NUMERICAL AND EXPERIMENTAL EXCITABILITY ANALYSIS OF MULTI-PENDULUM MECHATRONICS SYSTEM [1]

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Abstract: An experimental mechatronics set-up for research and education is described. The set-up consists of single and coupled double pendulum-like electromechanical system connected to the personal computer. The mathematical models of mechanical and electromagnetic processes in the system, data exchange interface and software for laboratory experiments are described. The analysis and control design problems for the system are difficult because only period of oscillations is available for online measurement and only pulse control torque can be applied. The algorithms for typical analysis and design problems (swinging and parameter estimation) are presented and studied both numerically and experimentally. Pulse-width modulated algorithm with the time shift for swinging the pendulums is described. Influence of the second pendulum on the excitability of the first one is investigated. The results of the paper lead to better understanding features of the *excitability index* – new characteristics of resonance properties of nonlinear dynamical systems.

Keywords: Nonlinear models, Nonlinear control, Excitation, Oscillation

1. INTRODUCTION

Control problems for complex oscillatory systems attract significant interest during recent years. One reason of it is possibility of a variety of applications (vibrational and laser technologies, telecommunications, etc). An important role for the development and testing of new control methods is played by laboratory equipment. Mechanical oscillatory systems (e.g. pendulum systems) are also of special interest for control education as examples of simple systems that may exhibit complex nonlinear behavior. That is why mechanical pendulum-like controlled toys drew much attention of control community recently (Åström and Furuta, 2000), (Christini *et al.*, 1996), (Dunnigan, 1998), (Furuta *et al.*, 1999), (Miroshnik and Bobtsov, 2000), (Shiriaev *et al.*, 2001).

An interesting example of one- and two-pendulum computer-controlled systems ("chaotic toys") has been developed in the Institute for Problems of Mechanical Engineering in St.Petersburg (Konoplev and Konjukhov, 1997), (Andrievsky *et al.*, 1999), (Andrievsky and Boykov, 2001). Each toy consists of two coupled pendulums, one inside another. The outer pendulums can be joined by means of an elastic link.

In this paper the system of (Andrievsky *et al.*, 1998), (Andrievsky and Boykov, 2001) reconsidered and used for numerical and laboratory experiments. The novelty of the paper is taking into account pecularities of the electromagnetic subsystem. The mathematical

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Owing to interaction of mechanical and magnetic processes the system possesses complex dynamics. Therefore fine tuning of the algorithm parameters during numerical experiments is necessary. To improve quality of the system, parameter estimation of the mathematical model was performed during laboratory experiments and the obtained estimates were used for tuning the control algorithm during numerical experiments.

In Section 2 a brief description of the set-up and its mathematical model are given, including electrical and magnetic subsystem, data exchange interface and software tools. Their description is given in subsections 2.2, 2.3. Section 3 demonstrates usage of laboratory experiments for estimation of the model parameters. The problem of the real-time state estimation, based the accessible flow data and the system model was considered in (Andrievsky and Boykov, 2001). In the papers (Andrievsky *et al.*, 1996), (Fradkov, 1999) it is shown that the control algorithm, ensuring prescribed energy level of the mechanical system can be designed by the *speed-gradient* technique. In the Section 4 this technique is applied for evaluation of excitability index for one-pendulum and two-pendulum systems. Excitability index was introduced in (Fradkov, 2000) as a measure of resonant properties of nonliear systems. It is defined as a family of L_∞ gains for different levels of input; it allows to formulate stability criteria and to measure damping ability of the system.

2. LABORATORY SET-UP DESCRIPTION

Laboratory set-up includes: *mechanical part*; *electrical part* (with *computer interface facilities*), and *the personal computer Pentium-3* for experimental data processing, representation of the results and the real-time control. The special *exchange routine* for data exchange via standard In-Out ports of the computer has been written.

2.1 *Mechanical system*

The mechanical part of the system, consists of two similar subsystems (mechanical toys), each including two coupled pendulums. Subsystems are coupled with the elastic link, see figure 1.

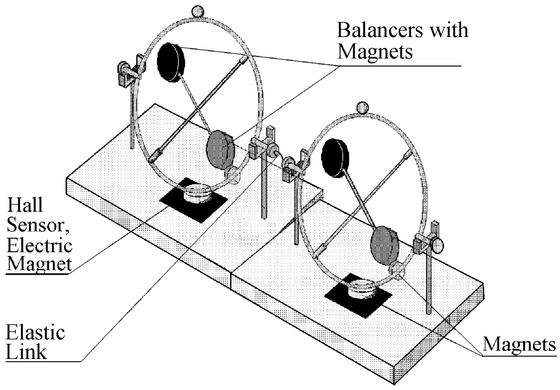

Fig. 1. Pendulums coupled with the elastic link.

Pendulums have biased centers of masses of the parts and sliding support of rotation axes. An external part of each subsystem is metal ring with a massive ball and two cylindrical magnets located on it. The ball and magnets displace the center of weight of the ring. Magnets transmit control forces to both links. Two opposite directed half-axes, ensuring its support on two flat platforms with terminators of a course, are located on the outer surface of the ring. System is fixed on the massive basement, in center of which the Hall sensor of the first link zero state and electrical magnet transmitting control efforts to cylindrical magnet are located.

The second (internal) pendulum consists of two cylindrical loads with magnets mounted on the axis symmetrically to its center. This part rotates inside an external ring with an axis of rotation fixed on it by means of cylindrical hinges. The hinges are mounted on the axis turned by 45 degrees with respect to the axis of the external ring rotation.

Each mechanical subsystem has three degrees of freedom:

(1) Translation of an external ring along support of the basis;
(2) Rotations of an external ring with the above-stated half-axes;
(3) Rotation of the second link with respect to external ring.

The following control efforts apply:

- Force of an electromagnet of the basis on one of magnets, established on an external ring;
- Force of the second magnet, established on an external ring on magnets of second (internal) link.

Measured variable is the time interval between passings of the external ring magnet above electromagnet of the base. These events are determined by means of the Hall sensors.

2.2 *Hardware description*

Oscillation control is provided on the basis of combined hard/soft-ware implementation. The energy for excitation is transmitted by the pulse-width modulated (PWM) signal with the constant level and variable duty cycle. From the programming point of view, hardware represents by the *write-only registers* (WO) for putting in the prescribed duty cycle of control signal from the computer, and the *read-only registers* (RO) for transferring oscillations of half-period duration values to the computer. The PWM based method provides more precise control than the numeric-pulse one, because of integration the high frequency pulses by the mechanical subsystem (the weight of one discrete of a control has less value, than on numeric-pulse control method).

Hardware has two identical channels for each mechanical subsystem and the common *peripheral controller*. The measuring unit is built using the *quartz oscillator* to calibrate the main clock pulses. This pulses after the frequency divider to 1000 Hz determine sampling time of 1 ms. Maximum measurable time interval, provided by 12-bit counter varies from 0.001 to 4.095 *s*. The beginning and the end of counting interval is determined by the *Hall sensors* and *zero-crossing detector*. For data transfer the peripheral controller uses the *Standard Parallel Port* (SPP) in byte bi-directional mode.

The *control unit* generates the exciting action applied to the pendulums via the opposite magnetic fields. It includes bi-channel *asinchronous pulse-width modulator* (APWM), logical command interpreting automaton and the *power amplifiers* to drive the electromagnets.

2.3 Software

As a basic development software for plant control and measured signal processing the programming package MATLAB^R, combined with the *Parallel In/Out Toolbox* (PIO-Toolbox) was used. The communications protocol based on the *sub-addressing method*, arrays (vectors in MATLAB) are considered as data units for transfer.

Main software routine is PIO (Parallel In/Out) function. This function makes available vector elements output through the parallel 8-bit channel (commonly used for a printer). There are some advantages of *pio*-function over the similar one from the Realtime Toolbox^R (HUMUSOFT, 1998). Function *pio* is written in assembly language using the means of *Borland C* compiler to connect with MATLAB and also may be re-compiled for SCO UNIX, LINUX as ELF- or DEE-modules.

2.4 *Simplified mathematical model of the mechanical system*

First consider a single subsystem. Its detailed model in the *aggregative form* was given in (Konoplev and Konjukhov, 1997), (Andrievsky *et al.*, 1998). For the purposes of this paper the simplified model was created. We take into account that the inner pendulum weakly influences the motion of the external ring. After some transformations, plant model can be written as:

$$\begin{aligned}&J_e\ddot{\varphi} = -m_1 g(R\sin\varphi - (\varrho+r)\cos\varphi) - \\ &m_2 g(f\sin\varphi - (\varrho+r)\cos\varphi) + \\ &m_p g(\varrho+r)\cos\varphi + m_3 g(R_m\sin\varphi + \\ &(\varrho+r)\cos\varphi) + M(\varphi,u) + M_f(t),\end{aligned} \tag{1}$$

where $J_e = m\varrho^2 + J_0$ is *the equivalent moment of inertia* of the plant. The coefficients m_i, r, R_m, ϱ are determined through mass-geometric parameters of the system. $M(\cdot)$ describes the torque of *the electromagnetic forces*, and the last term $M_f(t)$ stands for disturbances, caused, in the first place, motion of the inner pendulum. Electromagnetic attraction excites by *residual magnetization* of the core. Repulsive electromagnetic force is caused by *the controlling signal* $u(t)$, applied to the clips of the electromagnet. Assuming that the electromagnetic force changes as the inverse square of the distance between the magnets, we arrive at the following formulas for $M(\varphi, u)$:

$$\begin{aligned}&f_m(\varphi) = \left((R(1-\cos\varphi)+\delta)^2 + R^2\sin^2\varphi\right)^{-1}, \\ &\gamma(\varphi) = \pi - \arctan\frac{R(1-\cos\varphi)+\delta}{R\sin\varphi}, \\ &\mu(\varphi) = -f_m(\varphi)\cos\gamma, \\ &M(\varphi, u) = (A_c - A_u u)\mu(\varphi),\end{aligned} \tag{2}$$

where δ is the minimal value of the gap between the magnets, R denotes the radius of the external ring. A_c, A_m are assumed to be constant. Their values depend on the residual magnetization and the properties of the magnetoelectric circuit. Rewriting the model (1) in the state-space form, and taking the viscous friction into account we obtain the following equations:

$$\begin{cases}\dot{\varphi}(t) = \omega(t), \\ \dot{\omega}(t) = -a_1\sin(\varphi(t)-\psi) - a_2\omega(t) + \\ \quad (a_c + a_u u(t))\mu(\varphi) + f(t).\end{cases} \tag{3}$$

Plant parameters are found by preliminary mass-geometrical examination and via the off-line identification procedure on the base of the experimental data set (Andrievsky and Boykov, 2001). This procedure is briefly described in the Section 3. Preliminary mass–geometrical examination gives the following rough values of the model (3) parameters: $a_1 \approx 44\ s^{-2}$; $\psi \approx 0.13\ rad$; parameter a_2 belongs to the interval $[0.1 \div 0.5]\ s^{-1}$. Parameters a_c, a_m are obtained from relations $a_c = A_c/J_e, a_m = A_m/J_e$. Their values should be found experimentally. Now let us consider composite mechanical system with elastic link between subsystems. Assuming resilience of the link

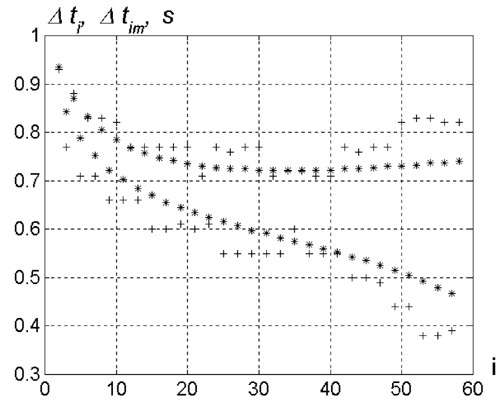

Fig. 2. The sequences of measured (+) and modelling (*) intervals Δt_i, Δt_{i_m} between zero-crossing.

to be linear, we obtain the model of the mechanical system in the following form:

$$\begin{cases} \dot{\varphi}_1(t) = \omega_1(t), \\ \dot{\omega}_1(t) = -a_1 \sin(\varphi_1(t) - \psi) - a_2\omega_1(t) + \\ k(\varphi_2 - \varphi_1) + (a_c + a_u u_1(t))\mu(\varphi_1) + f_1(t), \\ \dot{\varphi}_2(t) = \omega_2(t), \\ \dot{\omega}_2(t) = -a_1 \sin(\varphi_2(t) - \psi) - a_2\omega_2(t) - \\ k(\varphi_2 - \varphi_1) + (a_c + a_u u_2(t))\mu(\varphi_2) + f_2(t). \end{cases} \quad (4)$$

where $\varphi_i(t)$, $(i = 1, 2)$ are the rotation angles of pendulums; $u(t)$ is the external torque, (control action), applied to the first pendulum; f_1, f_2 stand for disturbances; k is the coupling strength (e.g. stiffness of the string).

3. PARAMETER IDENTIFICATION BASED ON THE LABORATORY EXPERIMENT

The model (3) parameters were refined based on the results of the laboratory experiment. The problem is nontrivial because only the time intervals between zero-crossings are available for measurement. For making experiment the field magnet was switched off ($u(t) \equiv 0$), the conjunctive spring was removed and initial ring deflection at the angle of 150 degrees was set. As a result, the sequence of N intervals $\{\Delta t_i\}$ (where $i = 1, 2, \ldots, N$), was obtained. This sequence is shown in figure 2.

This experimental sequence is compared with the simulation results, taken by means of the model (3) with specified parameter values. After calling numerical optimization procedure the revised parameter values are found. When searching two cost functionals are used: $Q_1 \triangleq \frac{1}{N}\sum_{i=1}^{N}(t_{k_i} - t_{m_i})^2$ is the mean square value between real and modelling zero-cross instants. This lost functional is considered as a main one. An additional functional $Q_2 \triangleq \frac{|N_m - N|}{N}$, where N_m is a number of zero-crossing in the modelling realization during the same period of time. Functional Q_2 is considered as a restriction. Finally, the lost functional Q is defined as:

$$Q = \begin{cases} 10^4 \cdot Q_2, & \text{when } Q_2 > 0.05, \\ Q_1, & \text{else.} \end{cases}$$

Using the standard optimization routine the following parameter estimates for the first pendulum were found: $a_c^* = 0.89\ s^{-2}$, $a_1^* = 27\ s^{-2}$; $a_2^* = 0.061\ s^{-1}$; $\psi^* = 0.082\ rad$ (Comparative results are presented in figure 2.) Analogously, for the second pendulum, $a_c^* = 0.87\ s^{-2}$, $a_1^* = 28\ s^{-2}$; $a_2^* = 0.063\ s^{-1}$; $\psi^* = 0.076\ rad$

4. EXCITABILITY ANALYSIS

4.1 *Excitability index*

In the papers (Fradkov, 1999), (Fradkov, 2000), the concept of *feedback resonance* and *excitability index* were introduced. Consider a system described by state-space equations

$$\dot{x} = F(x, u), y = h(x) \quad (5)$$

where $x \in R^n$ is state vector, u, y are scalar input and output, respectively. It was shown that in ofder to create resonance mode in a nonlinear system (to find small force that leads to significant changes in system behavior), it is possible to solve an optimal control problem

$$Q(\gamma) = \limsup_{\substack{|u(s)|\leq\gamma, \\ 0\leq s\leq t, \\ x(0)=0, \\ t\geq 0}} |y(t)|^2. \quad (6)$$

If the system (5) is BIBO stable and $x = 0$ is equilibrium of the unforced system ($F(0,0) = 0, h(0) = 0$) then $Q(\gamma)$ will be well defined. Apparently, the signal providing maximum excitation should depend not only on time but also on system state, i.e. input signal should have a feedback form. Since for linear systems the value function of the problem (6) depends quadratically on γ, it is naturally to introduce the *excitability index* (EI) for the system (5) as follows:

$$E(\gamma) = \frac{1}{\gamma}\sqrt{Q(\gamma)}, \quad (7)$$

where $Q(\gamma)$ is the optimum value of the problem (6). For nonlinear systems $E(\gamma)$ is a function of γ that characterizes excitability (resonance) properties of the nonlinear system. For MIMO systems matrix excitability index can be introduced in a similar way computing E_{ij} for every pair of input u_i and output y_j.

The solution to the problem (6) is quite complicated in most cases. It was shown in (Fradkov, 2000) that approximate locally optimal (speed-gradient) solution can be used

$$u(x) = \gamma\,\text{sign}\left(g(x)^T \nabla h(x) h(x)\right), \quad (8)$$

where $g(x) = \left.\frac{\partial F(x,u)}{\partial u}\right|_{u=0}$. The value (8) is obtained by maximizing the principal part of instant growth rate of $|y(t)|^2$. An important consequence is that excitability index can be estimated directly by applying input (8) to the system. For real world systems it can be done experimentally. Otherwise, if a system model is available, computer simulations can be performed.

4.2 *Excitability index of the single pendulum system*

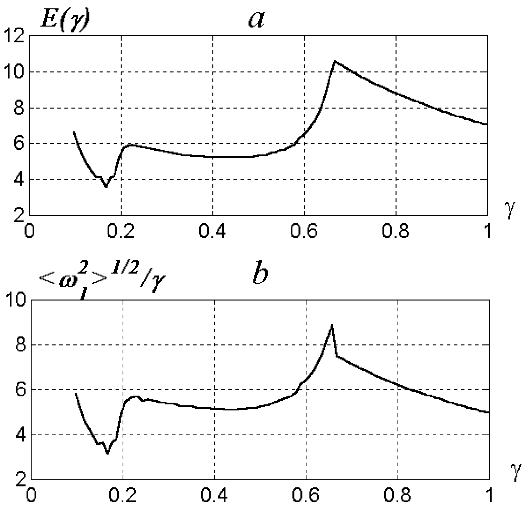

Fig. 3. The excitability indices of the single system (3) with respect to energy (*a*), and angular velocity (*b*).

Consider a single pendulum model (3). The specific design features of the considered system allow to apply only impulse control action at the time intervals $[t'_k, t''_k]$, where $t'_k = t_k - \gamma'$, $t''_k = t_k + \gamma''$, the time instants t_k ("the *zero-crossing instants*") satisfy the condition $\varphi(t_k) = 0$ ($k = 0, 1, 2, \ldots$). To find the excitability index, the following time-shifted pulse-width control action is applied:

$$u(t) = \begin{cases} u_0, & \text{if } t \in [t_k + (1-\gamma)T,\ t_k + T], \\ 0, & \text{otherwise,} \end{cases} \quad (9)$$

where $T = 0.255$ s is a maximal impulse duration; u_0 is a constant magnitude of the controlling impulse. For the sake of simplicity one can take $u_0 = 1$ and relate the electric and magnetic circuit properties to the generalized parameter a_u in Eqs. (3), (4). The on-off time ratio $\gamma = \gamma(t_k) \in [0, 1]$ is considered as a *controlling signal* of the overall system. If γ is small, the result of the action (9) approximates the locally optimal (speed-gradient) action (8). Figure 3 shows the excitability indices of the single electro-mechanical system with respect to energy $E_H(\gamma)$ (*a*), and with respect to angular velocity $E_\omega(\gamma) = \frac{1}{\gamma}\sqrt{\langle\omega^2\rangle}$ (*b*).

It is seen that the indices have the same qualitative behavior.

Control algorithm for locally optimal excitation has the form (9), where

$$u(t_k) = u_0 \operatorname{sign}\big(\omega_1(t_k)\big).$$

The excitability indices, shown in Fig. 3 has been found by simulation. Parameters of the model (4) was taken from the laboratory experiments and the LSE identification procedure. Figure 4 shows the excitability indices that have been found on the base of experiments.

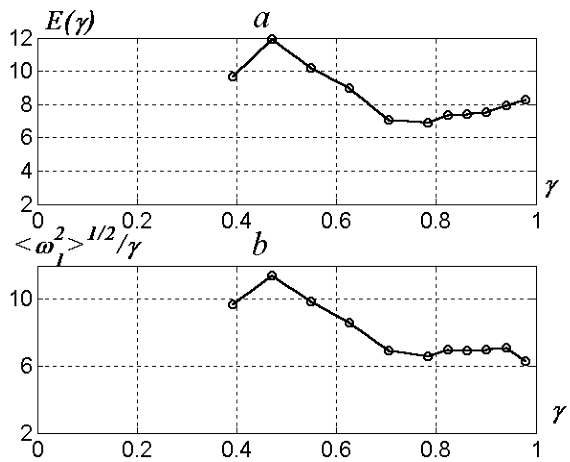

Fig. 4. The experimental excitability indices of the single pendulum system with respect to energy (*a*), and angular velocity (*b*).

5. CONCLUSION

The laboratory equipment for experiments with the controlled double link pendulums is described. The algorithms for typical analysis and design problems (parameter estimation and swinging to given energy level) are presented and studied both numerically and experimentally. Results of the numerical experiments are close to the laboratory ones.

Pulse-width modulated algorithm for swinging the pendulums is obtained based upon the speed-gradient approach. Influence of the second pendulum on the excitability of the first one is investigated. The results of the paper show that the described experimental set-up is useful for research and education in the field of nonlinear systems. They also lead to better understanding features of the excitability index – new characteristics of resonance properties of a nonlinear dynamical system.

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