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METHODS OF NONLINEAR AND ADAPTIVE CONTROL OF CHAOTIC SYSTEMS

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Abstract: The brief survey of recent (1993-1995) papers on control of chaotic systems is given. The problem statements and methods of synchronization, stabilization and control of bifurcations for systems admitting chaotic behavior are discussed.
Keywords: nonlinear control, adaptive control, oscillators, chaos.

1. INTRODUCTION: "PUTTING PHYSICS UNDER CONTROL"

During recent years a dramatic growth of interest was observed to the problem of controlling chaotic systems. The clear sign of it is the growth of number of publications. E.g., the bibliography on control and syncronization of chaos (Chen, 1996) contains more than 400 titles with about 300 published in 1993-1995 and expands by 3%-5% per month. However the rapid growth demands for regular surveying the field. The existing surveys (Ogorzalek, 1993), (Chen and Dong, 1993) reflect the papers published before 1993. The present paper is aimed at surveying the papers on control of chaos published mainly in 1993-1995 and those not mentioned in the previous surveys. Due to the paper length restrictions it is certainly uncomplete and only the main ideas of proposed methods are briefly characterized. A number of the recent publications tend to introduce the mathematical framework into the field previously attacted mainly at the "physical" level of rigour. An important role is played by the methods of nonlinear and adaptive control due to nonlinear dynamics of chaotic systems and presence of uncertainty in real problems. Therefore the second aim of this survey is to show links between new problems of control of chaos and the existing methods of nonlinear and adaptive control.

The most attention in the paper is paid to the problems of syncronization of chaotic systems (Section 2) where results are more systematic. Particularly the synchronization problem is formulated and such methods as the open loop, decomposition, high-gain feedback and coupling are considered. Other classes of problems addressed are stabilization of periodic orbits and equilibria (Section 3) and control of bifurcations (Section 4). Some applications and peculiarities of control of chaos are discussed in the Conclusion.

2. SYNCHRONIZATION

First we formulate the synchronization problem following (Blekhman, 1971, 1988)

Definition 2.1 (*Synchronization problem*).
Given equations of r interacting subsystems:

$$\dot{x}_i = F_i(x_i, u, t), \qquad i = 1 \ldots r, \qquad x_i \in R^{n_i} \tag{2.1}$$

and the equation of connection system

$$u = U(x_1, \ldots, x_r, t), \qquad u \in R^m \tag{2.2}$$

Find conditions of existence and stability of solution $\bar{x}(t)=(x_1(t),x_2(t),...x_r(t))$ of (2.1), (2.2) for $0 \le t < \infty$ which is called synchronous motion of (2.1), (2.2).
In case of arbitrary initial condition this problem will be referred to as the global (total) synchronization problem, while for certain acceptable initial conditions we will speak about conditional synchronization (Dedieu *et al.*, 1993).

Also the concept of stability may be understood in different senses. In original version of (Blekhman, 1971) standard concepts of Lyapunov or asymptotic stability of periodic solution $\bar{x}(t)$ were used. Later (Leonov *et. al.*, 1992) the concept of orbital stability of periodic $\bar{x}(t)$ was employed that is equivalent to stability in the sense of Zhukovsky allowing for different parametrizations $t'=t'(t)$ along periodic synchronous motion $\bar{x}(t')$. For our purposes more mild concept of stability is suitable: so called stability with respect to part of variables (Roumyantsev and Oziraner, 1987). For example, in case of asymptotic synchronization one can require convergence of type:

$$x(t)-\bar{x}(t) \to S \tag{2.3}$$

where S is a manifold of dimension less than $\sum_{i=1}^{r} n_i$, (special cases and examples can be found in (Afraimovich et. al., 1986; Wu and Chua, 1994)). If connection system is given the problem is one of analysis, while in case when we have to find (2.2) to achieve synchronization the problem is one of control system design and (2.2) can be regarded as control algorithm. The special case of the posed problem arises when one of subsystems, say the first one, does not depend on control: $\dot{x}_1 = F_1(x,t)$. If the solution $x_1(t)=\bar{x}_1(t)$ is bounded and the set S is just origin it may be regarded as external synchronizing signal and the problem is referred to as "external", or "master-slave", or "drive-response" synchronization. Note that in terms of control theory (2.4) is just reference model. In what follows we will briefly discuss different known approaches to solving the above problem.

Decomposition based method. Perhaps the first paper concerning chaotic synchronization was one by Pecora and Carroll (Pecora and Carroll, 1990) described a system of two oscillators of the following form:

$$\begin{matrix} \dot{z}_d = Z(z_d,y_d) \\ \dot{y}_d = Y(z_d,y_d) \end{matrix} \text{ drive sys. } \begin{matrix} \dot{z}_r = Z(z_r,y_d) \\ \dot{y}_r = Y(z_r,y_r) \end{matrix} \text{ response sys.}$$

where $(z_d,y_d)^T$ and $(z_r,y_r)^T$ are the state vectors of the drive and response systems. The goal of synchronization is

$$|z_d(t)-z_r(t)| \to 0, \text{ when } t \to \infty \tag{2.4a}$$

It was conjectured that (2.4a) is achieved if all Lyapunov exponents of the response system calculated along the trajectory of drive system are negative. As an example interconnected Lorenz systems were considered (Carrol, Pecora, 1993). Similar example was investigated in (Cuomo *et al*, 1993) where another variable of the Lorenz system was used to as the "drive signal". The case when the state of the response model is used as the drive signal for another slave system (cascade synchronization) was considered in (Carroll and Pecora, 1993). Consider the additional goal:

$$|y_d(t)-y_r(t)| \to 0, \text{ when } t \to \infty. \tag{2.4b}$$

It turns out that the conditions of synchronization in the sense (2.4a), (2.4b) can be established using concepts of input-to-state stability or more weak bounded input-bounded state stability (BIBS) (Sontag, 1990). To investigate system behavior consider the following error system:

$$\dot{e}_z = f_0(e_z,z_d,y_d) \quad \dot{e}_y = f_1(e_z,e_y,z_d,y_d),$$
$$e_z = z_d - z_r,\ e_y = y_d - y_r$$

It may be shown that the goals (2.4a), (2.4b) are (globally) achieved if $\dot{e}_z = f_0(e_z,z_d,y_d)$ and $\dot{e}_y = f_1(0,e_y,z_d,y_d)$ are (globally) asymptotically stable and the system $\dot{e}_y = f_1(e_z,e_y,z_d,y_d)$ is BIBS stable with e_z as input. It can be also shown that known applications of the Pecora-Carroll method for Chua's circuit (Murali and Lakshmanan, 1993), Lorenz equation (Cuomo et al, 1993) etc. can be analyzed in such a framework.

High-gain synchronization. Consider the following synchronization scheme:

$$\begin{aligned} &\dot{x}_1 = f_1(x_1)+\gamma K_1(y_1-y_2),\ y_1 = h_1(x_1),\quad y_2 = h_2(x_2) \\ &\dot{x}_2 = f_2(x_2)-\gamma K_2(y_1-y_2),\ x_1 \in R^{n_1}, x_2 \in R^{n_2}, \gamma \in R^1 \end{aligned} \tag{2.5}$$

First we assume that $f_1 = f_2, h_1 = h_2$ and the synchronization goal is $|x_1(t)-x_2(t)| \to 0$. Further assume that the error equation obtained from (2.5) after some coordinate transformation, is given by:

$$\begin{cases} \dot{e}_z = f_{e0}(e_z,e_y,z_1,y_1) \\ \dot{e}_y = f_y(e_z,e_y,z_1,y_1)+\gamma K(e_z,e_y,z_1,y_1)e_y \end{cases} \tag{2.6}$$

where $e_z = z_1 - z_2$, $e_y = y_1 - y_2$, and K is positive definite matrix. Using results of (Byrnes and Isidori, 1991) it can be shown that the error system can be made semiglobally exponentially stable by appropriate choice of gain γ if zero dynamics $\dot{e}_z = f_{e0}(e_z,0,z_1,y_1)$ are exponentially stable. Some authors also used linear feedback to synchronize chaotic systems e.g. Chen and Dong (1993a) applied it to synchronize two Duffing's oscillators.

Synchronization of two systems each of them is of Lur'e form (linear part with static nonlinearity in feedback loop) was considered in (Tesi *et al.*, 1994) where several parametrizations of linear part of the system are proposed which help improve synchronization property. An approach to synchronization problem for discrete-time systems based on the contraction mappings is presented in (Ushio, 1995).

Coupling. Synchronization scheme (2.5) for nonidentical f_1 and f_2 can describe connection (coupling) of different electrical circuits. Kapitanyak et. al (1993) considered coupling of Chua's circuit and linear oscillator. It was shown numerically that the synchronization in the sense that $|y_1 - y_2| \to 0$ can be observed.

Open-loop synchronization. Methods of open-loop synchronization do not require the measurement of plant state or output and therefore they are easy to implement. In the paper by Leonov (1986) the Lur'e system

$$\dot{x} = Px + B(\varphi(y) + \alpha \sin\omega t), \quad y = Cx$$

was considered with nonlinearity $\varphi(y)$ which graph lies inside stability sector for $|y| \geq y_1$ and outside otherwise. Using absolute stability theory the conditions wich ensure existence of bounded $2\pi/\omega$-periodic solution $\bar{x}(t)$ and synchronization goal $|x(t) - \bar{x}(t)| \to 0$ for any initial conditions were found. These conditions (which are in most cases valid for sufficiently large α) were relaxed by Churilova (1994) and extended to discrete-time systems by Gelig (1990). Also a number of papers on open-loop control by either harmonic or nonharmonic forcing signals which contain interesting computer simulations rather than rigorous results were published.

Adaptive synchronization. Adaptive synchronization problem can be posed as follows (Fradkov, 1994).

Definition 2.2. (Adaptive synchronization problem).

Given equations of r interacting subsystems:

$$\dot{x}_i = F_i(x_i, u, \xi, t), \qquad i = 1...r, \qquad x_i \in R^{n_i} \tag{2.7}$$

where $\xi \in \Xi$ is the vector of unknown parameters.

Find the equation of connection system

$$u = U(x_1, ..., x_r, \theta, t), \qquad u \in R^m \tag{2.8}$$

and adaptation algorithm

$$\theta = \Theta(x_1, ..., x_r, \theta, t)$$

ensuring the goal $|x_i(t) - \bar{x}_i(t)| \leq \Delta$ for $t > t_*$,

where $\bar{x}(t)$ is some solution, perhaps unknown *apriory*.

Some attempts have been made in order to achieve the goal of adaptive synchronization for chaotic systems (Vassiliadis, 1994) where known adaptive algorithms were employed for a new class of chaotic systems. Local synchronizing property of the system was established. System (2.7) with right hand sides consisting of linear passyfiable part and nonlinearities available for measurements was considered in (Fradkov, 1995). Conditions of adaptive synchronization achievement are based on feedback Kalman-Yakubovich lemma and speed-gradient (SG) algorithm (Fradkov, 1979). SG method is applicable also for other problems of synchronization and control of chaotic systems. It is based on reformulating of control goal as follows:

$$Q(x(t), t) \to 0, \text{ when } t \to 0 \tag{2.9}$$

where $Q(x,t) \geq 0$ is given objective function. Then the adaptation law is as follows:

$$\frac{d}{dt}(\theta + \psi(x, \theta, t)) = -\Gamma \nabla_\theta \omega(x, \theta, t) \tag{2.10}$$

where $\Gamma = \Gamma^T > 0$, $\omega(x,\theta,t) = \nabla_x Q F^T(x,\theta,t) + \partial Q/\partial t$ and $\psi(\cdot)$ satisfies pseudogradient condition:

$$\psi^T \nabla_\theta \omega(x, \theta, t) \geq 0 \tag{2.11}$$

Applicability of the method requires convexity of ω with respect to the vector of adjustable parameters. Moreover the main control law should be chosen in such a way to ensure that $Q(x)$ becomes Lyapunov function of the overall system for some "ideal" adjustable parameters. The SG method was applied to adaptive synchronization of Duffing's systems (Fradkov and Pogromsky, 1995; Pogromsky, 1995) and Chua's systems (Fradkov et al, 1995).

Notice that the problem of output adaptive synchronization encompasses the problems of adaptive output-feedback model-reference control and adaptive observer design. Therefore results of (Krstic et al., 1995; Marino and Tomei, 1992) can be employed.

3. STABILIZATION OF INHERENT SOLUTIONS OF CHAOTIC SYSTEMS

One of the first papers in the field of control of chaotic systems was the one by Ott, Grebogi and Yorke (Ott *et al*, 1990) where the idea to stabilize unstable periodic orbit embedded within the chaotic attractor was proposed. The idea is based on the linearization of the Poincare map in the neighbourhood of its fixed point, i.e. periodic orbit χ of the system. The paper produced a real boom in physics, because it makes possible to solve various applied problems, see Petrov et. al. (1994), Schiff et al. (1994), Weiss (1994) . In practice, linearization of Poincare map can be obtained by identification with probing control signals or using time-delayed coordinates.

Apparently linearization can be applied to stabilize periodic orbit of arbitrary system, but for systems with chaotic dynamics it has some advantages. Indeed, as we assumed the desired periodic orbit is embedded in attractor of the uncontrolled system, so we can design the control procedure in the following manner: wait until the trajectory of the uncontrolled system comes near the desired orbit and then apply one of the standard linear control algorithm for linearized control system. Once the trajectory falls in the neighborhood of the desired periodic orbit the local controller can stabilize it. So the question is whether the uncontrolled trajectory gets to this neighborhood without any control force. It certainly does in case when χ is ω – limit set of the uncontrolled system for all possible initial conditions, that is usually assumed in published papers.

The disadvantage of the approach is high sensitiveness to noise and disturbances due to the control is applied only at moments of intersection of the trajectory with surface Σ. An important feature is that the linearized model can be quite easy obtained from experimental data by solving the identification problem with probing signals.

Different control laws can be chosen to stabilize linearized discrete-time system on Poincare section Σ. For example Romeiras et al., (1992) applied pole-assignement procedure, while Hunt and Johnson (1993) used simple proportional output feedback. Gallias (1995) suggested to minimize the norm of the next value of the error vector by projection algorithm which essentially coincides with Kaczmarz algorithm , see e.g. (Aved'yan and Tsypkin, 1979). The convergence rate of OGY algorithm was evaluated in (Aston and Bird, 1995).

Another method was proposed by Pyragas (1992). It was suggested to use continuous-time control law

$$u(t) = -K(y(t) - \bar{y}(t)) \tag{3.1}$$

where y is the measurable output of the system and $\bar{y}(t)$ is the reconstructed inherent periodic solution of the uncontrolled system which can be obtained numerically from the time-delayed coordinates. Stability results for this kind of controller can be established similary to the case of high-gain synchronization. Since the desired trajectory $\bar{y}(t)$ is to be obtained via extensive preliminary experimentation another algorithm was proposed (Pyragas, 1992):

$$u(t) = -K(y(t) - y(t-\tau)) \tag{3.2}$$

where τ is the estimated period of $\bar{y}(t)$. Modification of this method using Fourier analysis was suggested in (Socolar et al., 1994). Computer simulations carried out for various examples (see ref. in Pyragas, 1995) demonstrate the ability of the control laws (3.1), (3.2) to stabilize the inherent periodic orbits.

General methods of nonlinear control were employed for stabilization of the inherent solutions of chaotic systems recently, e.g. feedback linearization (Alvarez-Gallegos, 1994), (Krishchenko, 1995), Lyapunov observer-based control for chaotic system (Nijmeijer and Berghius, 1995 a,b), variable structure control (Yu, 1995). Genesio and Tesi (1993) (see also Genesio *et. al*, 1993) employed frequency harmonic balance to formulate structural conditions which approximately express the occurrence of complex dynamics phenomena in Lur'e systems and suggested algorithms to control the distortion.

Algorithms for stabilizing higher periodic orbits were suggested by Paskota et al (1994, 1995). Chen and Dong (1995) and Lebender et al. (1995) developed neural network based algorithms. Finally, different problems of adaptive stabilization for chaotic systems were solved by speed-gradient method (Fradkov, 1994; Fradkov and Pogromsky, 1995; Pogromsky, 1995).

4.BIFURCATION CONTROL OF CHAOTIC SYSTEMS

One of the new approaches to the control of nonlinear systems is the bifurcation control (Wang and Abed, 1995; Lee and Abed, 1991, Abed et al, 1992;).

Assume that nonlinear system to control depends on some scalar parameter μ :

$$\dot{x} = F(x, \mu, u) \tag{4.1}$$

As this parameter slowly varies the uncontrolled system undergoes bifurcations which can result in changes of the system behavior.

There are two important problems arising in the control of each given bifurcation:
1) How to change the value of bifurcation parameter for which bifurcation occurs?
2) How to modify stability of bifurcated solutions?

To solve both of the problems Lee and Abed (1991) proposed to apply dynamic feedback based on the so called *washout filters* with the following transfer function:

$$G(s) = y(s)/x(s) = s/(s+d) \tag{4.2}$$

where d>0 is the filter parameter. As inputs of the filters the state x is used while feedback law u is based on the output y of the washout filters. The useful feature of the approach is that the control law does not result in any changes in the set of equilibria even in case of model uncertainty. The main idea of the approach is to find control of the following form:

$$u = -K_u y + y^T Q_u y + C_u(y, y, y) \tag{4.3}$$

where K_u is linear gain, Q_u is symmetric matrix, and C_u is a cubic form. Using various examples (Wang and Abed, 1995, Abed *et al.*, 1992) it was demonstrated that linear term in (5.3) solves the first problem while nonlinear (quadratic and cubic) terms modify the stability of the bifurcated solution. Similar approach can be also applied to discrete-time systems (see e.g. Abed *et al.*, (1992) , where the control of period doubling bifurcation was proposed).

CONCLUSION: CHAOS FACILITATES CONTROL

The reason of increasing interest in control of chaos is that control gives a perspective both of better understanding chaotic behavior and of modifying it. Various applications were reported such as eliminating multimode regimes in lasers (Gills et al., 1994); increasing the reaction rate in chemical technology by means of chaotic stirring (Petrov et. al., 1994); providing secure communications by using chaotic carrier signals (Cuomo et. al., 1993; Dedieu et al., 1993); treating ventricular tachycardia (Weiss et al.,1994); controlling neuronal activity in the preparation from the rat brain (Schiff et al, 1994).

Note in conclusion that chaotic behavior of uncontrolled plant can also facilitate control of it at least by two ways. The first way is based on idea that for many nonlinear systems chaos is easy to create using open-loop control and the resulting chaotic system will, through time, drive the trajectory of the system to vicinity of the goal set of the closed-loop overall system where control goal may be achieved by control signal of small level. This advantage of chaotic motion was used by (Vincent, 1995) to control a ball bouncing on a vibrating plate and by (Pogromsky, 1995) to stabilize inherent unstable equilibria of Lorenz system. The second way is related to identification based adaptive control where *persistency of excitation* ensures convergence of the parameter estimates to their true values. Although in general the presence of chaos does not imply persistency of excitation, in many cases it does. Therefore introducing chaotic behavior in originally nonchaotic system can help solve various control problems.

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