectory will return into a vicinity of the fixed point. It can be interpreted as a kind of approximate conservativity for the discretized system considered at sample instances. Therefore, in this case, it is also possible to achieve large changes in system behavior by means of small control.

Thus, the mechanism, possibilities and limitations of feedback are understood for the two broad classes of processes – conservative and chaotic oscillations – which are of importance in physics. It motivates further study of this phenomenon which belongs to the boundary area between physics and control science (in a broader sense – cybernetics) and may constitute a new field of physics: *cybernetical physics*. The subject of cybernetical physics is investigation of natural systems caused by (weak) feedback interactions with environment. Its methodology heavily relies on the design methods developed in cybernetics. However, the approach of cybernetical physics differs from conventional use of feedback in control applications (e.g. robotics, mechatronics, see [24]) aimed mainly at driving the system to the prespecified position or the given trajectory.

Other cyberphysical phenomena under investigation are: controlled synchronization, excitation of waves in nonlinear media, controlling energy exchange of subsystems, etc. We believe that the cybernetical methodology will also gain new insights in chemistry, biology and environmental studies.

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