Passification-based robust flight control design *

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Abstract

A passification-based robust autopilot for attitude control of the flexible aircraft under parametric uncertainty is designed. The high gain controller with forced sliding motions is used to secure good performance in the wide range of the aircraft model parameters. The shunting method is applied to ensure the closed-loop system stability under lack of the aircraft state information. The series reference model is used to assign the desired closed-loop system performance. An example illustrating typical design procedure for aircraft attitude control in the horizontal plane for different flight conditions is given. The simulation results demonstrate efficiency and high robustness of the suggested control system.

Key words: Flight control; Uncertain dynamic systems; Shunt compensation; Robustness; Passification

1 Introduction

Modern high-maneuverable aircrafts, such as fighters, operate over a wide range of flight conditions, which vary with altitude, Mach number, angle-of-attack, and engine thrust. The mechanical characteristics of the airframe, such as the center of gravity, change as well. The aircraft autopilot has to be able to produce a response that is accurate and fast despite severe variations in speed and altitude of the airframe or, in the other words, in the face of large parametric uncertainty [13,29,37,39]. The promising way to fulfill these requirements is application of the adaptive control technique. The adaptation method has to meet the conflicting requirements on the tuning rate and performance quality under the conditions of lack of the aircraft state measurements [6,14,23,28,35,37,41].

The term “passification-based adaptive control” was introduced in [36], though the structures of the passification based adaptive controllers for linear plants with underlying theory were introduced as early as in the 1970s [18,19] under different names. Initially they were named “Adaptive Systems with Implicit Reference Models” (ASIRM). Main results are presented in a number of books and surveys [1,17,22,23,26]. Later related structures were used in the so-called Simple Adaptive Control (SAC) systems [7,8,10,11,13,14,31,32]. Connection between two approaches was studied in [3]. Usage of passification-based flight control is motivated by its simplicity and its close relation to stability: if a system is passive with respect to some output $y$, it can be asymptotically stabilized by the output feedback $u = -ky$ for any $k > 0$. Applicability conditions of the method (necessary and sufficient conditions of passifiability) were provided in [18,19] for linear systems and in [15,25] for nonlinear systems.

In the present brief, the passification method is applied for robust attitude control of the flexible aircraft with high-order model. For ensuring the applicability conditions of the method, the parallel feedforward compensator (the shunt) [4,5,7,9,11,21,30,32,34] is introduced into the controller.

The brief is organized as follows. Some essentials of the passification method are outlined in Section 2. Section 3 is devoted to the application of passification method for robust control of flexible aircraft. Concluding remarks are given in Section 4.

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2 Preliminaries. Passification and Shunting Methods

2.1 Passification Theorem

Consider a linear time invariant (LTI) single-input multiple-output (SIMO) system

\[ \dot{x} = Ax + Bu, \quad z = Cx, \]  

(1)

where \( x = x(t) \in \mathbb{R}^n \) is a state vector, \( u = u(t) \in \mathbb{R}^l \) is a scalar control variable, \( z = z(t) \in \mathbb{R}^l \) is a measured output vector, \( A, B, C \) are constant real matrices of sizes \( n \times n, n \times l, l \times n \) respectively. Let \( G \) be \((1 \times l)\)-matrix.

Passification problem for the system (1) is understood here as finding an \((l \times 1)\)-matrix \( K \) such that the closed loop system with feedback \( u = -K^T z + v \) is strictly passive with respect to an auxiliary output \( \sigma = Gz \): inequality

\[ \int_0^T (\sigma v - \rho |v|^2) \, dt \geq 0 \quad \text{for some} \quad \rho > 0 \quad \text{and all} \quad T > 0 \]

holds for all trajectories of (1) starting from \( x(0) = 0 \). This is equivalent (as follows from Kalman–Yakubovich–Popov Lemma) to finding a matrix \( K \) satisfying the strict positive realness (SPR) condition: transfer function \( W(\lambda) = GC(\lambda I_n - A + BK^T C)^{-1} B \) of the closed-loop system \(^1\) from input \( v \) to the output \( \sigma = Gz \) satisfies the relations

\[ \text{Re} W(i\omega) > 0 \quad \text{for all} \quad \omega \in \mathbb{R}^l, \quad i^2 = -1 \]

and \( \lim_{\omega \to \pm \infty} \omega^2 \text{Re} W(i\omega) > 0 \). \hfill (2)

Definition 1 System (1) is called minimum phase with respect to the output \( \sigma = Gz \), if the polynomial

\[ \varphi_0(s) = \det \begin{bmatrix} sI_n - A & -B \\ GC & 0 \end{bmatrix} \]  

(3)

is Hurwitz; hyper minimum phase (HMP), if it is minimum phase and \( GCB > 0 \).

Theorem 2 (Passification Theorem, or Feedback Kalman–Yakubovich–Popov Lemma), \([22,26]\).

The following statements are equivalent:

(A1) There exist a positive definite \((n \times n)\)-matrix \( H \) and an \((l \times 1)\)-matrix \( K \) such that the relations

\[ H(A + BK^T C) + (A + BK^T C)^T H < 0, \quad HB = C^T G^T \]

hold.

(B1) The system (1) is hyper minimum phase with respect to the output \( \sigma = Gz \).

(C1) There exists a feedback

\[ u = K^T z + v \]  

(5)

rendering the closed-loop system (1), (5) strictly passive with respect to the output \( \sigma = Gz \). \hfill (B1)

Note, that if the condition (B1) is satisfied then the matrix \( K \) in (4) can be found in the form \( K = -\kappa G^T \) where \( \kappa \) is a sufficiently large positive real number. Extension of Theorem 2 to MIMO case can be found in \([22,26]\).

Remark 3 For MIMO case an additional requirement of symmetry \((GCB)^T = GCB \) is included in the HMP definition. It follows from the recent results by Barkana et al. \([12]\) that for \( l = m \) if the non-symmetric positive definite matrix \( W GCB \) is diagonizable, an unknown positive definite symmetric matrix \( R \) exists that makes the product \( R GCB \) positive definite symmetric. Therefore for \( l = m \) the original adaptive controllers can be used without additional \( G \) (or \( R \)).

Passification Theorem (Theorem 2) provides conditions for solvability of matrix inequalities related to feedback version of classical Kalman–Yakubovich–Popov (KYP) Lemma \([4,18,19]\). It provides also solvability conditions for the system passification problem by means of static output feedback. It has various applications in control design since the 1970s, e.g. design of adaptive controllers with Implicit Reference Model \([2,5]\).

2.2 Passification-based design of VSS and signal-parametric adaptive controllers

In this Section, an application of Passification Theorem to design of the variable-structure systems (VSS) \([40]\) and signal-parametric adaptive controllers \([1,2,38]\) is briefly described.

Consider the LTI plant (1) for \( m = 1 \) and the control objective \( \lim_{t \to \infty} x(t) = 0 \). Let the auxiliary objective be chosen as maintaining the sliding mode on the plane \( \sigma = 0 \), where \( \sigma = Gz \) is the auxiliary variable, \( G \) is the \((1 \times m)\)-matrix. Using speed-gradient method \([2,27]\) with the goal function \( \sigma^2 \), we arrive at the following control law:

\[ u(t) = -\gamma \text{sign} \sigma_1, \quad \sigma_1 = Gz(t) \]  

(6)

where \( \gamma > 0 \) is the gain parameter. As is shown in \([2,26]\), the goal \( x(t) \to 0 \) may be achieved in the system (1), (6) if there exist matrix \( H = H^T > 0 \) and vector \( A_* \) such that \( HA_* + A_*^T H < 0, HB = C^T G^T, A_* = A + BK_* C \). As is clear from Theorem 2, the mentioned condition is fulfilled if and only if the function \( W(s) \) is HMP, where \( W(s) = GC(sI_n - A)^{-1} B \) and the sign of the high frequency gain \( GCB \) is known. In that case for the sufficiently large \( \gamma \) the relation \( \lim_{t \to \infty} x(t) = 0 \) holds.

\(^1\) \( I_n \) denotes \( n \times n \) identity matrix.
To eliminate dependence of system stability on initial conditions and plant parameters, the following “signal-parametric”, or “combined”, adaptive control law may be used instead of (6) [1, 2]:

\[ u(t) = -K(t)^Tz(t) - \gamma \sigma_t, \quad \sigma_t = Gz(t) \]  
\[ \dot{K}(t) = \sigma_t \Gamma z(t), \]  
where \( \Gamma = \Gamma^T > 0, \gamma > 0 \) are design parameters.

It should be noticed that convergence of \( \sigma_t \) to zero in a finite time is essential for the VSS-like systems. It can be shown (see, e.g. [20, 26]) that this property is valid for any bounded region of initial conditions for the system (1), (7), (8). To ensure boundedness of the gain \( K(t) \) in practice, the parametric feedback may be added to the algorithm. Such a robustification of the adaptation algorithm (8) leads to the following adaptation law:

\[ \dot{K}(t) = \sigma_t \Gamma z(t) - \alpha (K(t) - K_0), \quad K(0) = K_0, \]  
where \( \alpha > 0 \) is the parametric feedback gain, \( K_0 \) is some initial “guessed” value of the gain matrix \( K \).

Application of the signal-parametric algorithm (7)–(9) to flight control design is demonstrated in [1, 24]. In the present brief we focus our attention to application of the control law (6).

2.3 Parallel feedforward compensator (shunt)

Note that the HMP condition is valid only for the case of plant relative degree \( k = n - m = 1 \). The design and analysis for general case \( k > 1 \) involve well known difficulties. Standard solutions based on explicit reference models [16, 17, 33] provide adaptive controllers of high order which are both difficult to implement and sensitive to noise. It was shown in [4, 5, 21] that the passification approach allows to design the simplified adaptive controller based on the so called “shunt” (parallel feedforward compensator, see also [9, 30, 32, 34]). Below the algorithm of [4, 21] is described containing few design parameters even for MIMO case. The solution is based on the following statement (see [21]):

\begin{theorem}
Assume the plant with transfer function \( GW(s) \) is minimum phase with scalar relative degree \( k > 1 \) for some \( l \times m \) matrix \( G \), the matrix \( -GCA^{-1}B \) being Hurwitz. Let \( P(s), Q(s) \) be Hurwitz polynomials of degrees \( k - 2, k - 1 \), correspondingly and all three polynomials \( P(s), Q(s), \varphi(s) = \delta(s) \) det \( GW(s) \) have the same signs of coefficients. Denote

\[ W_a(s) = GW(s) + \kappa \varepsilon (\varepsilon s + 1)^{k-2} / (s + \lambda)^{k-1} \]  
\[ W_c(s) = \frac{\kappa \varepsilon (\varepsilon s + 1)^{k-2}}{(s + \lambda)^{k-1}} \]  
where \( k \) is the relative degree of the plant transfer function \( k = n - m \).

Consider the LTI SISO plant \( (A_p, B_p, C_p) \) with the state vector \( x_p(t) \in \mathbb{R}^n \), the scalar control and output signals \( u(t), y_p(t) \). The plant transfer function is

\[ W_p(s) = C_p(sI_n - A_p)^{-1}B_p = \frac{B_p(s)}{A_p(s)} \]  
where \( s \in \mathbb{C} \) denotes the Laplace transform variable, \( A_p(s) = s^n + a_n s^{n-1} + \ldots + a_0, B_p(s) = b_0 s^m + b_1 s^{m-1} + \ldots + b_m \); \( k = n - m \) is the plant relative degree.

The following Theorems provide the desired property of the augmented plant (10) transfer function \( W_a(s) \) [6, 23]:

\begin{theorem}
Let \( W_p(s) \) (12) be minimum-phase with the relative degree \( r > 1 \) and \( b_0 > 0 \). Then there exist \( \kappa_0 > 0 \) and function \( \varepsilon_0(\kappa) > 0 \) such that the AP transfer function \( W(s) = W_p(s) + W_c(s) \) is HMP for all \( \kappa > \kappa_0 \) and \( 0 < \varepsilon < \varepsilon_0(\kappa_0) \).
\end{theorem}

\begin{theorem}
[6]. Let \( W_p(s) \) be stable \( (A_p(s) \) is a Hurwitz polynomial) with the relative degree \( k > 1 \) and \( b_0 > 0 \). Then for every \( \varepsilon > 0 \) there exists a (sufficiently large) \( \kappa_0 \) such that \( W(s) = W_p(s) + W_c(s) \) is HMP for all \( \kappa \geq \kappa_0 \).
\end{theorem}

Theorem 5 shows that one can introduce the shunt (11) with order \( \text{deg}(A_c(s)) = k - 1 = n - m - 1 \) for sufficiently large \( \kappa \) and small \( \varepsilon \) the augmented plant (10) satisfying the HMP condition for arbitrary given minimum-phase plant parameters domain. As follows from the Theorem 6, another way of shunt (11) parameters choosing provides the HMP condition for stable (and, possibly, nonminimum-phase) plants. For this case, the shunt equation can be simplified; namely \( W_c(s) = \frac{\kappa}{s + \lambda} \) may be taken instead of (11).

\begin{remark}
Theorems 4–6 provide less restrictive applicability conditions for VSS-like control design than the commonly used ones [40].
\end{remark}

\begin{remark}
The behavior of the augmented plant is close to that of the initial one for small \( \kappa \).
\end{remark}

3 Robust flight control design

In this Section, the yaw controller for flexible aircraft is designed-based on the approach presented in Sec. 2.
3.1 Airframe and onboard equipment modeling

In the sequel the following model of the lateral motion of the aircraft as a rigid body is used:

\[
\begin{align*}
\dot{\beta}(t) &= r(t) + a_2^\beta \beta(t) + a_3^\beta \delta_v(t), \\
\dot{r}(t) &= a_m^\beta \beta(t) + a_m^r r(t) + a_m^\delta \delta_v(t), \\
\dot{\psi}(t) &= r(t),
\end{align*}
\]  

where \(\psi(t), r(t)\) are the yaw angle and the yaw rate respectively, \(\beta(t)\) denotes the sideslip angle; \(\delta_v(t)\) is the rudder angle; \(a_i\) denote the aircraft model parameters. Values of \(a_i\) depend on the flight conditions (such as flight altitude, Much number, etc.) and are changing in the wide range during the flight.

The rudder actuators control loop modeled as the following second order LTI system

\[
W_{\text{servo}}(s) = \frac{\delta_v(s)}{\sigma_\psi(s)} = \frac{k_{\text{servo}}}{T_{\text{servo}}^2 s^2 + 2T_{\text{servo}} T_{\text{servo}} s + 1},
\]

where \(k_{\text{servo}}\) is an actuator steering servosystem gain (further on \(k_{\text{servo}} = 1\) is taken), \(T_{\text{servo}}\) stands for the servosystem response time factor, \(\sigma_\psi\) is the damping ratio, \(\delta_v\) denotes the commanded rudder deflection angle, generated by the autopilot, \(s \in \mathbb{C}\) stands for the Laplace transform variable.

The first mode of the airframe bending is taken into account and modeled as

\[
W_{\text{bend}}(s) = \frac{\Delta \psi(s)}{\delta_v(s)} = \frac{k_{\text{bend}}}{T_{\text{bend}}^2 s^2 + 2T_{\text{bend}} \xi_{\text{bend}} T_{\text{bend}} s + 1},
\]

where \(k_{\text{bend}}\) is the bending mode transition factor; \(T_{\text{bend}}\) is the response time factor, \(T_{\text{bend}} = \omega_{\text{bend}}^{-1}\), where \(\omega_{\text{bend}}\) is the 1st bending mode natural frequency; \(\xi_{\text{bend}}\) is the damping ratio (\(\xi_{\text{bend}} \approx 0\)).

The signal \(\psi_k\), measured by the gyros, is the sum of the yaw and bending angles:

\[
\psi_k(t) = \psi(t) + \Delta \psi(t).
\]

Equations (13)–(16) describe the seven-order plant model with the uncertain parameters \(a_i\). Note that the aircraft is the weathercock unstable if \(a_i^\psi < 0\).

3.2 Shunt transfer function

The plant (13)–(16) relative degree \(k = n - m = 4\), therefore the HMP condition does not valid for the considered system. Let us apply the shunting method of Sec. 2.3 and pick up the shunt transfer function (11) in the form

\[
W_k(s) = \frac{y_k(s)}{\sigma_\psi(s)} = \frac{\kappa}{s + \lambda},
\]  

where \(\kappa, \lambda\) are the shunt parameters, \(\lambda > 0\), \(\sigma_\psi = \text{sign} \ k_{\text{servo}}\). The extended plant output \(y(t)\) is the sum \(y(t) = \psi_k(t) + y_k(t)\), where \(y_k(t)\) is the shunt (17) output. The extended plant (13)–(17) transfer function \(W(s)\) of order \(n = 8\), the numerator \(B(s)\) has degree \(m = 7\), hence the relative degree \(k\) is equal to one, \(k = n - m = 1\). Therefore, to secure the HMP requirement for the extended plant model (13)–(17) only the Hurwitz condition for the numerator \(B(s)\) must be fulfilled.

3.3 HMP analysis

Let us find the HMP domain numerically for a typical area of the light jet aircraft model (13)–(16) parameters. The rudder servosystem and 1st bending mode parameters are taken as follows: \(T_{\text{servo}} = 0.05\ \text{s}, \ T_{\text{servo}} = 0.7, \ T_{\text{bend}} = 1.5 \cdot 10^4\ \text{s}, \ \omega_{\text{bend}} = 65\ \text{s}^{-1}, \ \xi_{\text{bend}} = 0.01\). The following shunt (17) parameters are taken: \(\lambda = 14\ \text{s}^{-1}, \ \kappa = -2\). The “tightened” HMP domain is found, which corresponds to the strengthened Hurwitz condition of polynomial \(B(s)\) with the stability margin \(\eta = 0.5\). This domain in the space of the principal aerodynamic parameters of the aircraft model (13) is shown in Fig. 1 (an interior of the domain bounded by the surfaces). It is seen that the HMP domain is wide and covers the cases of weathercock stable and weathercock unstable aircrafts as well.

3.4 Variable-structure robust autopilot design

The fulfillment of the HMP condition makes possible to use the VSS or the high-gain robust autopilot for the
aircraft attitude control (see above Sec. 2.2). Let us use the following VSS control law:

\[ \sigma = \bar{\sigma} \text{sat} \left( Ke(t) \right), \quad e(t) = y(t) - r(t), \tag{18} \]

where \( \bar{\sigma} \) denotes the maximal rudder deflection angle, \( \text{sat} \) stands for the saturation function, \( K \) is the high gain coefficient, \( y^*(t) \) is the reference signal, \( y(t) \) is the extended plant output. The control law (18) ensures the stable sliding motion with an accurate tracking of the reference input \( y^*(t) \) by the output \( y(t) \).

For ensuring the desired closed-loop system performance with respect to the command yaw angle \( \psi^*(t) \), the following second-order sequential reference model is employed:

\[ W_M(s) = \frac{r(s)}{\psi^*(s)} = \frac{\Omega_M^2}{s^2 + 1.4\Omega_M s + \Omega_M^2}. \tag{19} \]

The natural frequency \( \Omega_M \) gives the desired transient time of the closed-loop attitude control system.

### 3.5 Simulation results

Yaw angle transient responses for closed-loop system (13)–(19), obtained by the simulation for various aircraft model parameters are pictured in Fig. 3. Parameter values are given in Table 1.

<table>
<thead>
<tr>
<th>No</th>
<th>( a_{\alpha}, \text{s}^{-1} )</th>
<th>( a_{\beta}, \text{s}^{-1} )</th>
<th>( a_{\alpha m}, \text{s}^{-2} )</th>
<th>( a_{\beta m}, \text{s}^{-2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3</td>
<td>0.75</td>
<td>33</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
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<td>1.0</td>
<td>22</td>
<td>6.5</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
<td>0.6</td>
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</tr>
<tr>
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<td>1.2</td>
<td>-11</td>
<td>8.3</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>0.8</td>
<td>-15</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Note that the sets of parameters Nos 1–3 correspond the weathercock stable aircraft, whereas the sets Nos 4, 5 correspond the unstable aircraft. The plant (13)–(16) properties in the frequency domain for different flight conditions are demonstrated by Figs. 2, where the corresponding Bode diagrams are depicted.

The following autopilot (18), (19) parameters are taken: \( \bar{\sigma} = 20 \text{ deg}, \ K = 100, \ \Omega_M = 1 \text{ s}^{-1} \). The step signal of one degree in a magnitude is taken as a command yaw angle \( \psi^* \). It should be noticed that since the shunt parameter \( \kappa \) is small, the steady-state error is close to zero despite the fact that the extended plant output \( y(t) \) does not equal to the yaw angle \( \psi(t) \) due to adding the shunt output \( y_s(t) \).

One may compare the results obtained with those for employing the standard PD-control law with unchangeable parameters. The step responses for the same parameter values and a typical PD-controller

\[ \sigma(t) = \kappa \left( \psi^*(t) - \psi_g(t) \right) - \dot{k}_\psi \dot{\psi}(t) \tag{20} \]

are depicted in Fig. 4. The autopilot gains \( k_\psi = 2, \ k_\dot{\psi} = 1.5 \text{ s} \) has been found satisfying closed-loop system performance requirements for the regime No 4, but these parameters give unsatisfactory results for other regimes.

### 4 Conclusions

The Passification-based method applied for robust flight control system design. The shunting method is used ensuring the closed-loop system stability in the face of the lack of the aircraft state information. An example illustrating a typical design procedure for aircraft attitude control in the horizontal plane for wide range of the aircraft parameters is given, demonstrating efficiency and high robustness of the suggested control method.
Fig. 4. Standard PD-controller (20). Yaw angle and rudder deflection angle step responses for various aircraft model parameters.

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References


