FEEDBACK PASSIFICATION OF INTERCONNECTED SYSTEMS

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Abstract. This paper makes steps towards extending the feedback passification technique for stabilization to more general interconnected systems. Some connections to the backstepping process and adaptive versions are discussed.

Key Words. Nonlinear systems, stability, interconnected systems, passivity.

1. Introduction

An important technique in the development of nonlinear control theory is synthesis of stabilizing controls (Sontag, 1990; Kokotovic, 1992). A recent approach to stabilizability is based on the concept of passifiability (feedback passivity) (Byrnes, Isidori, Willems, 1991). This has been applied to simple cascades of nonlinear subsystems. In this paper, extension of some stabilization techniques using passivity concepts is explored.

Some of the recent investigations take into account the plant uncertainty in the problem formulation. This leads to development of nonlinear adaptive control techniques with passivity connections. One of the general techniques is the so-called speed-gradient (SG) method (Fradkov, 1979, 1990). Using the SG-methodology gives an opportunity to pose and solve the problem of adaptive passification (Seron, Hill and Fradkov, 1994).

A very fruitful idea in this field is iterative design of control algorithms or backstepping, (Kokotovic, 1992). In particular, results by Kanellakopoulos, Kokotovic and Morse (1992), allow one to attack problems with a higher relative degree and weakened matching conditions. Connections between backstepping and passivity have been established (Kokotović, Krstić and Kanellakopoulos, 1992).

For practical applications, it is important to establish semiglobal stabilization techniques as discussed by Sussman and Kokotovic (1991), Byrnes and Isidori (1991).

The present paper is aimed at further investigating possibilities and limitations of passification of interconnected systems and to extend the adaptive passification results started in Seron, Hill and Fradkov (1994). A key idea of large scale control system theory is the representation of a complex system as a collection of interacting subsystems e.g., see Michel and Miller (1977). Therefore the passification problem becomes one of establishing a passivity property of an interconnected system provided its subsystems have the appropriate properties. This approach is clearly related to the backstepping procedure, but may allow more natural system structures.

For the sake of brevity, all proofs are omitted. Further details are found in Fradkov, Hill, Jiang and Seron (1995).

2. Definitions and problem formulations

Definition 2.1 (Passivity) A system with input \( v \), output \( y \) and state \( x \in X \subseteq \mathbb{R}^n \) is said to be \( C^r \)-passive if there exists a \( C^r \) nonnegative real-valued function \( V(x) \), \( V(0) = 0 \), such that \( \forall x(0) = x^0 \in X, \forall t \geq 0, \) the following dissipa-
tion inequality (DI) holds
\[
\int_0^t v^T(\tau) y(\tau) d\tau \geq V(x(t)) - V(z^0)
\] (2.1)

Passive systems are a particular class of the more general dissipative systems.

Definition 2.2 (Dissipativity) A system with input \( v \), output \( y \) and state \( z \in X \) is said to be \((Q,R,S)\)-dissipative if there exists a \( C^1 \) nonnegative real-valued function \( V(x) \), \( V(0) = 0 \), such that \( \forall z(0) = z^0 \in X, \forall t \geq 0 \), the following DI holds
\[
\int_0^t \left[ y^T(\tau)Qy(\tau) + u^T(\tau)Ru(\tau) + y^T(\tau)Su(\tau) \right] d\tau \geq V(x(t)) - V(z^0)
\] (2.2)

Many of the results that we will present are related to output strict passive systems, or briefly OSP-systems, defined as \((-\rho I,0,1)\)-dissipative systems, where \( \rho \) is a positive constant (Hill and Moylan, 1976).

Systems that can be rendered passive via state feedback are called feedback passive systems. These systems were introduced in Byrnes, Isidori and Willems (1991), who considered a class of affine (linear in input) systems:
\[
\begin{align*}
\dot{z} &= f(z) + g(z)u \\
y &= h(z)
\end{align*}
\] (2.3)

Definition 2.3 (Feedback-Passive Systems) We say that the system (2.3) where \( z \in X \subset R^n \), \( u,y \in R^m \) and \( f, g, h \) are smooth functions, \( f(0) = h(0) = 0 \), is locally (resp. globally) \( C^r \) feedback-passive if there exists a \( C^r \) smooth control
\[
u = \alpha(x) + \beta(x)u
\] (2.4)

with \( \alpha(0) = 0, \beta(0) \neq 0 \), (resp. \( \beta(x) \neq 0 \forall x \in X \)), s.t. for the closed loop system (2.3), (2.4) the DI (2.1) is satisfied. For \( r \geq 1 \) the DI in differential (infinitesimal) form can be checked instead of (2.1):
\[
\dot{V}(z) = \frac{\partial V(x)}{\partial x}(f(z) + g(z)u) \leq y^T v
\] (2.5)

where \( V : X \rightarrow R \) is a \( (C^r) \) positive definite function.

Feedback dissipative systems are defined analogously.

A different problem involves not only the search for a state feedback but also the selection of an output function for which the system becomes passive (dissipative). More specifically:

Definition 2.4 (Feedback Passification Problem) For the system
\[
\dot{z} = f(z) + g(z)u
\] (2.6)

where \( z \in X \subset R^n \), \( u \in R^m \) and \( f, g \) are smooth functions, \( f(0) = 0 \), find a control law
\[
u = \alpha(x) + \beta(x)u
\] (2.7)

and output function
\[
y = h(x), \quad h(0) = 0
\] (2.8)

such that the input-output operator verifies the dissipation inequality (2.1).

When \( f \) and \( g \) in (2.3) are also functions of some vector of unknown parameters \( \xi \), i.e.
\[
\begin{align*}
\dot{z} &= f(z, \xi) + g(z, \xi)u \\
y &= h(z, \xi)
\end{align*}
\] (2.9)

with \( z, u \) and \( y \) as before and \( \xi \in \Xi \subset R^n \) a vector of unknown parameters, we can state the following problem, following Seron, Hill and Fradkov, (1994).

Definition 2.5 (Adaptive Feedback Passivity Problem) Design a two level control algorithm
\[
\begin{align*}
u &= U(x, \theta, v) \\
\theta &= \Theta(x, \theta)
\end{align*}
\] (2.10)

such that the closed loop (2.9)-(2.10) satisfies the DI (2.1) for some function \( V(x, \theta, \xi) \) with specified positivity properties.

The semiglobal versions of Definitions (2.4), (2.5) are motivated by related concepts of stabilization (Sussman and Kokotovic, 1991; Byrnes and Isidori, 1991) and extensions to the feedback dissipativity case are introduced in the similar way. e.g., system (2.3) is called semiglobally feedback passive if, for every bounded set \( D \subset R^n \), there exists a bounded set \( E \) with \( D \subset E \) and smooth feedback
\[
\begin{align*}
u &= \alpha_D(z) + \beta_D(z)u
\end{align*}
\] (2.11)

such that dissipation inequality in the form (2.1) (or (2.5)) is valid for system (2.3), (2.11) with initial conditions \( z(0) \in D \) and for input functions \( v(t) \) ensuring \( z(t) \in E, t \geq 0 \).
3. Feedback passifiability of an interconnection of two subsystems

Suppose the two state feedback passive systems (i.e., systems that may be rendered passive by state feedback) are given:

\[ \begin{align*}
S_1: & \quad \dot{x}_1 = f_1(x_1) + g_1(x_1)u, \\
& \quad y_1 = h_1(x_1), \quad i = 1, 2
\end{align*} \]  
where \( x_i \in \mathbb{R}^{n_i}, y_i \in \mathbb{R}^{l_i}, u_i \in \mathbb{R}^{m_i}, i = 1, 2 \). Let us examine the state feedback passivity of the basic (primary) types of interconnections of \( S_1 \) and \( S_2 \); cascade (series), parallel, and feedback. The state vector of the interconnection is composed of the state vectors of \( S_1 \) and \( S_2 \) in all cases, i.e., \( z = (x_1^T, x_2^T)^T \). We assume that vector dimensions allow the interconnection.

First note that the feedback connection can be reduced to the cascade one. Indeed, since the connection is described as \( u_1 = u + y_2 \), we can make the change \( u = y_2 + v = -h_2(x_2) + v \), so opening the loop.

As to the parallel interconnection, its feedback passivity does not follow from the feedback passivity of the subsystems \( S_1 \), \( S_2 \) even in the linear case.

Fig. 1. Cascade connection

The last case of the cascade connection shown in Figure 1 is the most interesting one. For this case, the following result holds.

**Theorem 3.1** Let \( S_1 \) have local relative degree 1, i.e.,

\[ \det L_{s_1}h_1(0) \neq 0 \]  
and satisfy \( h_1(x_1) = 0 \) iff \( x_1 = 0 \). Let \( S_2 \) be locally feedback passive, i.e., there exist smooth functions \( V_2 \) positive definite, \( \alpha_2(x_2), \alpha_2(0) = 0, \beta_2(x_2), \beta_2(0) \neq 0 \) such that,

\[ L_{f_2} + \alpha_2 V_2 \leq 0, \quad L_{f_2} \beta_2 V_2 = h_2^T. \]  

Then the cascade system is locally feedback passive with respect to output \( y_1 - \alpha_2 \). Further, the feedback law

\[ u = [L_{h_1}g_1(x_1)]^{-1}[\dot{\alpha}_2 - y_2 - \lambda(y_1 - \alpha_2) - L_{h_1}f_1 + v], \]  

\[ \lambda > 0, \]  
renders the system output strict passive.

**Corollary 3.1** Consider systems \( S_1, \ldots, S_n \) where \( S_1, \ldots, S_{n-1} \) have relative degree 1, and \( S_n \) is feedback passive. Then the cascade system shown in

Figure 2 is feedback passive.

Fig. 2. Cascade System

It is easy to see that global versions of Theorem 3.1 and Corollary 3.1 are valid.

4. Passification and speed-gradient algorithms

The next question after analyzing the solvability of the problem is how to solve it? The passifying feedback laws suggested in Byrnes, Isidori and Willems (1991) and in the previous section are not always appropriate because they depend essentially on the stabilizing (passifying) feedback for the subsystems: \( \alpha_1(x_1), \alpha_2(x_2) \). In many cases more convenient algorithms can be designed using the idea of speed gradient (Fradkov, 1979).

**Definition 4.1** Given the system

\[ \dot{z} = F(x, u, t) \]  
and the objective function \( Q(x, t) \), the vector function \( QF(x, u, t) = \nabla_u Q \) is called the speed-gradient of \( Q(\cdot) \) with respect to the system (4.1).

In general, \( QF(x, u, t) = \nabla_u \left[ \nabla_x Q^T F(x, u, t) + \frac{gQ}{dt} \right] \), evaluating \( \dot{Q} \) along the trajectories of system (4.1).

For affine systems \( \dot{x} = f(x) + g(x)u \) and time-invariant objectives \( Q = Q(x) \) the speed gradient reduces to the Lie derivative:

\[ QF(x) = L_g Q = \nabla_x Q^T g(x) \]  

Now let us formulate the main result of this section for affine system (2.3).

**Theorem 4.1** Assume that there exists continuous feedback \( u = \alpha(x) \) for system (2.3) such that the closed loop system is globally asymptotically stable, i.e., there exists a radially unbounded smooth function \( V(x) \) and continuous function \( \rho(x); \rho(x) > 0 \) for \( x \neq 0 \), such that

\[ \nabla_x V(x)^T [f(x) + g(x)\alpha(x)] \leq -\rho(x) \]  

and the following inequality holds:

\[ \lim_{x \to 0} \frac{\|\alpha(x)\|^2}{\rho(x)} < \infty. \]  

Then, for any bounded set \( \mathcal{D} \subset \mathbb{R}^n \), there exists scalar \( \gamma_D > 0 \) such that the SG-feedback

\[ u = -\gamma_D \nabla_u \dot{V} + v \]  

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makes system (2.3), (4.5) semiglobally state strictly passive with respect to the output \( y = g^T \nabla V(x) \). In other words, inequality (2.5) holds for all initial conditions \( x(0) \in D \).

**Corollary 4.1** For system (2.3), (4.5) with \( v = 0 \), the origin is asymptotically stable and \( D \) lies in its domain of attraction.

Theorem 4.1 and the corollary assert that if the system is asymptotically stabilizable then it is both SG-stabilizable and SG-passifiable; i.e., there exists a speed-gradient feedback law making the system both asymptotically stable and passi-

5. Passification and backstepping

The aim of this section is to apply the iterative design known as backstepping to derive passifying controllers for interconnected systems. The backstepping procedure has been successfully em-
ployed for the (adaptive) stabilization of systems in strict-feedback form, i.e. systems in which each new nonlinear integrator depends only on the state variables that are “fed back”. See, for example, Kolesnikov (1987); Kanellakopoulos, Kokotovic and Morse (1992); Jiang and Praly, (1991); Kokotovic, Kristic and Kanellakopoulos, (1992); Seto, Annaswamy and Baillieul (1992) and Druzhinina and Fradkov (1994a). The key idea of backstepping is based on the derivation of a controller for a basic interconnected structure having one integrator, and then repeat the known procedure adding one more integrator at the time. We start by considering systems of the form

\[ \dot{x} = f_0(x) + p(x)y \]  
\[ \dot{y} = f(z, y) + g(z, y)u \]

where \( x \in R^l \), \( u, y \in R^m \) and \( f_0, p, f \) and \( g \) are smooth functions and the \( m \times m \) matrix \( g(z, y) \) is nonsingular. Using the control \( u = g^{-1}(z, y)[v - f(z, y)] \) converts system (5.1), (5.2) into a cascade of the form considered in Theorem 3.1. With the \( z \) system (driven by \( y \)) feedback pas-
vive, Theorem 3.1 gives that the cascade is feedback passive. In Seron, Hill and Fradkov (1994), a passifying control was derived that makes system (5.1)–(5.2) OSP w.r.t. the output \( y \) under the assumption of minimum-phaseness, i.e. stability of the subsystem (5.1) when \( y = 0 \). Here we replace the minimum-phaseness requirement by the weaker assumption of stabilizability through \( y \) of the first subsystem (5.1). We state this stabiliz-
ability condition formally as follows.

**Assumption 5.1** (Stabilizability)

There exists a state feedback

\[ y = \alpha(z) \]  
\[ \dot{W}(z) = \frac{\partial W(z)}{\partial z} [f_0(z) + p(z)\alpha(z)] \leq -\eta(z) \leq 0 \]

Note that if (5.4) holds for \( \alpha(z) = 0 \), \( \eta(z) = 0 \), we recover the weakly-minimum phaseness assumption of Seron, Hill and Fradkov (1994), and if it holds for \( \alpha(z) = 0 \) and \( \eta(z) \) a positive definite function, Assumption 1 implies minimum-
phaseness of (5.1)–(5.2) w.r.t. the output \( y \).

On the other hand, if \( \alpha(z) \) is nontrivial, Assumption 1 implies minimum-phaseness of (5.1)–(5.2) w.r.t. the new output \( y_1 = y - \alpha(z) \). We can then state the following passifying result.

**Theorem 5.1**

Consider the system (5.1)–(5.2), where the matrix \( g(z, y) \) is nonsingular for all \( (z, y) \in R^l \times R^m \). Assume that the subsystem (5.1) is stabilizable by smooth feedback, i.e. it satisfies Assumption 1. Then (5.1)–(5.2) is passifiable (OSP) by means of the feedback

\[ u = g^{-1}(z, y) \left[ -f(z, y) + v - \rho \delta(y - \alpha(z)) \right. \\
+ \left. \frac{\partial \alpha(z)}{\partial x} [f_0(z) + p(z)y] \right. \\
- \delta \left. \frac{\partial W(z)}{\partial z} p(z) \right] \]

where \( \rho > 0 \), \( \delta > 0 \).

**Corollary 5.1**

Consider the strict cascade

\[ \dot{z} = f_0(z) + p(z)y \]  
\[ \dot{y} = f(y) + g(y)u \]

where \( z \in R^l \), \( u, y \in R^m \) and \( f_0, p, f \) and \( g \) are smooth functions.

If (5.7) has vector-relative degree one (i.e. the \( m \times m \) matrix \( g(y) \) is nonsingular), and (5.6) is stabil-
izable through \( y \) (i.e. it satisfies Assumption 1, then the strict cascade (5.6)–(5.7) is passifiable.

**Corollary 5.2 Integrator Backstepping, (Kanella-
kopoulos, Kokotovic and Morse, 1992)**

Assume the first subsystem of the cascade

\[ \dot{z} = f_0(z) + p(z)y \]  
\[ \dot{y} = u \]
where \( z \in \mathbb{R}^l, u, y \in \mathbb{R} \), satisfies Assumption 1. Then the feedback control

\[
\begin{align*}
  u &= -(y - \alpha(z)) + \frac{\partial \alpha(z)}{\partial z} [f_0(z) + p(z)y] \\
  &\quad - \frac{\partial W(z)}{\partial z} p(z)
\end{align*}
\]  

(5.9)

stabilizes the system (5.8) in the sense that the closed loop (5.8)-(5.9) is Lyapunov stable and

\[ \eta(z(t)) \to 0, \quad y(t) - \alpha(z(t)) \to 0 \quad \text{as} \quad t \to \infty \]

Remark 5.1 A more general interconnected cascade, where the first subsystem is not affine in \( y \), was considered in Drusinina and Fradkov (1994a, 1994b), where a stabilizing controller was derived for the form

\[
\begin{align*}
  \dot{z} &= F(z, y, t) \\
  \dot{y} &= \Phi(z, y, u)
\end{align*}
\]  

(5.10)

It is clear that the controller in Theorem 1 is no more than the extension of the backstepping control (5.9) to the case when \( u \) is a vector. Obviously, the procedure can be iterated to cope with many integrators in a MIMO strict feedback form.

6. Adaptive passification

We consider the system with linear parameter dependence given by

\[
\begin{align*}
  \dot{z} &= f_0(z) + p(z)\xi + p_0(z)y \\
  \dot{y} &= f(z,y)\xi + g(z,y)u
\end{align*}
\]  

(6.1)

where \( z \in \mathbb{R}^l, u, y \in \mathbb{R}^m, \xi \in \Xi \subset \mathbb{R}^p \), and

\[
p(z) = [p_1(z), \ldots, p_r(z), \ldots, p_p(z)]
\]

where \( p_i(z), i = 1, 2, \ldots, p \) are \( \ell \times 1 \) vectors,

\[
f(z,y) = [f_1(z,y), \ldots, f_l(z,y), \ldots, f_p(z,y)]
\]

where \( f_i(z,y), i = 1, 2, \ldots, p \) are \( m \times 1 \) vectors.

We reformulate the stabilizability condition of Assumption 5.1, specifying the structure of the stabilizing feedback as follows.

**Assumption 6.1 (Parametric Stabilizability)**

There exists a state feedback

\[
\alpha(z) := \alpha_0(z) + \bar{\alpha}(z)\xi
\]  

(6.2)

where

\[
\bar{\alpha}(z) = [\alpha_1(z), \ldots, \alpha_i(z), \ldots, \alpha_p(z)]
\]

with \( \alpha_i(z), i = 1, 2, \ldots, p \) are \( m \times 1 \) vector functions, and a smooth Lyapunov function \( W(z) \) such that

\[
\dot{W}(z) = \frac{\partial W(z)}{\partial z} [f_0(z) + p(z)\xi + p_0(z)\xi + \bar{\alpha}(z)\xi]
\]

\[
\leq -\eta(z) \leq 0
\]  

(6.3)

Then a passifying controller is given by the following theorem.

**Theorem 6.1** Consider the system (6.1), where the matrix \( g(z,y) \) is nonsingular for all \( (z,y) \in \mathbb{R}^l \times \mathbb{R}^m \). Assume that the first subsystem is stabilizable by the smooth feedback (6.2), i.e. it satisfies Assumption 6.1. Then (6.1) is output strict passifiable (OSP) by means of the dynamic feedback

\[
\dot{\theta} = \left\{ \frac{1}{\delta} [y - \alpha_\theta(z)]^T [f(z,y) - \Delta \alpha p(z)] + \frac{\partial W(z)}{\partial z} \right\} \Gamma
\]

\[
u = g^{-1}(z,y) \left\{ -f(z,y)\theta - \delta p_0(z)\Gamma \frac{\partial W(z)}{\partial z} \\
+ v - \delta \theta [y - \alpha_\theta(z)] + \Delta \alpha [f_0(z) + p(z)\theta + p_0(z)y + \bar{\alpha}(z)\theta] \right\}
\]  

(6.4)

where \( \theta := [\theta_1, \ldots, \theta_l, \ldots, \theta_p]^T \) is a \( p \times 1 \) vector of adjustable parameters, \( \alpha_\theta(z) := \alpha_0(z) + \bar{\alpha}(z)\theta, \delta > 0, \Gamma > 0 \) and

\[
\Delta \alpha(z, \theta) = \left( \frac{\partial \alpha_0(z)}{\partial z} + \sum_{i=1}^p \frac{\partial \alpha_i(z)}{\partial z} \theta_i \right)
\]

Remark 6.1 A solution for the multi-input-single-output case was given in Kokotovic, Kanellopoulos and Morse (1991). The algorithm given in Theorem 6.1 is simpler and it covers the case where the plant has vector-parameter uncertainty.

Theorem 6.1 is not a good basis for iterative design; the overall passifying adaptive controller has overparameterization since a separate estimator for \( \xi \) is generated at each step. We now proceed to show how this can be avoided.

**Theorem 6.2** For the \( z \) system with \( y \) considered as input, suppose there exist two smooth functions \( \alpha_0, \tau_0 \) and a nonnegative smooth function \( W_1 \) s.t.

\[
\begin{align*}
  \dot{z} &= f_0(z) + p(z)\xi + p_0(z)y \\
  \dot{\theta} &= \tau_0(z, \theta) + \tau \\
  y &= \alpha_0(z, \theta) + \bar{y}
\end{align*}
\]  

(6.5)

the following augmented system
\[ V_1(z, \tilde{\theta}, \xi) = W_1(z, \theta) + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \]  

(6.6)

the system (xiiilinear) with input \( u \) also satisfies this property for some functions \( W_2, V_2 \).

**Remark 6.1** Using this theorem repeatedly on strict-feedback systems generates a passifying adaptive nonlinear controller without overparametrization. This result recovers the main result of Kristić, Kanellakopoulos and Kokotović, (1992) via backstepping.

**References**


