19th IFAC Symposium on Automatic Control in Aerospace
September 2-6, 2013. Würzburg, Germany

Hidden oscillations in stabilization system of flexible launcher with saturating actuators ⋆

B.R. Andrievsky [∗,∗∗] **N.V. Kuznetsov** [∗,∗∗∗] **G.A. Leonov** [∗] **S.M. Seledzhi** [∗]

[∗] *Saint-Petersburg State University, 28 Universitetsky prospekt, 198504, Peterhof, Saint Petersburg, Russia, leonov@math.spbu.ru, kuznetsov@math.spbu.ru*
[∗∗] *Institute for Problems of Mechanical Engineering, the Russian Academy of Sciences, 61 Bolshoy prospekt, V.O., 199178, Saint Petersburg, Russia, boris.andrievsky@gmail.com*
[∗∗∗] *University of Jyväskylä, PO Box 35, FI-40014, Finland*

Abstract: In the paper the attitude stabilization system of the unstable flexible launcher with saturating input is considered. It is demonstrated that due to actuator saturation the system performance can significantly degrade. The analytical-numerical method is applied to demonstrate possibility of hidden oscillations and localize their attractor.

Keywords: launcher, flexible structure control, actuator saturation, oscillations

1. INTRODUCTION

It is well known, that due to control input saturation the closed-loop system performance may be significantly worsen, up to the stability loss (Åström and Rundqwist, 1989; Hippe, 2006; Tarbouriech et al., 2011). Particularly, oscillating behavior of the system, which is well damped at the nominal (non-saturated mode) may appear (see, e.g., (Tarbouriech et al., 2011; Leonov et al., 2012a; Andrievsky et al., 2012)).

Remark that simple simulation of such systems is an unreliable tool and can lead to wrong conclusions. For numerical computation of possible limit solutions in a dynamic system, all initial conditions need to be evaluated, and in a non-autonomous systems with input, in addition, all possible inputs need to be considered. Here to get reliable results of simulations one need to verify analytically a condition of uniqueness of limit solution (i.e. convergent property of systems (van den Berg et al., 2006; Pogromsky et al., 2009; Leonov et al., 2012a)) or to apply special analytical-numerical procedures, which allow to compute hidden oscillations (Leonov and Kuznetsov, 2011; Bragin et al., 2011; Leonov et al., 2011, 2012b).

The paper is organized as follows. The analytical-numerical procedure for localization of hidden attractors in dynamical systems is briefly described in Sec. 2. Application of this procedure to analysis of the stabilization system of flexible launcher is presented in Sec. 3, where the anti-windup augmentation is also outlined. Concluding remarks are given in Sec. 4.

⋆ Partially supported by the Ministry of Education and Science of Russian Federation, Saint Petersburg State University, Academy of Finland, and RFBR

2. HIDDEN ATTRACTORS LOCALIZATION BY MEANS OF ANALYTICAL-NUMERICAL PROCEDURE

In this section a brief exposition of the analytical-numerical procedure for hidden attractors localization in dynamical systems is given. More details may be found in (Leonov and Kuznetsov, 2011; Bragin et al., 2011; Leonov et al., 2011, 2012b), see also a survey (Leonov and Kuznetsov, 2013).

In the first half of the last century during the initial period of the development of the theory of nonlinear oscillations a main attention was given to analysis and synthesis of oscillating systems, for which the problem of the existence of oscillations can be solved with relative ease. The structure of many applied systems considered was such that the existence of oscillations was “almost obvious” — the oscillation was excited from an unstable equilibrium (so called self-excited oscillation). From the computational point of view this allows one to use a *standard computational procedure, in which after a transient process a trajectory, started from a point of unstable manifold in a neighborhood of equilibrium, reaches a state of oscillation therefore one can easily identify it.*

Note that a numerical analysis of oscillatory solutions by standard numerical procedures requires the existence of an attraction domain for such trajectories. In this case the numerical simulation can be utilized to characterize the oscillatory solution.

A further study showed that the self-exited periodic and chaotic oscillations did not give exhaustive information about the possible types of oscillations. In the middle of 20th century several examples of periodic and chaotic

978-3-902823-46-5/2013 © IFAC

10.3182/20130902-5-DE-2040.00040

oscillations of another type, so called (Leonov et al., 2011), *hidden oscillations* and *hidden attractors* were found. The oscillatory trajectory is called hidden oscillations if *its basin of attraction does not intersect with sufficiently small neighborhoods of equilibria.* Numerical localization, computation, and analytical study of hidden attractors are much more difficult problems since in this case no information about the equilibria can be directly used with standard computational procedures. Thus the hidden attractors cannot be numerically found by using conventional numerical methods. Furthermore, in this case it is unlikely that the integration of trajectories with random initial data furnishes hidden attractor localization since a basin of attraction can be relatively small and the dimension of hidden attractor can be much less than the dimension of the considered system.

Further, an example of effective analytical-numerical approach, for hidden oscillations localization, which is based on continuation, the method of small parameter and a modification of the describing function method (DFM) [1], is demonstrated.

The considered modification of the describing function method, based on the method of small parameter, permits one to obtain a strict justification of the existence of periodic solution and to define the initial data of this solution. Then a particular multi-step numerical procedure, based on the continuation principle, permits one to follow numerically the transformation of an initial periodic solution to some auxiliary system, defined analytically, to the periodic solution or chaotic attractor of the original system.

Consider a system with one scalar nonlinearity

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}\mathbf{x} + \mathbf{q}\psi(\mathbf{r}^*\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^n. \tag{1}$$

Here $\mathbf{P}$ is a constant $(n \times n)$-matrix, $\mathbf{q}, \mathbf{r}$ are constant n-dimensional vectors, * is a transposition operation, $\psi(\sigma)$ is a continuous piecewise-differentiable scalar function, and $\psi(0) = 0$.

Define a coefficient of harmonic linearization k (suppose that such a k exists) in such a way that the matrix $\mathbf{P}_0 = \mathbf{P} + k\mathbf{q}\mathbf{r}^*$ has a pair of purely imaginary eigenvalues $\pm i\omega_0$ $(\omega_0 > 0)$ and the rest eigenvalues have negative real parts . Rewrite system (1) as

$$\frac{d\mathbf{x}}{dt} = \mathbf{P_0}\mathbf{x} + \mathbf{q}\varphi(\mathbf{r}^*\mathbf{x}), \tag{2}$$

where $\varphi(\sigma) = \psi(\sigma) - k\sigma$.

Introduce a finite sequence of functions

$$\varphi^0(\sigma), \varphi^1(\sigma), \ldots, \varphi^m(\sigma)$$

such that the graphs of neighboring functions $\varphi^j(\sigma)$ and $\varphi^{j+1}(\sigma)$ slightly differ from one another, the initial function $\varphi^0(\sigma)$ is small, and the final function $\varphi^m(\sigma) = \varphi(\sigma)$ (e.g., below it is considered $\varphi^j(\sigma) = \varepsilon^j\varphi(\sigma)$, $\varepsilon^j = j/m$). Since the initial function $\varphi^0(\sigma)$ is small over its domain of definition, the method of describing functions give mathematically sound conditions on the existence of a periodic solution for system

$$\frac{d\mathbf{x}}{dt} = \mathbf{P_0}\mathbf{x} + \mathbf{q}\varphi^0(\mathbf{r}^*\mathbf{x}) \tag{3}$$

see, e.g. (Leonov, 2010; Leonov et al., 2010b,a).

Its application allows to define a stable nontrivial periodic solution $\mathbf{x}^0(t)$ (a starting oscillating attractor denoted further by $\mathcal{A}_0$).

Two alternatives are possible. The first case: all the points of $\mathcal{A}_0$ are in an attraction domain of the attractor $\mathcal{A}_1$, which is an oscillating attractor of the system

$$\frac{d\mathbf{x}}{dt} = \mathbf{P_0}\mathbf{x} + \mathbf{q}\varphi^j(\mathbf{r}^*\mathbf{x}) \tag{4}$$

with $j = 1$. The second case: in the passing from system (3) to system (4) with $j = 1$ it is observed a loss of stability (bifurcation) and $\mathcal{A}_0$ vanishes. In the first case the solution $\mathbf{x}^1(t)$ can be found numerically by starting a trajectory of system (4) with $j = 1$ from the initial point $\mathbf{x}^0(0)$. If numerical integration over a sufficiently large time interval $[0, T]$ gives an evidence that the solution $\mathbf{x}^1(t)$ remains bounded and is not attracted by an equilibrium, then this solution reaches an attractor $\mathcal{A}_1$. In this case it is possible to proceed to the next system (4) with $j = 2$ and to perform a similar procedure of computation of $\mathcal{A}_2$ by starting a trajectory of system (4) with $j = 2$ from the initial point $\mathbf{x}^1(T)$ and computing a trajectory $\mathbf{x}^2(t)$.

Following this procedure, sequentially increasing j, and computing $\mathbf{x}^j(t)$ (a trajectory of system (4) with the initial data $\mathbf{x}^{j-1}(T)$), one can either find a solution around $\mathcal{A}_m$ (an attractor of system (4) with $j = m$, i.e. original system (2)), or, alternatively, observe, at a certain step j, a bifurcation where the attractor vanishes.

Let $\varphi^0(\sigma) = \varepsilon\varphi(\sigma)$ with ε being a small parameter. To define the initial data $\mathbf{x}^0(0)$ of the initial periodic solution, system (3) with the nonlinearity $\varphi^0(\sigma)$ is transformed by a linear nonsingular transformation $\mathbf{x} = \mathbf{S}\mathbf{y}$ to the form [2]

$$\begin{aligned} \dot{y}_1 &= -\omega_0 y_2 + b_1\varphi^0(y_1 + \mathbf{c}_3^*\mathbf{y}_3), \\ \dot{y}_2 &= \omega_0 y_1 + b_2\varphi^0(y_1 + \mathbf{c}_3^*\mathbf{y}_3), \\ \dot{\mathbf{y}}_3 &= \mathbf{A}_3\mathbf{y}_3 + \mathbf{b}_3\varphi^0(y_1 + \mathbf{c}_3^*\mathbf{y}_3). \end{aligned} \tag{5}$$

Here y_1, y_2 are scalars, $\mathbf{y}_3$ is $(n-2)$-dimensional vector; $\mathbf{b}_3$ and $\mathbf{c_3}$ are $(n-2)$-dimensional vectors, b_1 and b_2 are real numbers; $\mathbf{A}_3$ is an $((n-2) \times (n-2))$-matrix with all eigenvalues with negative real parts. Without loss of generality, it can be assumed that for the matrix $\mathbf{A}_3$ there exists a positive number $d > 0$ such that $\mathbf{y}_3^*(\mathbf{A}_3 + \mathbf{A}_3^*)\mathbf{y}_3 \le -2d|\mathbf{y}_3|^2, \forall\, \mathbf{y}_3 \in \mathbb{R}^{n-2}$.

Introduce the describing function

$$\Phi(a) = \int_0^{2\pi/\omega_0} \varphi\big(\cos(\omega_0 t)a\big)\cos(\omega_0 t)dt. \tag{6}$$

assume the existence of its derivative.

Theorem 1. (Leonov, 2010; Bragin et al., 2011) If there is a positive a_0 such that

$$\Phi(a_0) = 0, \quad b_1\frac{d\Phi(a)}{da}\bigg|_{a=a_0} < 0 \tag{7}$$

then there is a periodic solution

$$\mathbf{x}^0(0) = \mathbf{S}(y_1(0), y_2(0), \mathbf{y}_3(0))^*$$

with initial data $y_1(0) = a_0 + O(\varepsilon)$, $y_2(0) = 0$, $\mathbf{y}_3(0) = \mathbf{O_{n-2}}(\varepsilon)$.

[1] DFM is used here for determining of an initial periodic solution for continuation method. However, in many simple cases the initial periodic solution is a self-excited oscillation and can be obtained by standard computational procedure.

[2] Such transformation exists for nondegenerate transfer functions.

This theorem describes the procedure of the search for stable periodic solutions by the standard describing function method[3] (see, e.g. (Khalil, 2002)).

3. HIDDEN OSCILLATIONS IN THE LAUNCHER STABILIZATION SYSTEM WITH SATURATING ACTUATOR

In this Section, the attitude stabilization system for hypothetical flexible unstable launcher in the presence of actuator saturation is analyzed by application of the above results.

3.1 Flexible launcher dynamics

Consider the following linearized model of a flexible space launch vehicle, cf. (Hahs and Sorrells, 1991):

$$\ddot{\psi}(t) + a_y^{\dot{\psi}}\dot{\psi}(t) + a_y^{\psi}\psi(t) = a_y^{\delta_r}\delta_r(t) + f_y(t), \tag{8}$$

$$\ddot{\tilde{\psi}}(t) + 2\xi_1\omega_1\dot{\tilde{\psi}}(t) + \omega_1^2\tilde{\psi}(t) = l_1\omega_1^2\delta_r(t) + \tilde{f}_y(t), \tag{9}$$

$$\psi_g(t) = \psi(t) + \tilde{\psi}(t). \tag{10}$$

Equation (8) represents the launcher dynamics as a rigid body, where ψ is the yaw angle, δ_r denotes the angle of the rudder deflection; equation (9) describes the first flexible mode dynamics; ψ_g in (10) stands for the plant output, measured by the gyro sensor; f_y, $\tilde{f}_y$ represent external disturbances, omitted in this study.

Assuming that the ruder servo dynamics are fast, but taking into account the actuator output limitations, let us describe the rudder servo by the following nonlinear (saturation) function

$$\delta_r = M\,\mathrm{sat}\left(\frac{u}{M}\right), \tag{11}$$

where $u(t)$ is the control signal, generated by the angular stabilization system, M represents the magnitude bound on the rudder deflection.[4]

3.2 Nominal controller design

Let the following proportional-derivative (PD) controller be used for the launcher stabilization on attitude:

$$u(t) = -k_P\psi_g(t) - k_D\dot{\psi}_g(t), \tag{12}$$

where k_P, k_D are the proportional and derivative controller gains (respectively).

Let the plant l4, (9) state-space vector $\mathbf{x}$ be taken as $\mathbf{x} = (\psi, \tilde{\psi}, \dot{\psi}, \dot{\tilde{\psi}})^{\mathrm{T}}$. Then the closed-loop system (8)–(12) dynamics may be represented in form (1) with the following matrices:

$$\mathbf{P} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_y^{\psi} & 0 & -a_y^{\dot{\psi}} & 0 \\ 0 & 0 & -\omega_1^2 & -2\xi_1\omega_1 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ a_y^{\delta_r} \\ l_1\omega_1^2 \end{pmatrix}$$
$$\mathbf{r}^* = -\left(k_P, k_P, k_D, k_D\right), \tag{13}$$

nonlinearity $\psi(\cdot)$ in (1) has a form (11).

Let the following parameters of model (8)–(11) be given: $a_y^{\delta_r} = 7\ \mathrm{s}^{-2}$, $a_y^{\psi} = -4\ \mathrm{s}^{-2}$ (this means the weathercock instability of the launcher), $a_y^{\dot{\psi}} = 0.4\ \mathrm{s}^{-1}$, $l_1 = -0.06\ \mathrm{s}^{-2}$, $\xi_1 = 0.03$, $\omega_1 = 2\ s^{-1}$ (such a low natural frequency of the first flexible mode may occur for large-scale space launch vehicles), $M = 0.174$ rad (≈ 10 degrees).

At the first stage let us consider the nominal (non-saturated mode), and, assuming that $\delta_r \equiv u$ find the PD-controller gains, cf. (Biannic and Tarbouriech, 2007; Biannic et al., 2006; Tarbouriech and Turner, 2009; Tarbouriech et al., 2011). Applying the frequency loop-sharping technique (Grassi and Tsakalis, 1996) one can find that the gains $k_P = 6$, $k_D = 2$ s ensure: gain stability margin $G_m = 16$ dB, magnitude stability margin $\Phi_m = 84$ deg, H_∞-gain $= 1.18$ for the nominal system.

3.3 Saturation effect and hidden oscillations analysis

Presence of the actuator saturations may lead to the so-called "plant wind-up" phenomenon (Hippe, 2006; Tarbouriech et al., 2011), which dramatically changes the overall closed-loop system performance.

For the given parameter values, the matrices in (13) are numerically as follows:

$$\mathbf{P} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 4.0 & 0 & -0.4 & 0 \\ 0 & -4.0 & 0 & -0.12 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ 14 \\ -0.48 \end{pmatrix}$$
$$\mathbf{r}^* = -\left(6.0, 6.0, 2.0, 2.0\right). \tag{14}$$

Application of described above multistep localization procedure allows one to find numerically hidden oscillation (see Fig. 1-4) which coexists with stable stationary point.

4. CONCLUSION

In this paper existence of hidden oscillations in launcher stabilization system with saturating actuator are demonstrated numerically. Existence of such hidden oscillations shows that simple modeling of trajectories in the neighborhood of stationary point of with random initial data can lead to unreliable results. Thus, following (Lauvdal et al., 1997), one can conclude "*Since stability in simulations does not imply stability of the physical control system (an example is the crash of the YF22), stronger theoretical understanding is required.* ".

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[3] In engineering practice for the analysis of the existence of periodic solutions it is widely used classical harmonic linearization and describing function method (DFM). However this classical method is not strictly mathematically justified and can lead to untrue results (e.g., the DFM proves validity of Aizerman's and Kalman's conjectures on absolute stability, while counterexamples with hidden oscillation are well known, see, e.g., (Bragin et al., 2011; Leonov and Kuznetsov, 2013)).

[4] To simplify the exposition we refer the servo static gain to parameters $a_y^{\delta_r}$ and l_1 of the plant model.

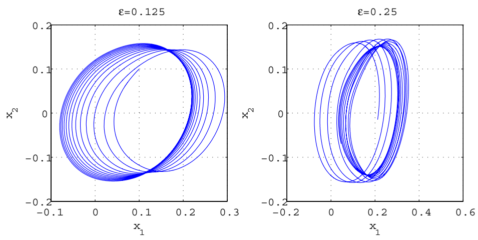

Fig. 1. Multistep localization of hidden oscillation. $\varepsilon = 0.125; 0.25$

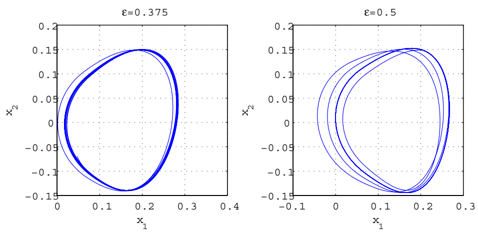

Fig. 2. Multistep localization of hidden oscillation. $\varepsilon = 0.375; 0.5$

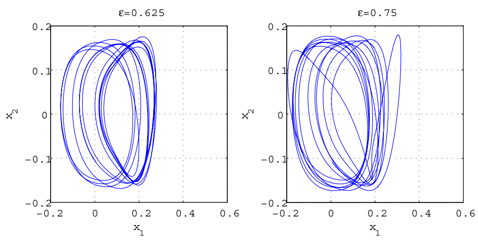

Fig. 3. Multistep localization of hidden oscillation. $\varepsilon = 0.625; 0.75$

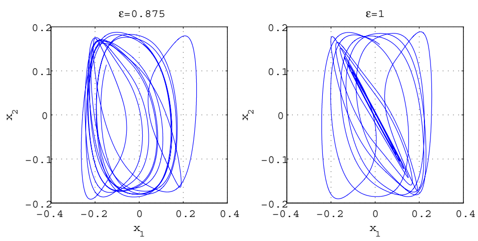

Fig. 4. Multistep localization of hidden oscillation. $\varepsilon = 0.875; 1$ At the last step ($\varepsilon = 1$, i.e. for the initial system) there is stable zero equilibrium point coexist with an oscillation (hidden oscillation).

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