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Discrete-Time VSS Control under Disturbances

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Abstract

Discrete-time VSS control algorithms proposed in [1-5] are analyzed for the controlled plant influenced by deterministic or stochastic disturbances of bounded intensity. For both cases, the bounds for transient and steady-state performance are obtained for minimum-phase plants.

1 Introduction

In papers [1-5], a new approach to design of robust and adaptive VSS controllers was proposed. This approach is based on the concepts of "sliding sector" and "β - equivalent control". An important problem is performance of new control systems under disturbances. In [5] the robustness of the new discrete-time VSS controllers was examined under input (additive) disturbances with known linear internal model and under bounded parametric (multiplicative) disturbances. However, the case of additive unknown disturbances still has not been investigated.

In the present paper some algorithms proposed in [1-5] are analyzed for the controlled plant influenced by deterministic or stochastic disturbances of bounded intensity. In the former case, disturbances are supposed to be uniformly bounded. In the latter case, the disturbances are assumed to bounded in the mean square. For both cases, the bounds for transient and steady-state performance are obtained for minimum-phase plants. The results are also extended to MIMO plants.

In Section 2 the state-feedback discrete-time VSS controller is introduced following [2.4] and the performance bounds for uniformly bounded disturbance are established. In Section 3 similar results for the disturbances bounded in the mean square sense are formulated. In the Conclusions some further results are outlined.

The work was performed while the first author was with Tokyo Institute of Technology.

2 Discrete-time VSS under bounded disturbances

Consider the linear discrete-time plant

$$x_{k+1} = Ax_k + Bu_k + D\varphi_k, k = 1, 2, \dots \tag{1}$$

where $x_k \in R^n$, $u_k \in R^1$ are the state vector and scalar input, respectively, $\varphi_k \in R^{n_\varphi}$ is the disturbances vector which is supposed to be bounded:

$$\|\varphi_k\| \le \Delta_\varphi, k = 1, 2, \dots \tag{2}$$

Define the discrete-time VSS controller as in [2,4]:

$$u_k = -(CB)^{-1}[CAx_k - \beta s_k - z_k] \tag{3}$$

where $1 > |\beta| > 0$,

$$s_k = Cx_k \in R^1. \tag{4}$$

Auxiliary control z_k is defined as

$$z_k = \begin{cases} 0, & |s_k| \le \delta(x_k) \\ -f_0\,(\mathrm{sign}\beta s_k)|x_k|, & |s_k| > \delta(x_k), \end{cases} \tag{5}$$

where $|x_k| = \sum_{i=1}^n |x_{ki}|$, $f_0 > 0$, $\delta(x_k) = f_0|x_k|/(2|\beta|)$.

The stability and performance properties of system (1)-(5) are formulated in the following theorem.

Theorem 1 *Let the undisturbed plant (1) be minimum phase with respect to output s_k, i.e. the polynomial*

$$C(\lambda) = W(\lambda)\alpha(\lambda)$$

be stable, where

$$\begin{aligned} W(\lambda) &= C(\lambda I - A)^{-1}B \\ \alpha(\lambda) &= det(\lambda I - A). \end{aligned}$$

Let $|\beta| < \bar\lambda$, where $\bar\lambda = max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_{n-1}|\}$, λ_i are zeros of the polynomial $C(\lambda)$. Then for any $\varepsilon > 0$, such that $\bar\lambda + \varepsilon < 1$ the state of system (1)-(5) satisfies inequality

$$\|x_k\| \le C_\varepsilon[\Delta_\varphi + (\bar\lambda + \varepsilon)^k \|x_0\|], \tag{6}$$

where the constant $C_\varepsilon > 0$ depends on matrices A, B, C, D.

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The proof employs the ideas of the proof of the theorem 1 [4] and the following key lemma.

Lemma 1 *Consider linear discrete-time controllable system*

$$x_{k+1} = Ax_k + Bu_k, s_k = Cx_k \qquad (7)$$

with transfer function $W(\lambda) = C(\lambda I - A)^{-1}B = \frac{C(\lambda)}{A(\lambda)}$ *If system (7)is minimum phase, and* s_k *satisfies inequalities*

$$|s_k| \le |\beta|^k |s_0| + \Delta, |\beta| < \bar{\lambda} = \max_i |\lambda_i|, \qquad (8)$$

where λ_i *are zeros of polynomial* $C(\lambda)$*, then for any* $\varepsilon > 0$*,such that* $\bar{\lambda} + \varepsilon < 1$ *there exists* R_ε *such that* x_k *satisfies the inequality:*

$$\|x_k\| \le R_\varepsilon(\bar{\lambda}+\varepsilon)^k(\|x_0\| + \frac{|s_0|}{(\bar{\lambda} - |\beta|)|c_0|}) + R_\varepsilon \frac{\Delta}{(1 - \bar{\lambda} - \varepsilon)|c_0|} \qquad (9)$$

Remark 1 *Similar result is valid for* $|\beta| > \bar{\lambda}$*. In this case the amount* $\bar{\lambda} + \varepsilon$ *should be replaced by* $|\beta|$*, and* $\varepsilon > 0$ *chosen to satisfy inequality* $\bar{\lambda} + \varepsilon < |\beta|$

3 Discrete-time VSS under stochastic disturbances, bounded in mean square

Suppose that the disturbances vector φ_k in (1) is stochastic vector, bounded in the mean square;

$$M\|\varphi_k\|^2 \le \sigma^2 \qquad (10)$$

In this case the same control law (3) provides mean square boundedness of the system trajectories. It is formulated in the following theorem.

Theorem 2 *Let the plant (1) be minimum phase with relative degree 1 with respect to output* s_k *(4).*

Let disturbances be mutually independent and satisfy (10). Then for any $\varepsilon > 0$ *there exist* $c'_\varepsilon > 0, c''_\varepsilon > 0$ *, such that*

$$M\|x_k\|^2 \le C'_\varepsilon \|x_0\| (\bar{\lambda} + \varepsilon)^k + C''_\varepsilon \sigma^2 \qquad (11)$$

To prove Theorem 2, the following bound can be derived similarly to the proof of Theorem 1:

$$M|s_k|^2 \le \bar{\beta}^k |s_0| + \frac{\bar{\beta}\|CD\|}{(1-\bar{\beta})(\bar{\beta} - |\beta|)}\sigma^2 \qquad (12)$$

And then the stochastic version of Lemma 1 can be applied, providing the mean-square bounds for solution of linear system excited by bounded in mean-square disturbance s_k with exponentially decaying transients.

4 Conclusions

We have derived hard bounds for both transient and steady-state behavior of discrete-time VSS control systems. The following extensions of the above results can be obtained similarly to the theorems of sections 2,3.

1. The tracking problem for the plants described by transfer functions (or difference equations)

$$A(q^{-1})y_k = q^{-d}B(q^{-1})u_k + \varphi_k \qquad (13)$$

can be treated in similar way to Theorems 1,2 after transforming the plant into the state space form (1). The bounds for tracking error $e_k = y_k - r_k$ can be obtained, using e.g.(18) of [5] (see also [2,3]).

Note that extension to the case of self-tuning control of disturbed plant is not straightforward. To deal with self-tuning discrete-time VSS systems one may introduce a dead zone into adaptation algorithm, like in the recursive goal inequalities approach [7,8].

2. The VSS approach may be extended to the nonlinear discrete-time plants using method of recursive goal inequalities [7.8].

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