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Feedback Resonance in 1-DOF and 2-DOF Nonlinear Oscillators *

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Abstract

The possibilities of studying nonlinear physical systems by small feedback action are discussed. Analytical bounds of possible system energy change by feedback are established. It is shown that for 1-DOF nonlinear oscillator the change of energy by feedback can reach the limit achievable for linear oscillator by harmonic (nonfeedback) action which corresponds to the resonance phenomenon. The feedback resonance phenomenon is demonstrated also for 2-DOF system consisting of two coupled pendulums and illustrated by computer simulation results.

1 Introduction: physics and control

Physics and mechanics provided generations of mathematicians with both problems to solve and inspiration for solution. Same is true with respect to the control theory which essential part is actually a branch of mathematics. However the reverse influence was not noticeable until recently.

The situation changed dramatically in 90s after it was discovered that even small feedback introduced into chaotic system can change its behavior significantly, e.g. turn the chaotic motion into the periodic one (Ott *et al.*, 1990). The seminal paper (Ott *et al.*, 1990) gave rise to an avalanche of publications demonstrating metamorphoses of numerous systems - both simple and complicated - under action of feedback. However the potential of modern nonlinear control theory (e.g. (Isidori, 1995; Pyragas, 1992; Nijmeijer and van der Schaft, 1990; Cook, 1994)) still was not seriously demanded although the key role of the system nonlinearity was definitely appreciated. On the other hand the new problems have some specific features for control theorists: the desired point or the trajectory of the system is not prescribed whilst the "small feedback" requirement is imposed instead. It took some time to realize that the problems of such kind are typical for control of more general oscillatory behavior and to work out the unified view of nonlinear control of oscillations and chaos (Fradkov and Pogromsky, 1998).

It needs only one more effort to make the next step and to start systematic studying the properties of physical (as well as chemical, biological, etc.) systems by means of small feedback actions.

The first consequences of such an approach for physics and mechanics were demonstrated in (Fradkov, 1998), where the mechanism of creating resonant behavior in oscillatory system by feed-

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back was examined for 1-DOF (one-degree-of-freedom nonlinear oscillator. This phenomenon was called *feedback resonance*.

In this paper we continue investigation of feedback resonance phenomenon. In Section 2 the speed-gradient method, useful for feedback design in oscillatory systems is outlined. In Sections 3,4 some results concerning feedback resonance in 1-DOF oscillators and its application to studying escape from a potential well are refined. In Section 5 the feedback resonance is demonstrated for 2-DOF system consisting of two coupled pendulums. The results are illustrated by computer simulations.

2 Exciting nonlinear oscillator

Consider the controlled 1-DOF oscillator modeled after appropriate rescaling by the differential equation

$$\ddot{\varphi} + \frac{\partial \Pi(\varphi)}{\partial \varphi} = u, \tag{1}$$

where φ is the phase coordinate, $\Pi(\varphi)$ is potential energy function, u is controlling variable. The state vector of the system (1) is $x = (\varphi, \dot{\varphi})$ and its important characteristics is the total energy $H(\varphi, \dot{\varphi}) = \frac{1}{2}\dot{\varphi}^2 + \Pi(\varphi)$. The state vector of the uncontrolled (free) system moves along the energy surface (curve) $H(\varphi, \dot{\varphi}) = H_0$. The behavior of the free system depends on the shape of $\Pi(\varphi)$ and the value of H_0. E.g. for simple pendulum we have $\Pi(\varphi) = \omega_0^2(1 - \cos\varphi) \geq 0$. Obviously, choosing $H_0 : 0 < H_0 < 2\omega_0^2$ we obtain oscillatory motion with amplitude $\varphi_0 = \arccos(1 - H_0/\omega_0^2)$. For $H_0 = 2\omega_0^2$ the motion along the separatrix including upper equilibrium is observed, while for $H_0 > 2\omega_0^2$ the energy curves get infinite and the system exhibits the permanent rotation with the average angular velocity $< \dot{\varphi} > \approx \sqrt{2H_0}$.

Let us put the question: is it possible to significantly change the energy (i.e. behavior) of the system by means of arbitrarily small controlling action?

The answer is well known when the potential is quadratic, $\Pi(\varphi) = \frac{1}{2}\omega_0^2\varphi^2$, i.e. the systems dynamics are linear:

$$\ddot{\varphi} + \omega_0^2\varphi = u. \tag{2}$$

In this case we may use the harmonic external action

$$u(t) = \overline{u}\sin\omega t \tag{3}$$

and for $\omega = \omega_0$ watch the resonance unbounded solution $\varphi(t) = -\frac{\overline{u}t}{2\omega_0}\cos\omega_0 t$.

However for nonlinear oscillators the resonant motions are more complicated with interchange of energy absorption and emission. It is well known that even for simple pendulum the harmonic excitation can even give birth to chaotic motions. The reason is, roughly speaking, in that the natural frequency of a nonlinear system depends on the amplitude of oscillations.

Therefore the idea comes: to create resonance in a nonlinear oscillator by changing the frequency of external action as a function of amplitude of oscillations. To implement this idea we need to make the value $u(t)$ depending on the current measurements $\varphi(t), \dot{\varphi}(t)$ which exactly means introducing the feedback

$$u(t) = U\left(\varphi(t), \dot{\varphi}(t)\right). \tag{4}$$

Now the problem is: how to find the feedback law (4) in order to achieve the energy surface $H(\varphi, \dot{\varphi}) = H_*$. This problem falls into the field of control theory. To solve it we suggest to use the so called Speed-Gradient(SG) method (Fradkov and Pogromsky, 1998; Fradkov, 1979; Fradkov, 1996; Shiriaev and Fradkov, 1998) which is outlined below.

3 Speed-gradient algorithms and energy control

Various algorithms for control of nonlinear systems were proposed in the literature, see e.g. (Isidori, 1995; Nijmeijer and van der Schaft, 1990; Cook, 1994; Vorotnikov, 1997). For purposes of "small control" design the following "speed-gradient" procedure is convenient (Fradkov and Pogromsky, 1998; Fradkov, 1979; Fradkov, 1996; Shiriaev and Fradkov, 1998).

Let the controlled system be modeled as

$$\dot{x} = F(x, u), \tag{5}$$

where $x \in R^n$ is the state and $u \in R^m$ is input (controlling signal). Let the goal of control be expressed as the limit relation

$$Q(x(t)) \to 0 \text{ when } \ t \to \infty. \tag{6}$$

In order to achieve the goal (6) we may apply the SG-algorithm in the finite form

$$u = -\Psi(\nabla_u \dot{Q}(x, u)), \tag{7}$$

where $\dot{Q} = (\partial Q/\partial x)F(x, u)$ is the speed of changing $Q(x(t))$ along the trajectories of (5), vector $\Psi(z)$ forms a sharp angle with the vector z, i.e. $\Psi(z)^{\mathrm{T}} z > 0$ when $z \neq 0$ (superscript "T" stands for transpose). The first step of the speed-gradient procedure is to calculate the speed $\dot{Q}$. The second step is to evaluate the gradient $\nabla_u \dot{Q}(x, u)$ with respect to controlling input u. Then the vector-function $\Psi(z)$ should be chosen to meet sharp angle condition. E.g. the choice $\Psi(z) = \gamma z, \gamma > 0$ yields the standard *proportional* (P) feedback

$$u = -\gamma \nabla_u \dot{Q}(x, u), \tag{8}$$

while the choice $\Psi(z) = \gamma \mathrm{sign} z$, where sign is understood componentwise, yields the *relay* algorithm

$$u = -\gamma \mathrm{sign}(\nabla_u \dot{Q}(x, u)). \tag{9}$$

The integral (I) form of SG-algorithm

$$\frac{du}{dt} = -\gamma \nabla_u \dot{Q}(x, u), \tag{10}$$

also can be used as well as combined, e.g. proportional-integral (PI) forms.

The underlying idea of the choice (8) is that moving along the antigradient of the speed $\dot{Q}$ provides decrease of $\dot{Q}$. It may eventually lead to negativity of $\dot{Q}$ which, in turn, yields decrease of Q and, eventually, achievement of the primary goal (6). However, to prove (6) some additional assumptions are needed, see (Fradkov and Pogromsky, 1998; Fradkov, 1979; Fradkov, 1996; Shiriaev and Fradkov, 1998).

Let us illustrate derivation of SG-algorithms for the Hamiltonian controlled system of the form

$$\dot{q} = \nabla_p H(q, p) + \nabla_p H_1(q, p)u, \quad \dot{p} = -\nabla_q H(q, p) - \nabla_q H_1(q, p)u, \tag{11}$$

where H is Hamiltonian of the free system, H_1 is interaction Hamiltonian. In order to control the system to the desired energy level H_*, the energy related goal function $Q(q, p) = (H(q, p) - H_*)^2$ is worth to choose. First step of speed-gradient design yields

$$\dot{Q} = 2(H - H_*)\dot{H} = 2(H - H_*)[(\nabla_q H)^{\mathrm{T}} \nabla_p H_1 - (\nabla_p H)^{\mathrm{T}} \nabla_q H_1]u = 2(H - H_*)\{H, H_1\}u$$

where $\{H, H_1\}$ is Poisson bracket. Since $\dot{Q}$ is linear in u, the second step yields $\nabla_u \dot{Q} = 2(H - H_*)\{H, H_1\}$. Now different forms of SG-algorithms can be produced. For example proportional (P) form (8) looks as follows

$$u = -\gamma(H - H_*)\{H, H_1\}, \tag{12}$$

where $\gamma > 0$ is gain parameter. For special case $H_1(q,p) = q,\ q = \varphi$ it turns into the algorithm (13) Analysis of the behavior of the system containing the feedback is based on the following theorem (proof see in (Fradkov and Pogromsky, 1998)).

Theorem. *Let functions H, H_1 and their partial derivatives be smooth and bounded in the region $\Omega_0 = \{(q,p) : |H(q,p) - H_*| \le \Delta\}$. Let the unforced system (for $u = 0$) have only isolated equilibria in Ω_0.*

Then any trajectory of the system with feedback either achieves the goal or tends to some equilibrium. If, additionally, Ω_0 does not contain stable equilibria then the goal will be achieved for almost all initial conditions from Ω_0.

Similar results are also valid for the goals expressed in terms of several integrals of motions and for the general nonlinear systems with SG-algorithms (see (Shiriaev and Fradkov, 1998)).

4 Feedback resonance in 1-DOF system

For the system (1) the SG-method with the choice of the goal function $Q(x) = [H(x) - H_*]^2$ produces simple feedback laws:

$$u = -\gamma\,(H - H_*)\,\dot{\varphi}, \tag{13}$$

$$u = -\gamma\,\mathrm{sign}\,(H - H_*)\cdot\,\mathrm{sign}\,\dot{\varphi}, \tag{14}$$

where $\gamma > 0$, $\mathrm{sign}(H) = 1$, for $H > 0$, $\mathrm{sign}(H) = -1$ for $H < 0$ and $\mathrm{sign}(0) = 0$. It follows from the Theorem 1 of Section 3 that the goal $H(x(t)) \to H_*$ in the system (1), (13) (or (1), (14)) will be achieved from almost all initial conditions provided that the potential $\Pi(\varphi)$ is smooth, its stationary points are isolated and there is no stable equilibria of the unforced system within initial energy layer $\{(\varphi,\dot{\varphi}) : H_0 \le H(\varphi,\dot{\varphi}) \le H_*\}$, where $H_0 = H(\varphi(0),\dot{\varphi}(0))$ is initial energy level (we assume $H_0 \le H_*$).

It is worth noticing that since the motion of the controlled system always belongs to an energy layer between H_0 and H_*, the right hand side of (13) is bounded. Therefore, taking sufficiently small gain γ we can achieve the given energy surface $H = H_*$ by means of *arbitrarily small* control. Of course this seemingly surprising result holds only for conservative (lossless) systems.

Let now the losses be taken into account, i.e. the system be modeled as

$$\ddot{\varphi} + \varrho\dot{\varphi} + \frac{\partial \Pi}{\partial \varphi} = u. \tag{15}$$

where $\varrho > 0$ is the damping coefficient. Then the upper bound of the energy level $\overline{H}$ reachable by the feedback of amplitude $\overline{u}$ can be calculated as

$$\overline{H} = \frac{1}{2}\left(\frac{\overline{u}}{\varrho}\right)^2. \tag{16}$$

Indeed, consider the 1-DOF oscillator with losses (15) controlled by the algorithm (14) (Extension to the n-DOF systems can be found in (Fradkov, 1999)). Evaluating the change of the goal function and substituting $u(t)$ from (14) with $\gamma = \overline{u}$ yields

$$\dot{H} = \frac{\partial H}{\partial p}(-\varrho p + u) = -\varrho p^2 + pu = |p|(\overline{u} - \varrho|p|).$$

Therefore $\dot{H} \ge 0$ in the region defined by the inequality $|p| \le \overline{u}/\varrho$ which is equivalent to the restriction on the kinetic energy $p^2/2 \le (\overline{u}/\varrho)^2/2$. The latter inequality holds if the condition

$$H \le \frac{1}{2}\left(\frac{\overline{u}}{\varrho}\right)^2 \tag{17}$$

is imposed on the total energy of the system. Hence the energy increases as long as (17) remains valid. It justifies the estimate (17).

In the case when the feedback (13) is used we obtain $\dot{H} = -\varrho p^2 - (H - H_*)\gamma p^2 = p^2(\gamma(H - H_*) - \varrho)$, and $\dot{H} \geq 0$ within the region $H \leq H_* - \varrho/\gamma$. It yields the estimate

$$\overline{H} = H_* - \frac{\varrho}{\gamma}. \tag{18}$$

However H_* cannot be taken arbitrarily large because of control amplitude constraint $|u| \leq \overline{u}$ which is equivalent to $\gamma|H - H_*|\ |p| \leq \overline{u}$. The above inequality is valid if

$$\gamma^2(H - H_*)^2 H \leq \frac{1}{2}\overline{u}^2. \tag{19}$$

Since (19) should be valid in the whole range of energies $0 \leq H \leq H_*$, it is sufficient to require it for $H = H_*/3$ providing maximum value of the left hand side of (19). Therefore the maximum γ consistent with (19) is $\gamma = \overline{u}/\left(\frac{2}{3}H_*\right)^{3/2}$. Substituting the above γ into (18) and taking maximum over H_* we obtain that the bound (16) is achieved with the choice $\gamma = \varrho/(2\overline{H}), H_* = 3\overline{H}$.

If damping is nonlinear then the maximum damping in the domain Ω should be taken into account in (16). Note, that although the estimate (16) holds for both feedback laws (13) and (14), to achieve it, the parameters of feedback should be chosen in different ways, namely, for feedback (13) $H_* = 3\overline{H}, \gamma_* = \varrho/(2\overline{H})$, while for (14) $\gamma_* = \overline{u}, H_* = \overline{H}$.

It is worth comparing the bound (16) with the energy level achievable for linear oscillator

$$\ddot{\varphi} + \varrho\dot{\varphi} + \omega_0^2\varphi = u(t), \tag{20}$$

where $\varrho > 0$ is the damping coefficient, by harmonic (nonfeedback) action. The response of the model to the harmonics $u(t) = \overline{u}\sin\omega t$ is also harmonics $\varphi(t) = A\sin(\omega t + \varphi_0)$ with the amplitude

$$A = \frac{\overline{u}}{\sqrt{(\omega^2 - \omega_0^2)^2 + \varrho^2\omega^2}}. \tag{21}$$

Let $\rho^2 < 2\omega_0^2$. Then A reaches its maximum for $\omega^2 = \omega_0^2 - \rho^2/4$. The energy averaged over the period is

$$\overline{H} = \frac{1}{2}\left(\frac{\overline{u}}{\varrho}\right)^2 + \mathrm{O}(\rho^2), \tag{22}$$

Comparison of (16) and (22) shows that for nonlinear oscillator affected by feedback the change of energy can reach the limit achievable for linear oscillator by harmonic (nonfeedback) action at least in the case of small damping.

5 Escape from a potential well

The feedback resonance phenomenon is related to escape from the potential wells which is important in many fields of physics and mechanics (Virgin and Cartee, 1991; Stewart *et al.*, 1995). Sometimes escape is an undesirable event and it is important to find conditions preventing it (e.g. buckling of the shells, capsize of the ships, etc.). In other cases escape is useful and the conditions guaranteeing it are needed. In all cases the conditions of achieving the escape by means of as small external force as possible are of interest.

In (Stewart *et al.*, 1995) such a possibility (optimal escape) has been studied for typical nonlinear oscillators (15) with a single-well potential $\Pi_e(\varphi) = \varphi^2/2 - \varphi^3/3$ (so called "escape equation") and a twin-well potential $\Pi_d(\varphi) = -\varphi^2/2 + \varphi^4/4$ (Duffing oscillator). The least amplitude of a harmonic external forcing $u(t) = \overline{u} \sin \omega t$ for which no stable steady state motion exists within the well was determined by intensive computer simulations. For example, for escape equation with $\varrho = 0.1$ the optimal amplitude was evaluated as $\overline{u} \approx 0.09$, while for Duffing twin-well equation with $\varrho = 0.25$ the value of amplitude was about $\overline{u} \approx 0.212$. Our simulation results agree with (Stewart *et al.*, 1995). The typical time histories of input and output for $\overline{u} = 0.209$ are shown in Fig. 1. It is seen that escape does not occur.

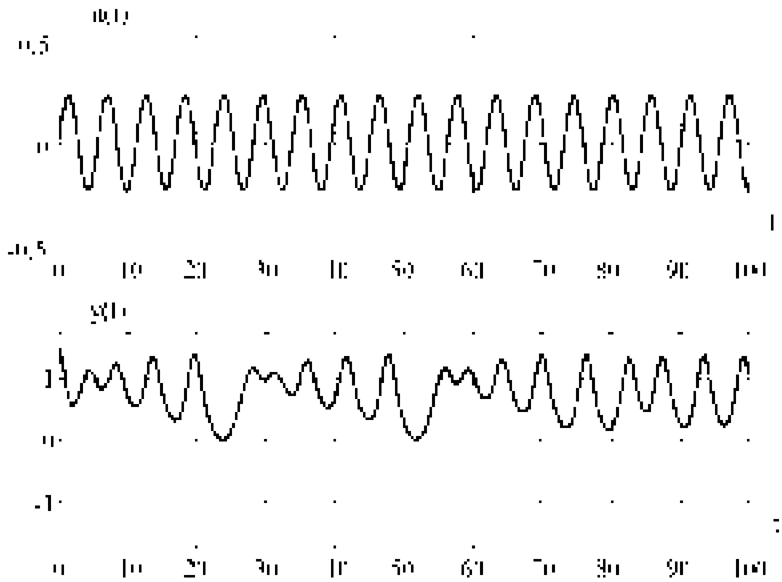

Figure 1: u(t), y(t) time histories for $u(t) = \overline{u} \sin \omega t$, $\overline{u} = 0.209$.

Using feedback forcing we may expect reducing the escape amplitude. In fact using the results of section 2, the amplitude of feedback (13), (14) leading to escape can be easily calculated, just substituting the height of the potential barrier $\max_{\Omega} \Pi(\varphi) - \min_{\Omega} \Pi(\varphi)$ for $\overline{H}$ into equation (16) where Ω is the well corresponding to the initial state. For example taking $H = 1/6, \varrho = 0.1$ for $\Pi_e(\varphi)$ gives $\overline{u} = 0.0577$, while $\overline{H} = 1/4, \varrho = 0.25$ gives $\overline{u} = 0.1767$, the values which are substantially smaller than those evaluated in (Stewart *et al.*, 1995). The less the damping, the bigger the difference between the amplitudes of feedback and nonfeedback signals leading to escape. Simulation exhibits still stronger difference: escape for Duffing oscillator occurs even for $\overline{u} = \gamma = 0.122$ if the law (14) is applied, see Fig. 2. Note that the oscillations in the feedback systems have both variable frequency and variable shape.

We also studied the dependence of escape amplitudes on the damping by means of computer simulations in the range of damping coefficient ϱ varying from 0.01 to 0.25. Simulations confirmed theoretical conclusion that the feedback escape amplitude is proportional to the damping (Fig. 3).

6 Exciting the two coupled pendulums

Consider the special case of the diffusively coupled oscillator model (used for modeling various physical and mechanical systems, see (Jackson, 1990)): the two pendulums model. For lossless case it has the form

$$\begin{cases} \ddot{\varphi}_1(t) + \omega^2 \sin \varphi_1(t) = & k(\varphi_2(t) - \varphi_1(t)) + u(t), \\ \ddot{\varphi}_2(t) + \omega^2 \sin \varphi_2(t) = & k(\varphi_1(t) - \varphi_2(t)), \end{cases} \tag{23}$$

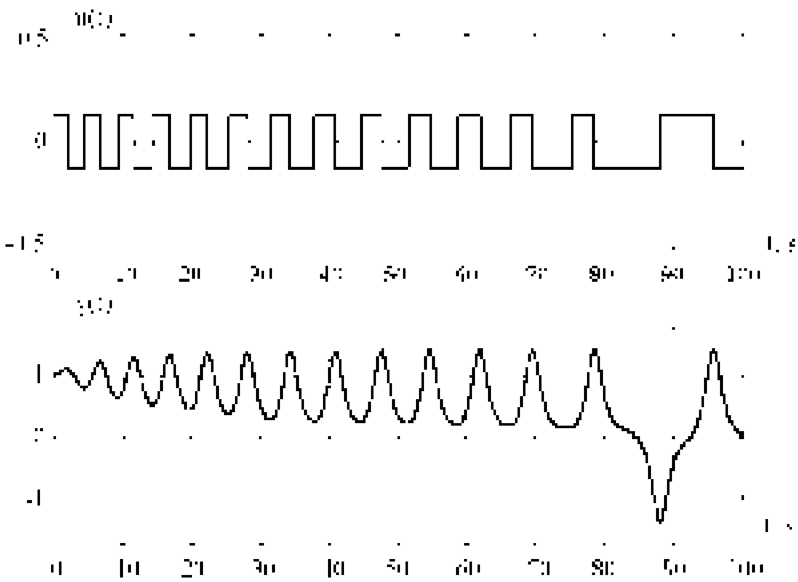

Figure 2: u(t), y(t) time histories for the law (14), $\gamma = 0.122$.

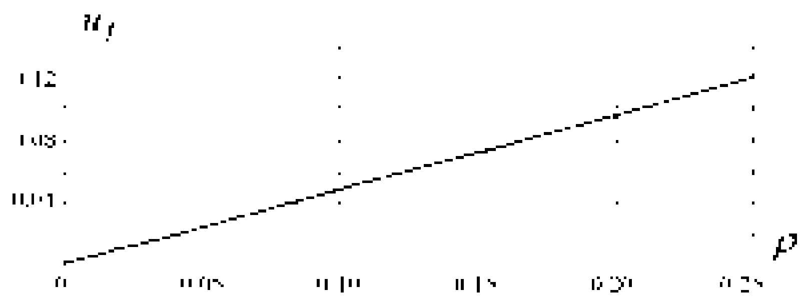

Figure 3: Dependence of the feedback escape amplitude $\overline{u}_f$ on the damping ϱ.

where $\varphi_i(t)$, $(i = 1, 2)$ are the rotation angles of pendulums, $u(t)$ is the external torque, (control action), applied to the first pendulum, ω, k are the system parameters: ω is the natural frequency of small oscillations, k is the coupling strength (e.g. stiffness of the string).

Introduce the state vector $x(t) \in \mathcal{R}^4$ as $x(t) \triangleq \text{col}\{\varphi_1,\ \varphi_2,\ \dot\varphi_1,\ \dot\varphi_2\}$. The total energy of the system (23) $H(x)$ can be written as follows

$$H(x) = \frac{1}{2}\dot\varphi_1^2 + \omega^2(1 - \cos\varphi_1) + \frac{1}{2}\dot\varphi_2^2 + \omega^2(1 - \cos\varphi_2) + \frac{k}{2}(\varphi_1 - \varphi_2)^2 \tag{24}$$

Consider the problem of excitation a "wave" with the desired amplitude by means of small feedback. The problem can be understood as achieving the given energy level of the system with additional requirement that pendulums should have the opposite phases of oscillation.

In order to apply the Speed-gradient procedure of Section 3, we introduce objective functions as

$$\begin{aligned} Q_\varphi(\dot\varphi_1, \dot\varphi_2) &\triangleq \frac{1}{2}(\delta_\varphi)^2 \\ Q_H(x) &\triangleq \tfrac{1}{2}(H(x) - H_*)^2. \end{aligned} \tag{25}$$

where $\delta_\varphi = \dot\varphi_1 + \dot\varphi_2$ and H_* is the prescribed value of the total energy.

The minimum value of the function Q_φ meets the "opposite phases" requirement (at least for small initial phases $\varphi_1(0), \varphi_2(0))$: $Q_\varphi(\dot\varphi_1, \dot\varphi_2) \equiv 0$ iff $\dot\varphi_1 \equiv -\dot\varphi_2$. The minimization of Q_H means achievement of the desired amplitude of the oscillations.

In order to design the control algorithm the weighted objective function $Q(x)$ is introduced as the weighted sum of Q_φ and Q_H, namely

$$Q(x) \triangleq \alpha Q_\varphi(\dot\varphi_1, \dot\varphi_2) + (1-\alpha)Q_H(x), \tag{26}$$

where α, $0 \le \alpha \le 1$ is a given weighting coefficient.

Performing calculations according to the Speed-gradient procedure of Section 3, we arrive to the following control law

$$\begin{aligned} u(t) &= -\gamma\left(\alpha\delta_\varphi(t) + (1-\alpha)\delta_H(t)\dot\varphi_1(t)\right), \\ \delta_\varphi(t) &= \dot\varphi_1(t) + \dot\varphi_2(t), \\ \delta_H(x(t)) &= H(t) - H_*, \end{aligned} \tag{27}$$

where $\gamma > 0$ is a gain coefficient.

Applying the results of (Fradkov and Pogromsky, 1998; Shiriaev and Fradkov, 1998) yields that sufficient conditions for the achievement of the control goal $Q(x(t)) \to 0$ are valid if the desired level of energy does not exceed the value $H_* = 2\omega^2$, corresponding to the upper equilibrium of one pendulum and lower equilibrium of another one.

Computer simulations were performed in order to complement the theoretical conclusions. The typical results are shown in Fig. 4 – 7 for the following values of parameters: $k = 5$, $\omega = 0.4\pi$, $\gamma = 0.8$, $\alpha = 0.7$, $H_* = 4.0$. All the initial conditions were taken equal to zero, except $\varphi_2(0) = 0.05\pi$.

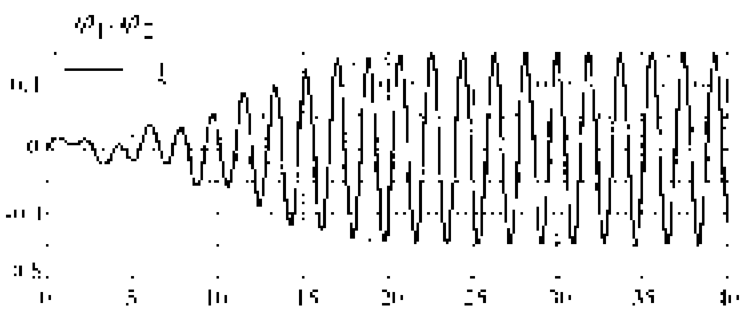

Figure 4: The phase angles φ_1, φ_2 time history.

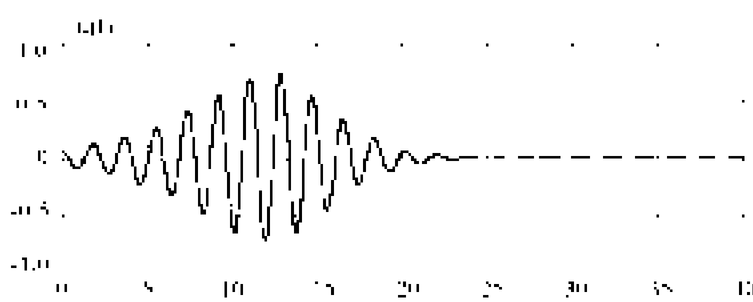

Figure 5: The control action $u(t)$ time history

It is seen that after some transient time both pendulums oscillate with the opposite phases while both goal functions approach the prescribed values. The transient time for both H and for Q_φ is about 100 time units. The relation between transient times for H and for Q_φ can be changed

Figure 6: The total energy $H(x(t))$ time history.

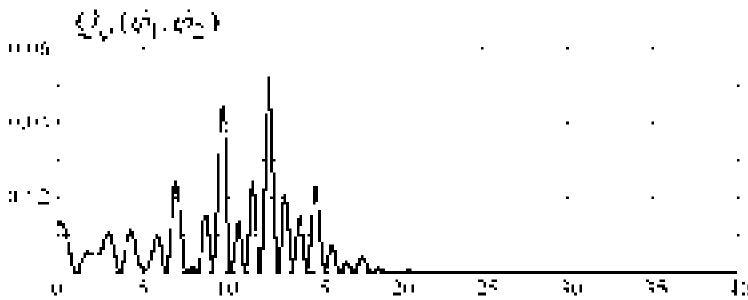

Figure 7: The qoal function $Q_\varphi(\dot\varphi_1, \dot\varphi_2)$ time history.

by means of changing the weight coefficient α. The control amplitude can be arbitrarily decreased by means of decreasing the gain γ (in the case when damping is not taken into account).

Discussion

The fundamental question of physics, mechanics and other natural sciences is: what is possible and why? In this paper we attempted to investigate what is possible to do with a physical system by feedback. It was shown that if system is close to conservative, its energy can be changed in a broad range by small feedback. Moreover, for multidimensional (e.g. 2-DOF) systems some additional symmetry (synchronization) properties can be preserved by feedback.

The nature of such an efficiency of feedback is very similar to the case of control of chaos (Ott *et al.*, 1990; Ott *et al.*, 1994). The method of (Ott *et al.*, 1990) and other related methods apply small control on the cross sections when the trajectory passes near an (unstable) periodic orbit which trace on the section is just the fixed point. By virtue of recurrence property of chaotic motion the trajectory will return into the vicinity of the fixed point. It can be interpreted as a kind of approximate conservativity for the discretized system considered at the sample instances. Therefore in this case it is also possible to achieve large changes in system behavior by means of small control.

Thus the mechanism, possibilities and limitations of feedback are understood for the two broad classes of processes — conservative and chaotic oscillations — which are of importance in physics. It motivates further study of this phenomenon which belongs to the boundary area of physics and control science (in a broader sense — cybernetics) and In fact one may constitute this field as the new field of physics: *cybernetical physics*. Its subject is investigation of the features of the natural system by admitting (weak) feedback interactions with the environment. Its methodology heavily relies on the design methods developed in cybernetics. However the approach of cybernetical physics differs from the conventional usage of feedback in control applications (e.g. robotics, mechatronics, see (van Campen, 1997)) aimed at driving the system to the prespecified position or the given trajectory.

Other related phenomena which are already under investigation are: controlled synchronization, excitation of waves in nonlinear media, controlling energy exchange of subsystems, etc. We believe that the cybernetical methodology will also gain new insights in chemistry, biology and environmental studies.

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