CONTROL OF OSCILLATIONS IN HAMILTONIAN SYSTEMS

A.L. Fradkov, I.A.Makarov, A.S.Shiriaev, O.P.Tomchina
Institute for Problems of Mechanical Engineering
61, Bolshoy V.O., St.Petersburg, 199178, RUSSIA
e-mail: alf@ccs.ipme.ru

Abstract. The speed-gradient method of control design for oscillatory nonlinear systems is extended to multi-objective and constrained problems including the problem of suppressing resonances.

Key Words. Nonlinear control, energy control, suppression of resonances, spherical pendulum, TORA example.

1 Introduction

Control of nonlinear oscillatory systems has attracted much attention of control theorists recently [18, 5, 8, 9, 11, 15, 17]. It is motivated by various potential application in mechanics, physics, vibrational technology, etc. Another reason of increasing interest is that control of oscillatory systems requires achieving nonclassical control goals (swinging, synchronization) as well as describing and analyzing complex motions of the closed loop system. The problem of swinging the Hamiltonian system up to the desired level of energy was solved in [8, 9, 11] using speed-gradient method [7]. It was shown that speed-gradient control algorithms allow to achieve the desired energy hypersurface with arbitrary small control intensity. However the motion along the energy hypersurface may be rather complex. For example it has been shown by computer simulation [14] that the controlled double pendulum (two-link manipulator) may perform chaotic motions with different levels of energy.

The present paper is devoted to the further study of possibilities and limitations of speed-gradient control of Hamiltonian systems.

In this work, we consider the controlled Hamiltonian system (CHS) [16] with a phase vector $x = (p, q) \in R^{2n}$ and Hamiltonian function $H(q,p,u) = H_0(q,p) + \sum_{j=1}^m H_j(q,p)u_j$, being linear in input functions $u_j, j = 1, \ldots, m$. The corresponding CHS with $u = 0$ is called unforced CHS. Given the set of the conserved quantities $F_i, i = 1, \ldots, k$ of the unforced CHS, we pose the control goal as directing each of them to the prespecified value:

$$\lim_{t\to\infty} F_i(x(t)) = f_i, i = 1, \ldots, k. \qquad (1.1)$$

The proposed control algorithms, ensuring the control aim (1.1), are obtained by means of speed-gradient method. The conditions are found ensuring that the $(2n - k)$-dimensional surface in the phase space of CHS defined by the desired values of the conserved quantities f_i is attracting set of the closed-loop CHS. Moreover it is shown that in the case when $k = n > 1$ each trajectory of the closed-loop CHS approaches some quasiperiodic trajectory and therefore can not be chaotic.

The obtained results then are extended to the class of the generalized Hamiltonian systems and to the systems with additional inequality constraint. The last problem is important in mechanics for suppression of resonances when spinning up nonbalanced rotors [6, 13].

2 The problem statement

Let M^{2n} be smooth $2n$-dimensional Poisson manifold with the system of local coordinates $x = (q,p) \in M^{2n}$ and standard Poisson structure (see [1],[16]). The notations $C^\infty(M^{2n})$, $V^\infty(M^{2n})$ stand for the Lie algebras of smooth functions and smooth vector fields on M^{2n}, correspondingly. Now let $F, G \in C^\infty(M^{2n})$, then we define their Poisson bracket as follows

$$\{F,G\}(q,p) = \sum_{i=1}^n \left(\frac{\partial F}{\partial p_i}\frac{\partial G}{\partial q_i} - \frac{\partial F}{\partial q_i}\frac{\partial G}{\partial p_i}\right)(q,p).$$

Same notation is used when F or G or both are vectors. Consider an affine controlled Hamiltonian system [16]

$$\left\{\begin{array}{rcl} \dot{q} & = & \nabla_p H_0(q,p) + \sum_{j=1}^m \nabla_p H_j u_j \\ \dot{p} & = & -\nabla_q H_0(q,p) - \sum_{j=1}^m \nabla_q H_j u_j \\ \end{array}\right\} \quad (2.1)$$

where $H_0 \in C^\infty(M^{2n})$ is a Hamiltonian function of unforced CHS (2.1); $H_j \in C^\infty(M^{2n}), j = 1,\ldots,m$ are interaction Hamiltonians being independent functions (in the sense that the corresponding one-forms $dH_j \in T^*(M^{2n})$ are linearly independent) [16]; $u_j, j = 1,\ldots,m$ are control inputs of CHS. The equations (2.1) provide a convenient mathematical description for various controlled physical and mechanical systems. The $q-$ and $p-$ components of phase vector will be called "generalized coordinates" and "generalized momenta", correspondingly.

Let the set of independent functions $F_i \in C^\infty(M^{2n}), i = 1,\ldots,k, k \leq n$ and the set of real numbers $f_i, i = 1,\ldots,k$ be given. Then the *level set* of functions F_i

$$M_f = \{(q,p) \in M^{2n} : \; F_i(q,p) = f_i, \; i = 1,\ldots,k\} \quad (2.2)$$

is $(2n-k)$-dimensional submanifold of M^{2n} by virtue of implicit function theorem. It is well known [1] that the condition

$$\{H_0, F_j\} = 0, \quad ,j = 1,\ldots,k \quad (2.3)$$

implies that the manifold M_f is invariant set of the unforced CHS. Moreover, in the case when $k = n$ (we assume that $F_1 = H_0$) and the manifold M_f is compact and connected it is diffeomorphic to a n-dimensional torus. Then the unforced CHS (2.1) is integrable and its motion on M_f is quasiperiodic (and hence is not chaotic). To achieve the control goal (1.1) we must design the input functions of the corresponding CHS in such a way that the level set M_f of functions $F_i, i = 1,\ldots,k$ should be attracting set for the controlled CHS (2.1).

Choose the set of admissible controls $\mathcal{U}$ as $\mathcal{U} = \{u = (u_1,\ldots,u_m)^T : u_j(q,p) \in C^\infty(M^{2n}), \; u_j(q,p) = 0, \forall (q,p) \in M_f, j = 1,\ldots,m\}$. The problem is to design the control algorithm

$$u = u(q,p) \in \mathcal{U} \quad (2.4)$$

for CHS (2.1) in such a way that for any given numbers f_i, $i = 1,\ldots,k$ the manifold M_f is an invariant attracting set for the trajectories of the closed-loop system (2.1), (2.4), (1.1).

3 Control algorithm design

To solve the posed problem we use the speed gradient method [7, 8], which suggests the following control algorithm

$$u(q(t),p(t)) = -\gamma \nabla_u \dot{Q}(q(t),p(t)) \quad (3.1)$$

or, more generally

$$u(q(t),p(t)) = -\Psi(\nabla_u \dot{Q}(q(t),p(t))), \quad (3.2)$$

where $Q(q,p)$ is the goal functional, $\dot{Q}$ is the full derivative of $Q(q,p)$ along the solutions of (2.1) and $\Psi(z)$ is a vector-function forming sharp angle with z, i.e. $\Psi(z)^T z > 0$ for $z \neq 0$. Take the goal functional as follows

$$Q(q,p) = \frac{1}{2}[F-f](q,p)^T \, R \, [F-f](q,p), \quad (3.3)$$

where $F(q,p) = (F_1(q,p),\ldots,F_k(q,p))^T$, $f = (f_1,\ldots,f_k)^T$ and R is symmetric positive definite constant matrix. Then the corresponding speed-gradient control algorithm (3.1) has the following form

$$u = -\gamma\{\bar{H}, Q\} = -\gamma[\bar{H}, F](q,p) \cdot R \cdot P(q,p), \quad (3.4)$$

where $\bar{H}$ stands for column vector with components H_j

The general algorithm (3.2) looks as follows

$$u = -\Psi(\{\bar{H}, Q\}). \quad (3.5)$$

Introduce the set

$$S(q,p) = \text{span}\{\text{ad}^s_{H_0}\{\bar{H}, F\}, \; s = 0,1,\ldots\}.$$

where for every $H, G \in C^\infty(M^{2n})$ we define inductively $\text{ad}^0_H G = G, \text{ad}^1_H G = \{H,G\}, \text{ad}^{s+1}_H G = \{H, \text{ad}^s_H G\}$.

Proposition 3.1 *Consider the controlled Hamiltonian system (2.1) defined on a smooth $2n$-dimensional manifold M^{2n} with Hamiltonians $H_0(q,p), H_j(q,p)$ bounded together with their first* *and second partial derivatives on the set* $\Omega_\varepsilon = \{(p,q) : Q(p,q) < \varepsilon\}$ *for some* $\varepsilon > 0$. *Let* $F_i \in C^\infty(M^{2n}), i = 1,\ldots,k$ *be conserved quantities of the unforced CHS (2.1). Assume that there exists* $\delta > 0$ *such that each connected component of the set*

$$D_\delta = \Omega_0 \cap \{(p,q) : \det A^T A < \delta\}$$

is compact, where $A = \{\bar{H}, F\}$ *and*

$$\dim S(q,p) \geq k \quad \forall (q,p) \in \Omega_\varepsilon, \qquad (3.6)$$

Then the control goal (1.1) is achieved for any trajectory of the system (2.1), (3.5) with initial conditions from the set Ω_ε.

Remark. If the condition (3.6) hold everywhere in Ω_ε except some set of isolated points (p_*, q_*) then (p_*, q_*) is an equilibrium of the unforced system. Therefore it follows from the center manifold theory that if the unforced system has only isolated equilibria (p_*, q_*) in Ω_ε and each of them is unstable (in sense that corresponding Jacobi matrix has at least one eigenvalue with positive real part) then the Lebesgue measure of the initial conditions for which the goal (1.1) is not achieved is equal to zero.

The proof follows lines of [9] and [16] (compared with Lemma 10.7 of [16] the goal equilibrium is replaced by the goal manifold).

In fact the Proposition 3.1 states that if we have avoided convergence to the stable equilibrium then the control goal (1.1) will be achieved for almost all initial conditions under observability-like condition (3.6).

In the case when one of the goal functions F_j is proper function the explicit conditions ensuring the goal (1.1) can be given.

Proposition 3.2 *Let* F_1 *be proper function, i.e. the set* $\{(p,q) : F_1(p,q) \leq c\}$ *is compact for all* $c \in R^1$. *Let* $\dim S(p,q) \geq k$ *for all* $(p,q) \in \Omega_0$, *where* $\Omega_0 = \{(p,q) : |F_1(p,q) - f_1| < \varepsilon\}$. *Then the goal (1.1) is achieved in system (2.1), (3.4) for all initial conditions* $(p(0), q(0)) \in \Omega_0$.

Remark 1. The simple condition eliminating convergence to a stable equilibrium is just absence of stable equilibria in the connected component of the set $\{(q,p) : Q(q,p) \leq Q(q(0), p(0))\}$ containing $Q(q(0), p(0))$. To satisfy it the proper choice of the goal function $Q(q,p)$ i.e. proper choice of values f_i and weighting matrix R may help.

Remark 2. In case when $k = n$ and each two of functions F_i are in involution, i.e.

$$\{F_i, F_j\} = 0, \quad i,j = 1,\ldots,k, \qquad (3.7)$$

it can be proved that each trajectory of the closed loop CHS tends to some trajectory of the unforced system which is either quasiperiodic one, or equilibrium point [1]. Hence, the behavior of the closed loop control system (2.1), (3.5) can not be chaotic.

4 Example: spherical pendulum

For example consider spherical joint manipulator with a single link of length l and a point mass m associated to the free end of link modelled as spherical pendulum. Choose the spherical coordinates of free end of link $q = (q_1, q_2)^T$ as the generalized coordinates. Then the Hamiltonian of unforced CHS takes the following form

$$H_0(q,p) = \frac{1}{2}(1/\omega)(p_1^2 + p^2/\sin^2 q_1) + mgl\cos q_1, \qquad (4.1)$$

where $\omega = 1/(ml^2)$, $p = (p_1, p_2)^T$ is the vector of the generalized momenta. Note, that the Hamiltonian $H_0(q,p)$ does not depend on q_2. Hence this coordinate is cyclic and therefore, p_2 is conserved quantity of the system under consideration. Thus the set of function

$$F_1(q,p) = H_0(q,p), F_2(q,p) = p_2$$

forms the full number of conserved quantities, i.e $k = n = 2$.

In the case when $H_1(q,p) = q_1,\ H_2(q,p) = 0$ ($k = 2, m = 1$), the condition (3.6) is not fulfilled. Note, that this case can be reduced to the problem of energy stabilization of a simple pendulum $k = n = m$, considered in early works [8, 9, 14]).

In the case when $H_i(q,p) = q_i, i = 1,2$ ($k = m = 2$), or $H_1(q,p) = 0,\ H_2(q,p) = q_2$ ($k = 2, m = 1$), the condition (3.6) is fulfilled on $\{(q,p) \in R^4 : q_1 \neq 0, p_1 \neq 0\}$ and

$$\begin{aligned} u_1 &= -\gamma \cdot (\omega p_1 (H_0(q,p) - f_1) + \\ &\quad \omega p_2 (p_2 - f_2)/\sin^2 q_1) \qquad (4.2) \\ u_2 &= -\gamma \cdot (p_2 - f_2), \end{aligned}$$

In the case when $H_1(q,p) = 0,\ H_2(q,p) = q_2$ ($k = 2, m = 1$), then $S = \{(q,p) \in R^4 : q_1 \neq 0\}$ and

$$\begin{array}{l} u_1 = -\gamma \cdot (\omega p_2 / \sin^2 q_1 \cdot (H_0(q,p) - f_1), \\ u_2 = -\gamma \cdot (p_2 - f_2). \end{array} \qquad (4.3)$$

By virtue of Proposition 3.1, the presented control algorithms (4.2) and (4.3) ensure the control aim realizing for any initial condition of the control system (2.1) belonging to $W_0 \subset R^4 \backslash \{(q,p) \in R^4 : q_1 \neq 0, p_1 \neq 0\}$ in both cases of input functions.

5 Control of generalized Hamiltonian systems

The proposed approach applies also to the so called generalized Hamiltonian systems [17] which are described in the canonical local coordinates as follows

$$\begin{cases} \dot{q} = \nabla_p H_0(q,p,s) + g_q(q,p,s)u, \\ \dot{p} = -\nabla_q H_0(q,p,s) + g_p(q,p,s)u, \\ \dot{s} = g_s(q,p,s)u, \end{cases} \qquad (5.1)$$

where $q \in R^n, p \in R^n, s \in R^l, H_0, g_q, g_p, g_s$ are some smooth functions.

Obviously the function H_0 in (5.1) is an invariant of the unforced system

$$\begin{cases} \dot{q} = \nabla_p H_0(q,p,s), \\ \dot{p} = -\nabla_q H(q,p,s), \\ s = \mathrm{const}. \end{cases} \qquad (5.2)$$

Suppose that some set of the invariants $H_1, \ldots, H_m$ of the unforced system is given. Then we may pose the problem of achieving the goal

$$\lim_{t\to\infty} H_i(q(t), p(t), s(t) = H_{i*}, i = 1, \ldots, k \qquad (5.3)$$

and design the speed-gradient algorithm (3.2) as follows. Choose partial goal functional

$$V_i = (H_i - H_{i*})^2/2.$$

Its derivative along (5.1) is as follows

$$\dot{V}_i = (H_i - H_{i*})[\frac{\partial H_i}{\partial p}\dot{p} + \frac{\partial H}{\partial q}\dot{q} + \frac{\partial H_i}{\partial s}\dot{s}]$$

and

$$\nabla_u \dot{V}_i = (H_i - H_{i*})[\frac{\partial H_i}{\partial p}g_p + \frac{\partial H}{\partial q}g_q + \frac{\partial H_i}{\partial s}g_s]^T.$$

Therefore the algorithm (3.1) reads as

$$u = -\gamma \sum_{i=1}^{m} \nabla H_i^T g(H_i - H_{i*}), \qquad (5.4)$$

where $g = \mathrm{col}(g_q, g_p, g_s)$. The conditions which guarantee achievement of the goal (5.3) can be derived similarly to Proposition 3.1.

A special case of (5.1) is a mechanical system with kinematic constraints. Consider the Lagrange-Euler system with the Hamiltonian

$$H = \frac{1}{2}\dot{q}M(q)\dot{q} + \Pi(q), \qquad (5.5)$$

where $M(q)$ is positive definite matrix of kinetic energy, $\Pi(q)$ is potential energy. Suppose there are k kinematic constraints on the generalized velocities of the form

$$A(q)^T\dot{q} = 0, \qquad (5.6)$$

where the $k\times n$ matrix $A(q)$ has rank k. Performing the procedure of eliminating k dependent generalized coordinates (see e.g. [10, 17]) we arrive to a generalized Hamiltonian description in the space of reduced dimension $2n - k$. The control algorithm ensuring the goal (5.3) can be derived similarly to (5.4).

6 Suppression of resonances

Consider the problem of controlling translational oscillations of rotational actuator (TORA) which has become recently a benchmark example for nonlinear control [6, 13, 2, 12]. In [2, 12] the problem of global stabilization of TORA was considered. However initially this system has been used as a simplified model to study the resonance capture phenomenon [6, 13]. The capture phenomenon represents the failure of a rotating mechanical system to be spun up by a torque-limited rotor to a desired rotational velocity due to its resonant interaction with another part of the system [6, 4, 3].

We consider the problem of suppressing the resonances by means of control. The key point of our solution is to pose the problem as spinning the system up to the desired energy level under restriction on the energy of its specified subsystem. To explain this idea consider the TORA example The approach applies however to much more general systems. The TORA system consists of a cart attached to a wall by a spring. On the cart a rotating eccentric mass (debalanced rotor) is actuated by a DC motor. The goal is to achieve the desired average angular velocity of the motor under constraint imposed on the translational oscillations of the cart. The motor torque is assumed to be a control variable $u = u(t)$.

The model of the system is given by

$$\begin{aligned}(M+m)\ddot{z} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + kz &= 0,\\ ml^2\ddot{\theta} + m\ddot{z}\cos\theta &= u,\end{aligned}$$

where z is the displacement of the cart from its equilibrium position, θ is rotational angle of the rotor. The system has the state vector $x = \mathrm{col}(z, \dot{z}, \theta, \dot{\theta}) \in \mathbb{R}^4$.

The total energy of the system is as follows:

$$H = \frac{1}{2}(M+m)\dot{z}^2 + ml\dot{z}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2. + \frac{1}{2}kz^2. \tag{6.1}$$

We want to achieve the control goal

$$Q(x(t)) \to 0, \tag{6.2}$$

where $Q = (H - H_*)^2/2$ and H_* is the desired energy level under the constraint

$$Q_1(x(t)) \le \Delta, \tag{6.3}$$

where $\Delta > 0$ and Q_1 is the kinetic energy of the cart

$$Q_1(x) = \frac{1}{2}m\dot{z}^2. \tag{6.4}$$

Since we have two objective functions (Q and Q_1) speed-gradient algorithm cannot be applied directly. To solve the problem the projection of gradient-like control strategy is proposed:

a) If the condition (6.3) is satisfied then SG algorithm is used designed for objective function Q which is defined in (6.2).

b) When constraint (6.3) is broken the control strategy suggests to supprese vibrations of first subsystem by means of SG algorithm designed according to the objective (6.4). To ensure the achievement of the two goals (6.2) and (6.3) exploit the freedom of choosing function Ψ in general form of SG algorithm (3.2). Namely choose Ψ forming sharp angle with the speed-gradient of $-Q_1$ and strictly sharp angle with the speed-gradient of $-Q$. Then Q_1 will never increase while Q will decrease unless vectors $\nabla_u\dot{Q}$ and $\nabla_u\dot{Q}_1$ lie in the opposite directions. In that exceptional case right choice is just to put $u = 0$. However the exception may become a rule if the angular velocity of the rotor coincides with the angular frequency of free oscillations of the cart. Such an event means exactly appearance of the resonance which can be passed by small change of control along $-\nabla_u\dot{Q}$ followed by coming back to the previous strategy. The only thing remaining to describe the control algorithm precisely is just explicit expression of speed-gradients.

Calculating the speed-gradients of the goal functions Q and Q_1 in u gives

$$\nabla_u\dot{Q} = (H - H_*)\dot{\theta}, \qquad \nabla_u\dot{Q}_1 = \dot{z}/\cos\theta.$$

To meet the sharp angle condition we combine these expressions into the control algorithm

$$u = \begin{cases} -\gamma(H - H_*)\dot{\theta}, & \text{if } (\nabla_u\dot{Q})(\nabla_u\dot{Q}_1) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{6.5}$$

where $\gamma > 0$. Explicitly,

$$u = \begin{cases} -\gamma(H - H_*)\dot{\theta}, & \text{if } \mu(t) > 0 \\ 0, & \text{if } \mu(t) \le 0 \end{cases} \tag{6.6}$$

where $\mu(t) = (H(p(t), q(t)) - H_*)\dot{\theta}(t)\dot{z}(t)\cos\theta(t)$. Obviously $\mu(t) = (H(p(t), q(t)) - H_*)\dot{z}(t)\dot{z}_1(t)$ where $z_1 = l\sin\theta$ is the projection of the rotor position onto the axis of spring. Although the function Q_1 is not an invariant of the unforced system, the algorithm (6.6) does not allow it to increase and the constraint (6.3) will be held for all $t \ge 0$ provided that it holds at the initial time instant $t = 0$.

7 Conclusion

The obtained results establish possibilities and some limitations of SG algorithms for organizing oscillatory behavior of nonlinear Hamiltonian systems. The proposed algorithms ensure control goal (1.1) for arbitrary f_i, and therefore for arbitrary energy level of system. Moreover, the goal can be achieved with arbitrary small $\gamma > 0$, i.e. for arbitrary small control level (so called swinging property [9]). The results have been extended to the generalized Hamiltonian systems and systems with constraints. It is interesting to compare the above results with the KAM-theory [1] which in essence analyses the behavior of system with uncontrolled perturbed Hamiltonian. One of the core results of KAM-theory can be interpreted as follows: the perturbed system with Hamiltonian $H_\epsilon(q,p) = H_0(q,p) + \epsilon H_d(q,p)$ generically becomes chaotic when ϵ grows. Our results show that the controlled perturbed system with Hamiltonian $H(q,p) = H_0(q,p) + \sum_{j=1}^m H_j(q,p)u_j$ and SG feedback will never create chaos since the trajectories of the closed loop system for arbitrary gain tend to quasiperiodic motions.

The work was supported in part by RFBR (grant 96-01-01151) and INTAS (project 94-965).

References

[1] Arnol'd V.I. Mathematical methods of classical mechanics. Berlin, Springer-Verlag, 1978.

[2] Wan C.-J., Bernstein D.S., Coppola V.T. Global Stabilization of the oscillating eccentric rotor. Proc. 34th IEEE Conf.Dec.Contr., 1994, pp.4024-4029.

[3] Blekhman, I.I., *Synchronization of dynamic systems*, (Moscow-Nauka), 1971, (in Russian).

[4] Blekhman, I.I. *Synchronization in science and technology*, (ASME Press, New York), 1988.

[5] K.J.Åstrom and K.Furuta. Swinging up a pendulum by energy control. Proc 13th Congress of IFAC, v.E, pp.37-42.

[6] Ewan-Ivanovski R.M. *Resonance oscillations in mechanical systems.* Elsevier, Amsterdam, 1976.

[7] Fradkov A.L. Adaptive control in complex systems. Moscow, Nauka, 1990 (in Russian).

[8] Fradkov A.L. Nonlinear adaptive control: regulation-tracking-oscillation. Proc. of 1st IFAC Workshop on New Trends in Design of Control Systems. Smolenice, 1994, pp.426-431.

[9] Fradkov A.L. Swinging control of nonlinear oscillations. Intern. J. of Control, 1996, v.64, n.6, pp.1189-1202.

[10] Fradkov A.L. and E.V. Panteley. Equivalence of two types of algebraic-differential equations. In *Adaptive control of mechanical systems* ed. by A.L. Fradkov, Preprint IPME N71, St.Petersburg, 1992, pp. 41-44.

[11] A.L.Fradkov, P.Yu.Guzenko, D.J.Hill and A.Y.Pogromsky. Speed-gradient control and passivity of nonlinear oscillations. Proc. of the 3rd IFAC Symposium on Nonlinear Control Systems, Tahoe-Sity, USA, 1995, pp.655-659 (v.2)

[12] M. Jankovich, D. Fontaine, P.V. Kokotovic, TORA example: Cascade and passivity based designs, *IEEE Trans. Control Systems Technology*, 1996, v. 4, pp. 292-297.

[13] Kinsey R,J., Mingori D.L., Rand R.H. Nonlinear controller to reduce resonance effects during despin of a dual-spin spacecraft through precession phase lock Proc. *Proc. 31th IEEE CDC*, 1992, pp. 3025-3030.

[14] Konjukhov A.P., Nagibina O.A., Tomchina O.P. Energy based double pendulum control in periodic and chaotic mode. Proc. of 3rd Intern. Conf. on Motion and Vibration Control (MOVIC'96), Chiba, Japan, 1996, pp.99-104.

[15] B.M.Maschke and A.J.van der Schaft. A Hamiltonian approach to stabilization of nonholonomic mechanical systems. Proc. of the 33rd Conf. on Decision and Control, 1994, pp.2950-2954.

[16] H.Nijmeijer and A.J.van der Schaft. Nonlinear Dynamical Control Systems. New York: Springer-Verlag, 1990.

[17] A. van der Schaft. L_2-gain and passivity techniques in nonlinear control. Springer-Verlag, 1996.

[18] M.Wiklund, A.Kristenson, and K.J.Åstrom. A new strategy for swinging up an inverted pendulum. Proc. of the 12th IFAC World Congress, v.9, pp.151-154, 1993.