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Control of Chaos: Some Open Problems

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Abstract. A brief outline of the emerging field termed "Control of Chaos" is given. Three major branches of research, related to so called "non-feedback control", OGY method and Pyragas method are surveyed. It is demonstrated that some important problems of justification of existing control algorithms remain unsolved and still provide challenges for control theorists.

Keywords: Nonlinear control, chaotic systems

1 Introduction

"Control of Chaos" has become a hot topic for physicists and engineers during the last decade. Publication activity in this field has grown tremendously. Starting with a few papers in 1990, the number of publications exceeded 2700 in 2000, with more than a half published in 1997-2000. Although different interpretations of the term "control" are in use [1] the intensity of publications is unusually high. The control of chaos has been addressed recently in a few monographs, see References.

Surprizingly, the development of the field was triggered by essentially one paper, by E.Ott, C.Grebogi and J.Yorke from the University of Maryland (Ott et al, 1990), where the term "controlling chaos" was coined. Perhaps, the key achievment of the paper (Ott et al, 1990) was the demonstration of the fact that a significant change in the behavior of a chaotic system can be made by a very small, "tiny" correction of its parameters. This observation opened possibilities for changing behavior of natural systems without interferring with their inherent properties. The idea was quickly appreciated in physics and other natural sciences. It is worth noticing that, in spite of the enormous number of published papers, very few rigorous results are so far available. Most papers are written in a "physical style" and their conclusions are justified by computer simulations rather than analytical tools. Many problems still remain unsolved.

Describing some of the open problems is the aim of this brief survey. Three approaches to control of continuous-time chaotic systems will be surveyed: the so called "nonfeedback control", OGY method and Pyragas method. These approaches were historically the first in the field and produced the most number of publications.

In the Section 2 some preliminaries are given concerning used system models and control goals. The Section 3 is devoted to surveying three abovementioned approaches [2].

2 Models of Controlled Plant and Control Goals

We will consider continuous systems with lumped parameters described in state space by differential equations

$$\dot{x} = F(x, u), \tag{2.1}$$

where x is n-dimensional vector of the state variables; $\dot{x} = d/dt$ stands for the time derivative of x; u is m-dimensional vector of inputs (control variables). The vector–function $F(x, u)$ is usually assumed continuously differentiable. If external disturbances are present, more general

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[1]E.g. in some papers the term "control parameters" stands for bifurcation parameters, i.e. paper deals with analysis of chaotic system rather than with control of it.

[2]References to the Section 3 can be found in the bibliography on control of chaos (papers of 1997–2000) at www.rusycon.ru/chaos-con.html

time-varying models will be considered

$$\dot{x} = F(x, u, t). \qquad (2.2)$$

The model may also include the description of measurements $y = h(x)$, where y is l-dimensional vector of output variables y.

The typical goal for control of chaotic systems is stabilizing of an unstable periodic solution (orbit). Let $x_*(t)$ be the T-periodic solution of the free (uncontrolled, $u(t) = 0$) system (2.1) with initial condition $x_*(0) = x_{*0}$, i.e. $x_*(t + T) = x_*(t)$ for all $t \geq 0$. If the solution $x_*(t)$ is unstable it is reasonable to pose the goal as stabilization in some sense, e.g. driving solutions $x(t)$ of (2.1) to $x_*(t)$

$$\lim_{t\to\infty} [x(t) - x_*(t)] = 0 \qquad (2.3)$$

or driving the output $y(t)$ to the desired output function $y_*(t)$, i.e.

$$\lim_{t\to\infty} [y(t) - y_*(t)] = 0 \qquad (2.4)$$

for any solution $x(t)$ of (2.1) with initial conditions $x(0) = x_0 \in \Omega$, where Ω is given set of initial conditions.

The problem is to find a control function in the form of either an open loop, (or feedforward) control $u(t) = U(t, x_0)$ or in form of state feedback $u(t) = U(x(t))$ or output feedback $u(t) = U(y(t))$ to ensure the goal (2.3) or (2.4).

Such a problem is nothing but a tracking problem, very familiar to control theorists. However the key feature of the control of chaotic systems as claimed in (Ott et al, 1990) is to achieve the goal by means of sufficiently small (ideally, arbitrarily small) control. Solvability of this task is nontrivial since the solution $x_*(t)$ is unstable.

3 Methods of Controlling Chaos: Continuous Time

3.1 Feedforward (Open-loop) Control

The idea of *feedforward* control (also called *nonfeedback* or *open loop* control) is to change the behavior of a nonlinear system by applying a properly chosen input function $u(t)$ – external excitation. Excitation can reflect influence of some physical action, e.g. external force/field, or it can be some parameter perturbation (modulation). Such an approach is attractive because of its simplicity: no measurements or extra sensors are needed. It is especially advantageous for ultrafast processes, e.g. at the molecular or atomic level where no possibility of system variables measurement exists.

The possibility of significant changes to system dynamics by periodic excitation is known for almost a century. A number of authors discovered that high frequency excitation can stabilize unstable equilibrium of a pendulum (Stephenson, 1908C1; Kapitsa, 1951C1). This discovery triggered the development of vibrational mechanics (Blekhman, 2000C1). Analysis of general nonlinear systems affected by high frequency excitation is based on the Krylov-Bogoljubov averaging method (Bogoljubov and Mitropolsky, 1961C1). In control theory high frequency excitation and parameter modulation was studied within the framework of vibrational control (Meerkov, 1980C1; Bellman et al., 1984C1) and dither control (Zames and Shneydor, 1976C1). However the abovementioned works dealt only with the problem of stabilizing a given equilibrium or the desired (goal) trajectory.

Recently Morgul (1999C1a, 1999C1b) proposed the use of piecewise constant dither control to modify system dynamics (nonlinearity shape, equilibrium points, etc.) for systems in Lur'e form. In particular, the creation and elimination of chaotic behavior was studied using heuristic conditions for chaos suggested by Genesio and Tesi (1992C1). A vast literature is devoted to excitation with medium frequencies - those comparable with the natural frequencies of the system. The possibility of transformation of periodic motion into chaotic motion and vice versa was demonstrated by Alexeev and Loskutov (1987C1) for a 4th order system describing dynamics of two interacting populations. Matsumoto and Tsuda (1983C1) demonstrated the possibility of suppressing chaos in a Belousov-Zhabotinsky reaction by adding a white noise disturbance. These results were based on computer simulations. A first account of theoretical understanding of the phenomenon was given in (Pettini, 1988C1; Lima and Pettini, 1990C1), where the so called Duffing-Holmes oscillator

$$\ddot{\varphi} - c\varphi + b\varphi^3 = -a\dot{\varphi} + d\cos(\omega t) \qquad (3.5)$$

was studied by Melnikov's method. The right-hand side of (3.5) was considered as a small perturbation of the unperturbed Hamiltonian system. The Melnikov function related to rate of change of the distance between stable and unstable manifolds for small perturbations was calculated analytically and parameter values producing chaotic behavior of the system were chosen. Then additional excitation was introduced into the parameter of nonlinearity $b \to b(1+\eta\cos\Omega t)$ and the new Melnikov function was computed and studied numerically. It was shown that if Ω is close to the frequency of initial excitation ω then chaos may be destroyed. Experimental confirmation was made by a magnetoelastic device with two permanent magnets, electromagnetic shaker and optical sensor (Fronzoni et al., 1991C1). The results were surveyed in (Lima and Pettini, 1998C1) where some open problem were also posed.

Fronzoni and Giocondo (1998C1) showed by simulation of Josephson junction and liquid crystal models and by experiments with a bistable mechanical device that changing the phase and frequency of parameter perturbation can either decrease or increase the threshold of chaos.

Belhaq and Houssni(1999C1) considered the case of quasiperiodic excitations by reducing it to the periodic case, see also (Zhalmin, 1999C1; Belhaq and Houssni, 2000C1). Basios et al.(1999C1) studied the case of parametric noise excitation by Melnikov analysis. Tereshko and Schehinova(1998C1) suggested that the excitation frequency be chosen to resonate with the peak frequency of the power spectrum of one of the system variables. Mirus and Sprott (1999C1a, 1999C1b) attempted to achieve resonance of excitation with the frequency of the desired periodic excitation. Since a chaotic attractor contains trajectories close to periodic orbits with different periods, a proper choice should be made to minimize the amplitude of excitation. A numerical illustration of the approach was given for a Lorenz system (Mirus and Sprott 1999C1a) and for a high dimensional system of 32 diffusively coupled Lorenz systems. Harmonic excitation was introduced via modulation of parameter r. In the papers of Chizhevsky et al(1998C1), Pisarchik and Corbalan(1999C1) stabilization of unstable periodic orbits by means of periodic action with frequency much lower than the characteristic frequency of the system was demonstrated. Suppression of chaos in circular yttrium-ion-garnet films was discussed by Piskun and Wigen (1999K).

In a number of papers the choice of excitation function is based on tailoring it to the system nonlinearity. Let the controlled system be described by equations:

$$\dot{x} = f(x) + Bu, x \in \mathbb{R}^n, u \in \mathbb{R}^m. \qquad (3.6)$$

Now let $m = n$ and $\det B \neq 0$. If the desired solution of the controlled system is $x_*(t)$ then an intuitevely reasonable choice of excitation is

$$u_*(t) = B^{-1}(\dot{x}_*(t) - f(x_*(t)), \qquad (3.7)$$

because $x_*(t)$ will satisfy the equations of the excited system, see (Hübler and Lusher, 1989C1). The equation for the error $e = x - x_*(t)$ is then $\dot{e} = f(e + x_*(t)) - f(x_*(t))$. If the linearized system with matrix $A(t) = \partial f(x_*(t))/\partial x$ is uniformly stable in the sense that $A(t) + A(t)^T \leq -\lambda I$ for some λ and for all $t \geq 0$ then all solutions of (3.6), (3.7) will converge to $x_*(t)$ (more general convergence conditions can be found in (Fradkov and Pogromsky, 1998). In case $m < n$ and B is singular B the same result is valid under matching conditions: vector $\dot{x}_*(t) - f(x_*(t))$ is in the span of the columns of B. Then the control can be chosen to be $u_*(t) = B^+(\dot{x}_*(t) - f(x_*(t))$, where B^+ is the pseudoinverse matrix. Despite the fact that the uniform stability condition rules out chaotic (i.e. unstable) trajectories, it is claimed in a number of papers that some local convergence to chaotic trajectories is observed if the instability regions are not dominanting. Rajasekar et al.,(1997C1) compare this approach with other methods through 2nd order system example describing the so called Murali-Lakshmanan-Chua electronic circuit and FitzHugh-Nagumo equations describing propagation of nerve pulses in a neuronal membrane. Ramesh and Narayanan (1999C1) investigated (numerically) different schemes of nonfeedback excitation in the presence of noise.

In the paper of Fradkov et al.(1999C1) frequency-domain conditions for global convergence of the soltions of Lur'e systems to the steady-state mode with nonperiodic excitation were obtained. These analytical results are based on the corresponding results for the periodic case (Leonov, 1986C1) and allow for instability of the controlled system.

In summarizing then, a variety of different open-loop methods have been proposed. Most of them were evaluated by simulation for special cases and model examples. However, the general problem of finding conditions for creation or suppression of chaos by feedforward excitation still remains open.

3.2 Linearization of Poincaré Map (OGY Method)

In the seminal paper by Ott et al, (1990) two key ideas were introduced:

1. To use the discrete system model based on linearization of the Poincaré map for controller design.

2. To use the recurrent property of chaotic motions and apply control action only at time instants when the motion returns to the vicinity of the desired state or orbit.

The original version of the algorithm was described for discrete-time systems (iterated maps) of dimension 2 and for continuous-time systems of dimension 3 and required on-line computation of the eigenvectors and eigenvalues for the Jacobian of the Poincaré map. Numerous extensions and interpretations have been proposed by different authors in subsequent years and the method is commonly referred to as the "OGY method". According to the recent publications (Boccaletti et al, 2000; Grebogi and Lai, 1997C2a, 1997C2b; Grebogi et al, 1997C2a,1997C2b) the idea of the OGY method is as follows.

Let controlled system be described by the state space equations (2.1) where $x \in \mathbb{R}^n, u \in \mathbb{R}^1$. (Usually the variable u represents a changeable parameter of the system rather than a standard "input" control variable but it makes no diference from a control theory point of view). Obtain the desired (goal) trajectory $x_*(t)$ which is a solution of (2.1) with $u = 0$. The goal trajectory may be either periodic or chaotic: in both cases it is recurrent. Draw a surface (Poincaré section)

$$S = \{x : s(x) = 0\} \tag{3.8}$$

through the given point $x_0 = x_*(0)$ transversally to the solution $x_*(t)$ and consider the map $x \mapsto P(x,u)$ where $P(x,u)$ is the point of first return to S of the solution to (2.1) with constant input u started from x. The map $x \mapsto P(x,u)$ is called *the controlled Poincaré map.* It is well defined at least in some vicinity of the point x_0 owing to the recurrence property of $x_*(t)$ (The precise definition of the controlled Poincaré map requires some technicalities, see (Fradkov and Pogromsky, 1998). Iterating the map, we may define a discrete-time system

$$x_{k+1} = P(x_k, u_k), \tag{3.9}$$

where $x_k = x(t_k), t_k$ is the time of the kth crossing, $u_k = u(t)$ for $t_k < t < t_{k+1}$.

The next step of the control design is to replace the initial system (2.1) by the linearized one

$$\tilde{x}_{k+1} = A\tilde{x}_k + Bu_k, \tag{3.10}$$

where $\tilde{x}_k = x_k - x_0$ and find a stabilizing controller, e.g. $u_k = Cx_k$ for (3.9) Finally, the proposed control law is as follows:

$$u_k = \begin{cases} C\tilde{x}_k, & \text{if } \ |\tilde{x}_k| \le \Delta, \\ 0, & \text{otherwise,} \end{cases} \tag{3.11}$$

A key point of the method is to apply control only in some vicinity of the goal trajectory by introducing an "outer" deadzone. Though the law (3.11) looks like standard high-gain controller, the presence of the "outer" deadzone has the effect of bounding control action and makes it nontrivial to justify the method.

Nevertheless, numerous simulations performed by different authors confirmed the efficiency of such an approach. Often slow convergence was reported which is actually the price of achieving nonlocal stabilization of a nonlinear system by small control.

There are two important problems to solve for implementation of the method: lack of information about the system model and incomplete measurements of the system state. To overcome the first problem - uncertainty of the linearized plant model, Ott et al.(1990) and their followers suggested estimation of parameters in the state-space representation (3.10)(see survey papers Boccaletti et al.,(2000); Arecci et al.,(1998); Grebogi et al.,(1997C2)). However the detailed methods of extracting the parameters of the model (3.10) from the measured time series are yet to be presented.

The problem is of course well known in identification theory and is not straightforward, because identification in closed loop under 'good' control may prevent from 'good' estimation.

The second difficulty can be overcome by replacing the initial state vector x by the so called *delay coordinate vector* $X(t) = [y(t), y(t-\tau), \ldots, y(t-(N-1)\tau]^T \in \mathbb{R}^n$, where $y = h(x)$ is the output (e.g. one of the system coordinates) available for measurement and $\tau > 0$ is delay time. Then the control law has the form:

$$u_k = \begin{cases} \mathcal{U}(y_k, y_{k,1}, \ldots, y_{k,N-1}), \\ \quad \text{if } \ |y_{k,i} - y_*| \le \Delta, i = 1, \ldots, N-1 \\ 0, \ \ \text{otherwise,} \end{cases} \tag{3.12}$$

where $y_{k,i} = y(t_k - i\tau)$.

A special case of algorithm (3.12) introduced by Hunt(1991C2) was termed *occasional proportional feedback (OFP)* . OFP algorithm is used for stabilization of the amplitude of a limit cycle and is based on measuring local maxima(or minima) of the output $y(t)$, i.e. the Poincaré section is defined as (3.8) with $s(x) = \partial h/\partial x F(x,o)$, which corresponds to $\dot{y} = 0$. If y_k is the value of kth local maximum, then the OPF method suggests a simple control law

$$u_k = \begin{cases} K\tilde{y}_k, & \text{if } \ |\tilde{y}_k| \le \Delta, \\ 0, & \text{otherwise,} \end{cases} \tag{3.13}$$

where $\tilde{y}_k = y_k - y_*$ and $y_* = h(x_0)$ is the desired upper level of oscillations.

Further modifications and extensions to the OGY method have been recently proposed. Epureanu and Dowell (1997C2) used only data collected over a single period of oscillation. A quasicontinous extension of the OGY method has been proposed by Ritz et al,(1997C2). A multi-step version was studied by Holzhuter and Klinker(1998C2). Epureanu and Dowell (1998C2, 2000C2) suggested the use of a time-varying control function $u(t) = c(t)\bar{u}$ instead of a constant between crossings and choice of $c(t)$ to minimize control energy. Iterative refinement extending the basin of attraction and reducing the transient time was proposed by Aston and Bird (1997C2, 2000C2). Basins of attraction for the initial state and parameter estimates were evaluated by Chanfreau and Lyyjynen (1999C2), while transient behavior was also investigated by Holzhuter and Klinker(1998C2). New demonstrations of efficiency of the OGY method were obtained both by computer simulations for the Copel map (Agiza, 1999C2), the Bloch wall (Badescu et al, 1997C2), magnetic domain-wall system (Okuno et al, 1999C2) and by physical experiments with bronze ribbon (Schweinsberg et al, 1997C2), glow discharge (Braun, 1998C2) and nonautonomous RL-diode circuit (Bezruchko et al, 1999C2). The OPF method has been used for stabilization of the frequency emission from a tunable lead-salt stripe geometry infrared diode laser and implemented in an electronic chaos controller (Senesak et al, 1999C2). A modification of OPF was investigated by Flynn and Wilson (1998C2).

However, only partial results on justification of the algorithms (3.12) and (3.13) are available. The main problem is estimation of the accuracy of the linearized Poincaré map in the delayed coordinates:

$$y_k+, \dots, +a_{N-1}y_{k,N-1} = b_1 u_k+, \dots, +b_{N-1}u_{k-N-1}. \qquad (3.14)$$

In (Fradkov and Guzenko, 1997C2; Fradkov et al., 2000C2) a justification of the above method was given for the special case when $y_{k,i} = y_{k-i}, i = 1, \dots, n$. In this case the outputs are measured and control action is changed only at the instants of crossing the surface, see also (Fradkov and Pogromsky, 1998). For controller design an input-output model (3.14) was used containing fewer coefficients than (3.10). For estimation the method of recursive goal inequalities due to Yakubovich was used introdicing additional inner deadzone to resolve the problem of estimation in closed loop. Inner deadzone combined with outer deadzone of the OGY method, provides robustness of the identification-based control with respect to both model errors and measurements errors.

Justification of adaptive control methods for other cases is still an open problem.

3.3 Delayed feedback

During recent years there has been increasing interest in the method of time-delayed feedback (Pyragas, 1992C3). K.Pyragas, a Lithuanian physicist proposed to find and stabilize a τ-periodic orbit of the nonlinear system (2.1) by a simple control action

$$u(t) = K[x(t) - x(t-\tau)] \qquad (3.15)$$

where K is feedback gain, and τ is time-delay. If τ is equal to the period of an existing periodic solution $\bar{x}(t)$ of (2.1) for $u = 0$ and the solution $x(t)$ to the closed loop system (2.1), (3.15) starts from $\Gamma = \{\bar{x}(t)\}$, then it will remain in Γ for all $t \geq 0$. A puzzling observation was made, however that $x(t)$ may converge to Γ even if $x(0)\bar{\in}\Gamma$.

The law (3.15) applies also to stabilization of forced periodic motions in the system (2.1) with a T-periodic right-hand side. Then τ should be chosen equal to T.

An extended version of Pyragas method has also been proposed with

$$u(t) = K\sum_{k=0}^{M} r_k[y(t-k\tau) - y(t-(k+1)\tau] \qquad (3.16)$$

where $y(t) = h(x(t)) \in \mathbb{R}^1$ is the observed output and $r_k, k = 1, \dots, M$ are tuning parameters. For $r_k = r^k, |r| < 1$, and $M \to \infty$ the control law (3.16) becomes:

$$u(t) = K[y(t) - y(t-\tau)] + Kru(t-\tau) \qquad (3.17)$$

Although algorithms (3.15)-(3.17) look simple, analytical study of the closed loop behavior seems difficult. Until recently only numerical and experimental results concerning performance and limitations of Pyragas method were available.

In (Basso et al, 1997C3; Basso et al, 1998C3) the stability of a forced T-periodic solution of a Lur'e system (system represented as feedback connection of a linear dynamical part and a static nonlinearity) with a modified Pyragas law

$$u(t) = G(p)[y(t) - y(t-\tau)] \qquad (3.18)$$

where $G(p), p = d/dt$ is transfer function of the filter, was investigated. Using absolute stability theory (Fradkov and Pogromsky, 1998) sufficient conditions on the transfer function of the linear part of the controlled system and on the slope of nonlinearity were obtained under which there exist stabilizing $G(p)$. A procedure for "optimal" controller design, maximizing the stability bound was proposed in (Basso et al, 1998C3). Extension to systems with a nonlinear nominal part and a general framework based on frequency-domain tools are presented in (Basso et al, 1999C3).

Ushio (1996C3) established for a class of discrete-time systems that a simple necessary condition for stabilizability with a Pyragas controller (3.15) is that the number of real eigenvalues of matrix A greater than one should not be odd, where A is the matrix of the system model linearized near the desired fixed point. Proofs for more general and continuous-time cases were given independently by Just et al. (1997C3) and Nakajima (1997C3). The corresponding results for an extended control law (3.16) were presented in (Nakajima and Ueda, 1998C3; Konishi et al,1999C3), who applied Floquet theory to the system linearized near the desired periodic solution. Using a similar approach, Just et al.(1999C3) gave a more detailed analysis and established approximate bounds for a stabilizing gain K. Some bounds for K for a Lorenz system were obtained by Simmendinger et al (1997C3) using the Poincaré small parameter method.

Schuster and Stemmel (1997C3) noticed that for a scalar autonomous discrete-time system $y_{k+1} = f(y_k, u_k)$ a necessary condition for existence of a discrete version of the stabilizing feedback (3.16) is $\lambda < 1$, where $\lambda = \partial f/\partial y(0,0)$, following from the theorem of Giona (1991C3). They showed that restriction $\lambda < 1$ can be overcome by periodic modulation of the gain K.

The Pyragas method was extended to coupled (open flow) systems (Konishi et al., 1998C3; 2000C3a; 2000C3b), modified for systems with symmetries (Nakajima and Ueda, 1998C3). It was also extended to include an observer estimating the difference between the system state and the desired unstable trajectory (fixed point) (Konishi and Kokami, 1998C3).

Reported applications include stabilization of coherent modes of lasers (Bleich et al, 1997C3; Loiko et al, 1997C3; Naumenko et al,1998C3), magnetoelastic systems (Hai et al, 1997C3; Hikihara et al, 1997C3), control of cardiac conduction model (Brandt et al, 1997C3), control of stick-slip friction oscillations (Elmer, 1998C3), traffic models (Konishi et al, 1999C3,2000C3), PWM controlled buck convertor (Battle et al, 1999C3), paced excitable oscillator described by Fitzhugh-Nagumo model widely used in physiology (Bleich and Socolar,2000C3), catalytic reactions in bubbling gas-solid fluidized bed reactors (Kaart et al, 1999K). A comparison of delayed feedback and feedforward methods for control of chaos in lasers was presented by Glorieux(1998K).

A drawback of the control law (3.15) is its sensitivity to parameter choice, especially to the choice of the delay τ. Apparently, if the system is T-periodic and the goal is to stabilize a forced T-periodic solution, then the choice $\tau = T$ is mandatory. Alternatively an heuristic trick is to simulate the unforced system with initial condition $x(0)$ until the current state $x(t)$ approaches $x(s)$ for some $s < t$, i.e. until $|x(t) - x(s)| < \varepsilon$. Then the choice $\tau = t - s$ will give a reasonable estimate of a period and the vector $x(t)$ will be an initial condition to start control. However such an approach often gives overly large values of the period. Since chaotic attractors contain periodic solutions of different periods, an important problem is to find and to stabilize (with small control) the solution with the smallest period. This problem remains open.

Conclusions

Three major branches of research, related to "nonfeedback control", OGY method and Pyragas method are historically the first ones and currently florishing in the field of "Control of Chaos". It is seen, however that some important problems of justification of existing control algorithms remain unsolved and provide challenges for control theorists of the XXIst century.

References

Chen, G., and X. Dong, (1998), *From chaos to order: perspectives, methodologies and applications.* (World Scientific, Singapore).

Fradkov, A.L. and Pogromsky, A.Yu.(1998) *Introduction to control of oscillations and chaos.* (World Scientific, Singapore).

Handbook of Chaos Control (1999) /Ed. H.G.Schuster, (Wiley & Sons).

Controlling Chaos and Bifurcations in Engineering Systems(1999)/Ed.G.Chen, CRC Press.

Ott, E., C. Grebogi, and J. Yorke, (1990), Controlling chaos. *Phys. Rev. Lett.*, vol. **64**(11), pp. 1196–1199.