# ADAPTIVE CONTROL OF OSCILLATORY AND CHAOTIC SYSTEMS BASED ON LINEARIZATION OF POINCARÉ MAP

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## Abstract

The new method of output feedback adaptive control of oscillatory processes is suggested based on goal inequalities and input-output form of linearized controlled Poincaré map. Conditions of achieving the goal (tracking) with given accuracy are established. The method is applied to modifying behavior of model systems: brusselator in periodic self-oscillatory mode and brusselator in chaotic forced oscillatory mode.

## 1 Introduction

The importance of oscillatory processes and models has been recognized in many fields of science and technology[1],[2]. In many applications it is necessary to modify the behavior of the system, e.g. to change amplitude or period of oscillations, to transform chaotic motions into periodic ones ans vice versa. This problem can be understood as one of control. It was attacked recently by reseachers from different scientific communities. However the modern nonlinear control [5] and nonlinear adaptive control [8] theories have not been applied widely. Methods of adaptive control are of special interest for applications because they are intended to achieve the desired behavior of the system without complete knowledge of its mathematical model.

The paper is devoted to design and application of adaptive control algorithms based on linearization of Poincaré map and on the method of recursive goal inequalities[8]. The idea of using Poincaré map for control of chaotic oscillations was introduced by Ott *et.al.*[9] and got significant attention of reseachers (see [3],[15]). However it still has not been investigated from control point of view. Particularly, the problem of output feedback adaptive control design remains unsolved. Below the solution to this problem is given based on method of recursive goal inequalities suggested by V.Yakubovich in 1966. In Sec.2 general mathematical results concerning approximation of controlled Poincaré map of nonlinear system by means of simplified linear input-output model are formulated. The finite-convergent adaptive control algorithm based on estimation of model parameters is described in Sec.3. In the concequent sections theoretical results are applied to model chemical systems. In Sec.4 the method of Sec.3 is applied to the limit cycle stabilization of brusselator model. Similar problem for chaotic oscillations of forced brusselator is considered in Sec.5.

## 2 Problem Statement and Linearization of Controlled Poincaré Map

Consider nonlinear controlled system described by state space model

$$\dot{x} = F(x,u), \quad y = h(x), \tag{1}$$

where $x{=}x(t)$ is $n-$dimensional state vector of the system; $u{=}u(t)$ is scalar input (control action); $y{=}y(t)$ is scalar output available for measurement. The problem is to find control law (control algorithm) of form $u(t) = U\{y(\tau),u(\tau),0 \le \tau \le t\}$ such that $u(t){\in}\mathrm{U}$ (U is the convex set of admissible control values, for example, $\mathrm{U}{=}[-\overline{u},\overline{u}]$ for $\overline{u}{>}0$). and the following control goal is achieved

$$|y(t) - y_*(t)| < \Delta, \tag{2}$$

where $y_*(t){=}h(x_*(t))$ is the desired output function corresponding to desired periodic or recurrent trajectory (orbit) $x_*(t)$ of system (1) for $u(t) \equiv u_*$. Recall that the trajectory $x(t)$ is called recurrent[1] if for any $\varepsilon$ >0 it returns to $\varepsilon$-vicinity of arbitrary point not later than after some finite time $T_\varepsilon$. Recurrency allows for both periodic and chaotic behavior of the trajectory.

The difficulty of the above problem is caused by its significant nonlinearity. Moreover, in many applications some of the parameters of system (1) are unknown, i.e. the desired orbit $x_*(t)$ and "ideal" control $u_*$ are unknown too. Finally, sometimes the values $y_*(t)$ are specified and $y(t)$ can be measured only in some instances $t_k, k{=}1,2,...$

To solve the problem we reformulate it in the discretized version. Assume that the hypersurface $S_u$ in the state space is given which depends on the control value as on parameter and intersects the given base orbit $\overline{x}(t)$ in the point $\overline{x}_0=\overline{x}(0)$. Assume further that $S_u$ is transversal to $x(t)$ for all $u\in$U. It can be shown that there exists (smaller) open set $\overline{S}_u\subset S_u$ such that any trajectory of (1) starting at the point $x\in\overline{S}_u$ will meet again the surface $S_u$ at least once at the point $x'=P(x,u)$. The mapping $P:\overline{S}_u\times \mathrm{U}\rightarrow S_u$ is called the controlled Poincaré map. It defines the new discrete-time controlled system

$$x_{k+1}=P(x_k,u_k),\quad y_k=h(x_k),\quad k=1,2,\ldots \tag{3}$$

where $u_k\in$U, $x_k=x(t_k)\in S_{u_k}$, at least for $x_k$ close to $\overline{x}$. The trajectory of (3) coincides with trajectory of initial system (1) at time instances $t_k$ when $x(t)$ crosses the surface $S_{u_k}$, if control action is piecewise constant between crossings: $u(t)=u_k, t_k\leq t<t_{k+1}$. Let $z\in R^{n-1}$ be coordinate vector in the vicinity $\overline{S}$ of $\overline{x}_0$ in some coordinate frame $z(x)$. Without loss of generality we may assume that $z(\overline{x}_0)=0$ and consider the system (3) as one with state space $R^{n-1}$:

$$z_{k+1}=\tilde{P}(z_k,u_k),\qquad y_k=\tilde{h}(z_k) \tag{4}$$

where $\tilde{P}(0,0)=0$; $\tilde{h}(0)=0$. Note that the trajectories of (4) may be well defined not for all $z\in\overline{S}_u$ and all $k$=1,2,..., because $\tilde{P}(z_k,u_k)$ may leave $\overline{S}_u$ (e.g. if $\overline{x}(t)$ is unstable solution to (1)). However, for any $k>0$ there exists (smaller) set $S_u^k$ such that $z_i\in S_{u_i}^i$ for all $i\leq k$ and for all $u_i\in$U, where vectors $z_i$ are generated by system (4). (Just take $S_u^k=S_u^{k-1}\cap P^{-1}(S_u^{k-1},u)$, $k$=1,2,..., $S_u^0=\overline{S}_u$). Moreover, owing to recurrency of $\overline{x}(t)$ for any $k_*$, there exist $k>k_*$ such that $x_k\in\overline{S}_u$ and therefore $z_k,z_{k+1}$ are well defined. The typical case when the above setting is important appears when only local extremums (e.g. maximums) of $y(t)$ can be measured. Then the surface $S_u$ can be defined by equation $\dot{y}$=0 or $\nabla h(x)^T F(x,u)$=0, where $\nabla$ stands for the gradient of scalar function.

Introducing system (4) we reduce the problem of stabilizing of the orbit $x_*(t)$ to that of stabilizing the fixed point (origin) of the discrete-time system (4). To solve this new problem we need the following result.

**Theorem 1.** Let function $F(\cdot)$ be twice continuously differentiable and all the trajectories $x(t)$ of (1) for $x_0-\overline{x}_0$ sufficiently small and $u\in$U be recurrent.
Then there exist positive numbers $\Delta_z$,$\Delta_u$,$L_f$ such that for $||z||\leq\Delta_z$, $||u||\leq\Delta_u$ the following representation is valid

$$\tilde{P}(z,u)=Az+Bu+f(z,u), z\in\overline{S}_u, u\in U \tag{5}$$

where $A$,$B$ are matrices of size (n-1)×(n-1), (n-1)×1, respectively, and $||f(z,u)||\leq L_f(||z||^2+||u||^2)$. If, additionally, function $h(\cdot)$ is smooth, then the discrete-time system (4) can be described by input-output model:

$$y_{k+1}+..+a_{n-2}y_{k-n+2}=b_0u_k+..+b_{n-2}u_{k-n+2}+\varphi_k \tag{6}$$

where $a_i,b_i$ are coefficients of the transfer function of linearized system (4) : $\frac{B(\lambda)}{A(\lambda)}$=$C(\lambda I$-$A)^{-1}B$, $C$=$\frac{\partial\tilde{h}(0)}{\partial z}$, $B(\lambda)$=$\sum_{i=0}^{n-2}b_i\lambda^i$, $A(\lambda)$=$\lambda^{n-1}+\sum_{i=0}^{n-2}a_i\lambda^i$, and the disturbance $\varphi_k$ satisfies inequality

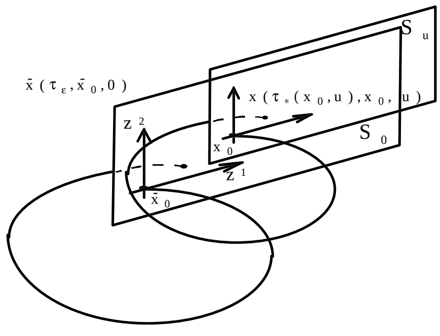


Fig. 1: Controlled Poincaré map

$$|\varphi_k|\leq L_\varphi(1+||A||)^{2n}(\Delta_z^2+\Delta_u^2) \tag{7}$$

*Proof.* Let $\delta(t)=x(t)-\overline{x}(t)$, where $x(t)$=$x(t,x_0,u)$- the solution to system (1) with initial condition $x_0$=$x(0)$, thus the linear part of (1) with respect to $\delta(t),u$ is as follows

$$\frac{d}{dt}\delta(t)=A(t)\delta(t)+B(t)u+\varphi(t) \tag{8}$$

where $A(t)$=$\frac{\partial F}{\partial x}(\overline{x}(t),0)$, $B(t)$=$\frac{\partial F}{\partial u}(\overline{x}(t),0)$, and the disturbance $\varphi(t)$ satisfies the inequality

$$||\varphi(t)||\leq L_\varphi(||\delta(t)||^2+||u||^2), \tag{9}$$

where $L_\varphi$ for example may be chosen as maximal Lipschitz constant for vector-function $F(x,u)$ components derivatives. Let $\tau_\varepsilon\leq T_\varepsilon$ be the time of the first falling of the recurrent base trajectory $\overline{x}(t,\overline{x}_0,0)$ onto $\varepsilon$-vicinity of $\overline{x}_0$ on the hypersurface $S_0$, as it shown in Fig.1 (if $\overline{x}(t)$ is periodic then $\tau_\varepsilon\equiv T_\varepsilon$ is simply the period). It's easy to seen that at least for some $\Delta_x>0$,$\Delta_u>0$ and for $||x_0$-$\overline{x}_0||<\Delta_x$, $||u||<\Delta_u$ the solution $x(t,x_0,u)$ intersects the hypersurface $S_u$ in some time instant $\tau_*(x_0,u)$ satisfying the inequality $|\tau_*(x_0,u)-\tau_\varepsilon|\leq L_\tau(||x_0$-$\overline{x}_0||+||u||)$. To improve the bound (9) let us estimate the value of $\mu(t)$=$||\delta(t)||$: $\mu(t)\leq\int_0^t||F(x(\tau,u)-F(\overline{x}(\tau,u)||d\tau+\mu(0)\leq L_F\int_0^t(||\delta(\tau)||+||u||)d\tau+\mu(0)= L_Ft||u||+\mu(0)+L_F\int_0^t\mu(\tau)d\tau$, where $L_F$ is the Lipschitz constant for function $F(x,u)$. It follows form the Gronwall's lemma that the following holds: $\mu(t)\leq(L_Ft||u||+||x_0-\overline{x}_0||)e^{L_Ft}\leq(L_F\tau_\varepsilon||u||+||x_0$-$\overline{x}_0||)e^{L_F\tau_\varepsilon}$, and $\varphi(t)$ satisfies the inequality

$$\varphi(t)\leq 2L_\varphi e^{2L_F\tau_\varepsilon}||x_0-\overline{x}_0||^2+L_\varphi(2L_F^2\tau_\varepsilon^2e^{2L_F\tau_\varepsilon}+1)||u||^2 \tag{10}$$

The solution to the inhomogeneous linear differential system (8) has the form $\delta(t)$=$\Phi(t,0)\delta(0)+\int_0^t\Phi(t,\tau)[B(\tau)u+\varphi(\tau)]d\tau$, where $\Phi(t,\tau)$ is the fundamental matrix of the system $\dot{x}$=$A(t)x$. Using $t$=$\tau_*(x_0,u)$ one can obtain

$$P(x_0,u)-\overline{x}_0=A(x_0,u)(x_0-\overline{x}_0)+B(x_0,u)u+\varphi(x_0,u), \tag{11}$$

where $A(x_0,u)=\Phi(\tau_*(x_0,u),\tau_\varepsilon)\Phi(\tau_\varepsilon,0)$,
$B(x_0,u)=\int_0^{\tau_*(x_0,u)}\Phi(\tau_*,\tau)B(\tau)d\tau$,
$\varphi(x_0,u)=\int_0^{\tau_*(x_0,u)}\Phi(\tau_*,\tau)\varphi(\tau)d\tau$.
Further, it may be shown that the following takes place: $||\Phi(\tau_*,\tau_\varepsilon)-I_n||\leq L_\Phi(||x_0$-$\overline{x}_0||+||u||)$. Then (11) becomes

$$P(x_0,u)-\overline{x}_0=\overline{A}(x_0,u)(x_0-\overline{x}_0)+\overline{B}(x_0,u)u+\overline{\varphi}(x_0,u), \quad (12)$$

where $\overline{A}$=$\Phi(\tau_\varepsilon,0)$, $\overline{B}$=$\int_0^{\tau_\varepsilon}\Phi(\tau_\varepsilon,\tau)B(\tau)d\tau$, and the disturbance $\overline{\varphi}(t)$ satisfies the inequality $\overline{\varphi}(t)\le\overline{L}(||x_0\text{-}\overline{x}_0||^2+||u||^2)$. Finally, projecting (12) onto the tangent hyperplane to $S_u$ and using that $\tilde{P}(z,u)\in S_u$ for $z\in S_u$, we obtain the representation (5). The remaining part of the theorem follows from the standard transformation of state-space system equations to input-output form. The theorem is proved.

It follows from Theorem 1 that the initial problem can be replaced by the discrete-time control problem for system (6) with the goal

$$|y_k-y_*|<\Delta_y,\ \Delta_y>0, \quad (13)$$

where $\Delta_y$ can be evaluated in terms of $\Delta$ in (2) and bounds of system parameters.

# 3 Main Result

If coefficients $a_i,b_i$ of the model (6) were known and the disturbances $\varphi_k$ were small, the following linear dynamic feedback controller would solve the problem:

$$u_k^*=[y_*+\sum_{i=0}^{n-2}a_iy_{k-i}-\sum_{i=1}^{n-2}b_iu_{k-i}]/b_0 \quad (14)$$

To overcome uncertainty we employ the idea of adaptive control and replace the true values $a_i$,$b_i$ by their estimates $\hat{a}_{ik}$,$\hat{b}_{ik}$ generated by adaptation algorithm. Additionally, we need to to keep controls inside the admissible set U. Introduce the vector of tunable parameters $\vartheta_k\in R^{2n-2}$ and vector of observations (regressor) $w_k=col\{y_*,y_k,...,y_{k-n+2},u_{k-1},...,u_{k-n+2}\}\in R^{2n-2}$. Choose the main loop control law as follows:

$$\begin{aligned} u_k' &= \vartheta_k^T w_k \\ u_k &= \begin{cases} u_k' & \text{if } |u_k|\le\overline{u} \\ u_{k-1} & \text{otherwise} \end{cases} \end{aligned} \quad (15)$$

It is clear that for $\vartheta_k$=$\vartheta^*$= $b_0^{-1}$col$\{1,a_0..a_{n-2},\text{-}b_1..\text{-}b_{n-2}\}$ control law (15) coincides with ideal law (14) (if $|u_k|$≤$\overline{u}$).

For adaptation algorithm design the method of goal inequalities [6],[8] is used. Adaptation algorithms based on goal inequalities contain the dead zones which suppress the influence of disturbances. In our case the disturbance in the model (6) becomes significant outside some vicinity of the base trajectory $\overline{x}(t)$. Therefore it is reasonable to introduce "inverse" deadzone (switching adaptation off for large values of $||x_k-\overline{x}(t_k)||$ or $||z_k-\overline{z}_k||$ exceeding some treshold), together with the "goal" deadzone (corresponding to the goal inequality (13). Finally, we propose the following adaptation algorithm:

$$\begin{aligned} \mu_{k+1} &= \begin{cases} 1 \text{ if } |y_{k+1}-y_*|>\Delta_y \text{ and} \\ \quad |y_{k-i}-\overline{y}(t_{k-i})|<\overline{\Delta}, i=0..N-1, \\ 0, \text{ otherwise;} \end{cases} \\ \vartheta_{k+1}' &= \begin{cases} \vartheta_k-\gamma\text{sign}(b_0)(y_{k+1}-y_*)w_k/|w_k|^2 \\ \quad \text{if } \mu_{k+1}=1, \\ \vartheta_k \text{ otherwise;} \end{cases} \\ u_{k+1}' &= \vartheta_{k+1}'^T w_{k+1} \\ \vartheta_{k+1} &= \begin{cases} \vartheta_{k+1}', \text{ if } |u_{k+1}'|\le\overline{u} \text{ and } \mu_{k+1}=1 \\ \vartheta_{k+1}'-(u_{k+1}'-\overline{u})/|w_k|^2, \\ \quad \text{if } u_{k+1}'>\overline{u} \text{ and } \mu_{k+1}=1 \\ \vartheta_{k+1}'-(u_{k+1}'+\overline{u})/|w_k|^2, \\ \quad \text{if } u_{k+1}'<-\overline{u} \text{ and } \mu_{k+1}=1 \\ \vartheta_k, \text{ if } \mu_{k+1}=0. \end{cases} \end{aligned} \quad (16)$$

where $\gamma$>0 is adaptation gain, $\overline{u}$ is the maximum absolute value of control, $\Delta_y$ is the maximal desired difference between $y_k$ and $y_*$, $\overline{\Delta}$ is connected with the size of the "tube" in the state space near the base trajectory $\overline{x}(t)$ where the input-output model (6) is well defined. We need to impose some version of the observability property: discrete nonlinear system (4) is called *strongly N-observable* if

$$\forall\varepsilon>0\ \exists\delta>0: |y_{k+i}|<\delta, i=0..N-1\Rightarrow ||z_k||<\varepsilon \quad (17)$$

The convergence properties of the proposed adaptive controller are formulated in the following assertion.

**Theorem 2.** Let $F$ in (1) be twice continuously differentiable, $h$ be continuously differentiable and assume that

**A1.** The base solution $\overline{x}(t)$ is recurrent;

**A2.** The system (4) is $N$-observable for some $N$>0;

**A3.** The sign of $b_0$ in (6) is known;

**A4.** Parameters of the system and the goal satisfy the following restrictions:

$$|b_0|-\sum_{i=1}^{n-2}|b_i|>0, |y_*|<\overline{u}\left(|b_0|-\sum_{i=1}^{n-2}|b_i|\right) \quad (18)$$

Then there exists $\Delta_0$>0 such that for any $\Delta_y$<$\Delta_0$ there exist $\overline{\Delta}$>$\Delta_0$, $\gamma$>0,$\lambda\in$(0,1) such that the goal (13) with $\lambda y_*$ and $\lambda\Delta_y$ is achieved for all sufficiently large $k$ >0 in the system (1),(15),(16) under restriction $|u_k|$<$\lambda\overline{u}$.

*Remark 1.* The condition A1-A4 cannot ensure that for arbitrary given $y_*$ and $\overline{u}$ the goal (13) can be achieved by the control law (15),(16). However it follows from Theorem 2 that there exist small positive multiplier $\lambda$ such that the simultaneous reduction of our goal and the restriction on control makes it possible to achieve the reduced goal.

*Remark 2.* The similar theorem may be proved for identification-based adaptation algorithm.

*Proof.* For any $\varepsilon$>0 and $\overline{x}_0$ we may choose $T_\varepsilon$ - the upper estimate of the return time for all $k$>0. Thus, all trajectories which started in $\varepsilon$-vicinity of $\overline{x}_0$ will be bounded. The constant $C_\varphi$ in the estimate $|\varphi_k|$≤$C_\varphi(||z_k||^2$+$||u_k||^2)$ will be bounded too. Due to condition A4 we may choose $\overline{\Delta}$>0 such that

$$|y_*|<\overline{u}\left(|b_0|-\sum_{i=1}^{n-2}|b_i|\right)-\overline{\Delta}\sum_{i=0}^{n-2}|a_i| \quad (19)$$

This choice ensures that the "ideal" control (14) is admissible if $|y_{k-i}|<\overline{\Delta}$ and $|u_{k-i}|<\overline{u}$ ($i$=0..n-1). Due to condition $\tilde{h}(0)=0$ and $h(\cdot)\in C^1$ we may choose $\Delta_z>0$ such that inequality $\|z\|<\Delta_z$ implies $|\tilde{h}(z)|<\overline{\Delta}$. Then decrease $\varepsilon$ if necessary until the inequality $\|z_k\|<\varepsilon$ implies $\|z_{k+i}\|<\Delta_z$ for $i$=1..N, where N is the number from the condition A2. After that we may choose $\Delta_0<\overline{\Delta}$ so that the following inequality holds:

$$|y_*| + \Delta_0 < \overline{u}\left(|b_0| - \sum_{i=1}^{n-2}|b_i|\right) - \overline{\Delta}\sum_{i=0}^{n-2}|a_i| \qquad (20)$$

Next we need to ensure that the model error $\varphi_k$ already satisfying the bound $|\varphi_k|\le C_\varphi(\|z_k\|^2+\|u_k\|^2)$ satisfies also the inequality $|\varphi_k|\le\varrho\Delta_y$, where $0<\varrho<1$ and $\Delta_y<\Delta_0$. Obviously it is sufficient to hold the following: $C_\varphi(\Delta_z^2+\overline{u}^2)<\varrho\Delta_y$. To ensure this we introduce $\lambda\in(0,1)$ and consider inequality

$$C_\varphi\Delta_z^2 < \lambda\varrho\Delta_y - \lambda^2 C_\varphi\overline{u}^2 \qquad (21)$$

For sufficiently small $\lambda$ the right-hand side of (21) is positive. Therefore decreasing once again $\varepsilon$ and $\Delta_z$ we may satisfy (21). Now we need not forget to provide inequality (20) for $y_*$ and $\Delta_y$ replaced by $\lambda y_*$ and $\lambda\Delta_y$. Apparently it can be done by replacing $\Delta_0$ and $\overline{\Delta}$ by $\lambda\Delta_0$ and $\lambda\overline{\Delta}$ respectively. Thus we can apply the theorems on finite convergence of algorithms solving goal inequalities [6], [7]. In fact, the plant corresponding the input-output model (6) is minimum-phase according to condition A4 and the model error ensures the existence of "ideal" vector of tunable parameters $\vartheta^*$ satisfying the control goal (13) with something to spare ($\varrho<1$). The reference to Theorem 2[7] finishes the proof .

The performance of the proposed algorithm is examined below by computer simulations using the universal simulation system ADAM [10] (Analysis of Differential-Algebraic Models) working in the MATLAB environment.

# 4 Control of Oscillations in the Brussellator

One of the most popular nonlinear oscillatory model of chemical kinetics is the so called brussellator: [2]

$$\begin{cases} dX/dt = A - (B+1)X + X^2Y \\ dY/dt = BX - X^2Y, \end{cases} \qquad (22)$$

where $X$ is the concentration of initial substance; $Y$ is the concentration of product; $A, B$ are parameters (constants of reaction rate). This system has the fixed point at $X$=$A$, $Y$=$B/A$, and for some values of parameters $A, B$ this fixed point is unstable and the system (22) has a stable limit cycle [2] as shown in Fig.2. Consider the following control problem. Let $t_k$ be the time when $Y(t)$ achieves its $k$-th local maximum, the corresponding maximal value being $y_k = Y(t_k)$. Let the control action $u(t)$ be a piecewise constant function changing the parameter $A$ at $t_k$: $A = A_0 + u(t)$, $u(t) = u_k$ for $t_k \le t < t_{k+1}$. The values of the system parameters $A_0$ and $B$ are supposed to be unknown.

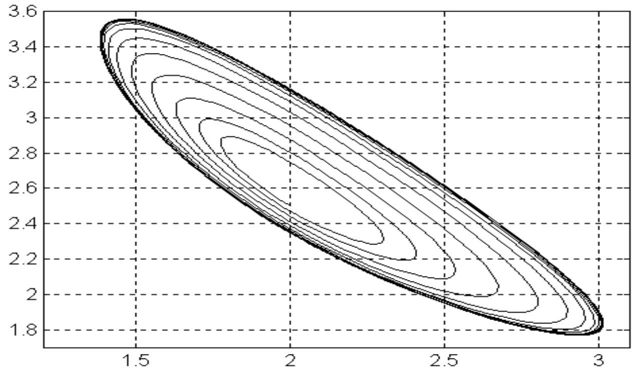

Fig. 2: Brusselator's stable limit cycle

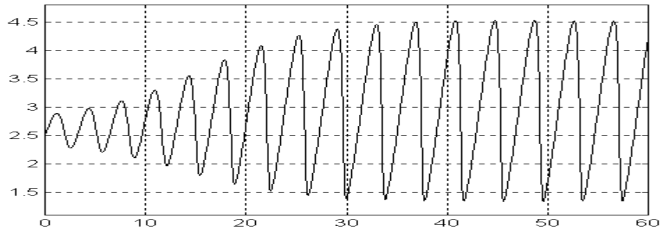

Fig. 3: Plot of $Y(t)$ vs $t$ for controlled brusselator

The control goal is to hold up the local maximum values of $Y(t)$ at the given level $y_*$ by varying $u(t)$ at the instants $t_k$. The linearized input-output model (6) is as follows

$$y_{k+1} = ay_k + bu_k + \varphi_k, \qquad (23)$$

where $a$ and $b$ are unknown coefficients, $\varphi_k$ is the bounded disturbance .

The adaptive control algorithm includes the main loop algorithm

$$u_k = [y_* - \hat{a}_k y_k]/\hat{b}_k, \qquad (24)$$

which calculates the new value of control action $u_k$, and the adaptation algorithm based on the results of Sec.2, which calculates the plant model (23) parameters estimations $\hat{a}_k, \hat{b}_k$ ($\gamma > 0$ is the adaptation gain):

$$\begin{aligned} \hat{a}_{k+1} &= \hat{a}_k - \gamma(y_k - y_*)y_k \\ \hat{b}_{k+1} &= \hat{b}_k - \gamma(y_k - y_*)u_k, \end{aligned} \qquad (25)$$

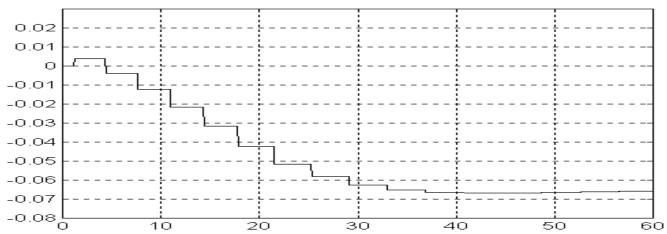

Fig. 4: Plot of $u(t)$ vs $t$ for controlled brusselator

Figs.3,4 show the plots of $Y(t)$ and $u(t)$ versus $t$ for the control goal (13) with $y_* = 4.5$ ($\max Y(t) \approx 3.55$ for the uncontrolled system). The corresponding phase portrait is shown in Fig.5. The following initial conditions and parameters were chosen: $A_0$=2; $B$=5.2; $X(0)$=2; $Y(0) = 2.5$; $\gamma$=0.095,$\hat{a}_0$=1,$\hat{b}_0$=100.

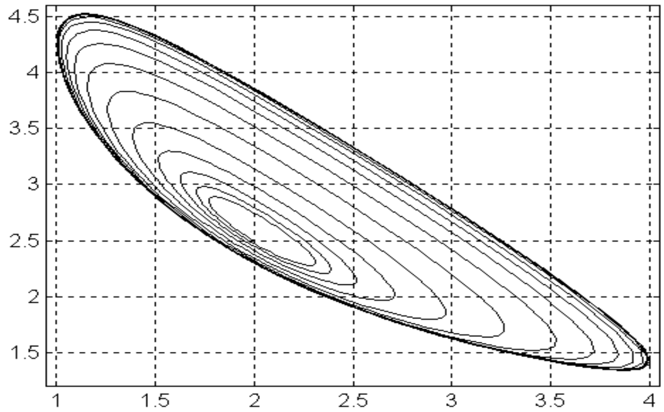


Fig. 5: Phase portrait of controlled brusselator ($y_*$=4.5)

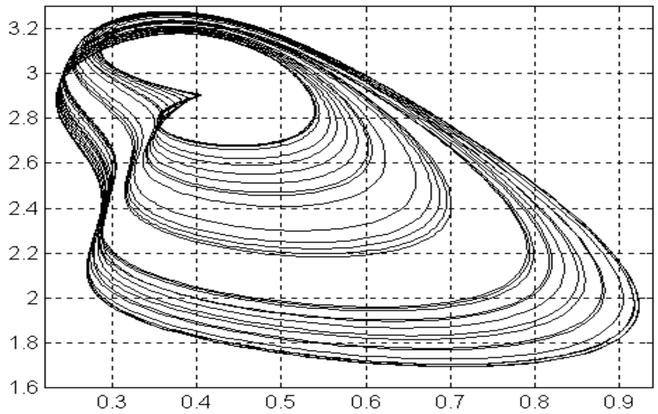


Fig. 6: Chaotic attractor of forced brusselator

# 5 Control of Chaos in the Forced Brussellator

It was found that the dynamics of the brussellator may become chaotic [1],[11] if the concentration of substance $A$ is modulated by the harmonic law: $A$=$A_0$+ $\tilde{a}\cos(\omega t)$. The corresponding parameter values are: [1] $A_0$=0.4, $B$=1.2, $\tilde{a}$=0.05 and $\omega$=0.81. For these values the brussellator has a chaotic attractor (Fig.6). The control problem statement and the adaptive control algorithm are the same as that described in Sec.3. In this case, $A = A_0 + \tilde{a}\cos(\omega t) + u(t)$, $y_k = Y(t_k)$. The values of the system parameters $A_0, B, \tilde{a}$ and $\omega$ are supposed to be unknown. The control goal is to hold up the local maximum values of $Y(t)$ at the given level $y_*$ by varying $u(t)$ at the instants $t_k$.

Figs.7,8 show the plots of $Y(t)$ and $u(t)$ versus $t$ for the control goal (13) with $y_*$=2.5 ($\max Y(t) \approx 3.2$ for the uncontrolled system). The corresponding phase portrait is shown in Fig.9. The following initial conditions were chosen for simulation: $X(0)$=0.5; $Y(0)$=1.0. The simulations show that the goal (13) is also achieved for other values of $y_*$, up to $y_*$=3.5 (Fig.10).

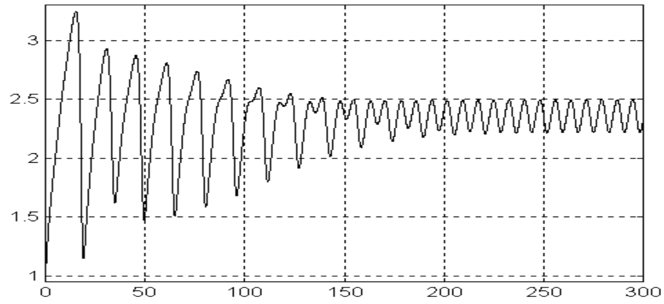


Fig. 7: $Y(t)$ vs $t$ for controlled forced brusselator($y_*$=2.5)

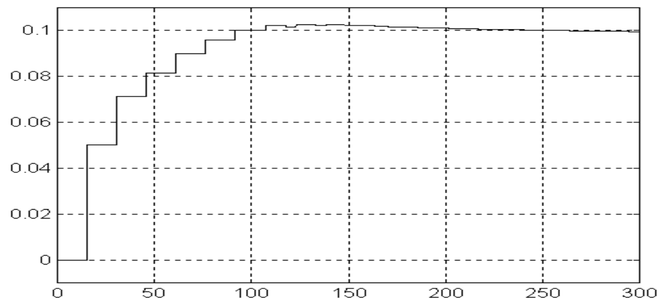


Fig. 8: $u(t)$ vs $t$ for controlled forced brusselator($y_*$=2.5)

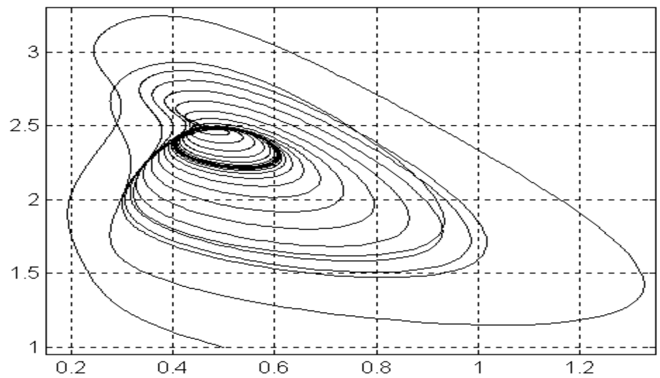


Fig. 9: Phase portrait of controlled forced brusselator($y_*$=2.5)

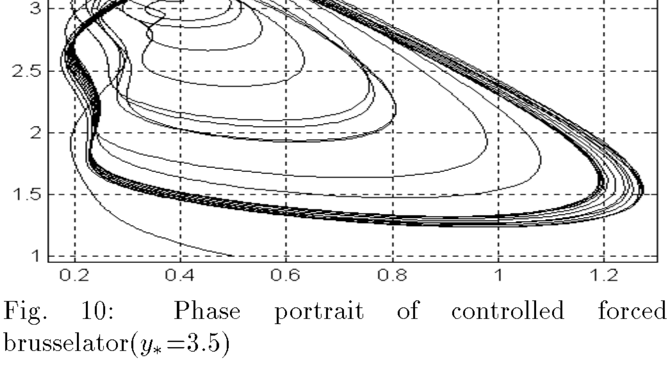


Fig. 10: Phase portrait of controlled forced brusselator($y_*$=3.5)

## 6 Conclusions

The efficiency of the proposed approach for the number of periodic and chaotic oscillation control problems has been demonstrated. Note that algorithm (15), (16) works well also for the case of forced oscillation although it has been derived for time invariant systems. The convergence rate is sufficiently high, while the value of control level is relatively small. The memory depth of the algorithm (the number of delayed coordinates) does not depend on the fractal dimension of the system attractor. Note that in some previous works the number of delayed coordinates is determined from Takens theorem [1] which in fact does not apply to controlled systems.

The conditions of the theorem 2 are fulfilled for arbitrary small $y_*$, $\overline{u}$ if the goal trajectory $x_*(t)$ is one of periodic orbits constituting dense set of periodic orbits embedded into attractor of system (1) and if the system (1) is locally controllable in sense of [13],[14]. Indeed in this case the recurrent base trajectory can be transformed into the periodic goal trajectory by means of arbitrary small control[15]. It follows from a controlled version of closing lemma (cf.[12]) which can be proved using the method of [13]. Therefore in this case the statement of the Theorem 2 holds for $\lambda$=1.

The proposed method can be readily extended to multi-input-multi-output systems and applied to practical control problems, e.g. in chemical engineering. [16]

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