Convergence Based Anti-windup Design Method and Its Application to Flight Control

Gennady A. Leonov*, Boris Andrievsky*†, Nikolay V. Kuznetsov*‡ and Alexander Yu. Pogromsky*§
*The Mathematics and Mechanics Faculty, Saint-Petersburg State University,
28 Universitetsky prospekt, 198504, Peterhof, Saint Petersburg, Russia
Tel: +7(812) 428-4210, Fax: +7(812) 428-6944, Email: leonov@math.spbu.ru, kuznetsov@math.spbu.ru
†The Institute for Problems of Mechanical Engineering of the Russian Academy of Sciences,
61 Bolshoy prospekt, V.O., 199178, Saint Petersburg, Russia,
Fax: +7(812) 321-4771, Tel: +7(812) 321-4766, Email: boris.andrievsky@gmail.com
‡University of Jyväskylä, PO Box 35, FI-40014, Finland
§Department of Mechanical Engineering, Eindhoven University of Technology,
Postbus 513 5600 MB Eindhoven, The Netherlands, Tel: +31 (040) 247-3464 Email: A.Pogromsky@tue.nl

***Abstract*—A convergence-based anti-windup control strategy is presented and demonstrated by the example of aircraft yaw control problem.**

***Index Terms*—integrator windup, flight control, anti-windup correction, convergent systems**

I. INTRODUCTION

The "*anti-windup*" (AW) control problem is a challenging one during the several last decades and a great number of works is devoted to it. Let us briefly recall the existing results in the field of AW methods for control of aircrafts. The windup problem arising from actuator rate and magnitude limits in the context of manual flight control for an open loop unstable aircraft was addressed in [1]. Input saturation may also cause a *Pilot Induced Oscillation* (PIO), which can be described as an inadvertent, sustained aircraft oscillation which is the consequence of an abnormal joint enterprise between the aircraft and the pilot [2]. PIOs are usually observed in the form of the "pitch flutter" i.e. the divergent oscillation on pitch attitude during aircraft landing. A terrifying illustration of this detrimental effect is given by the pilot-induced oscillations that entailed the *YF-22* crash in April 1992 and *Gripen* crash in August 1993 [3], [4]. To prevent such a degradation of the system performance, various AW techniques were proposed. State-of-the-art of the AW methods may be found in the monographs [5], [6] and the survey [7]. It is worth mentioning that the mathematical tools for analysis of various methods for aircraft flutter supression in 1940th has been developed by M. V. Keldysh [8]. In the present paper, Keldysh's methods are further extended and modified for the AW control problem.

The closed-loop quadratic stability and L_2 performance properties of linear control systems subject to input saturation is considered in [9]. These properties are examined within the context of the linear AW augmentation paradigm, which refers to designing a linear filter to augment a linear control system subject to a local specification, called the "unconstrained closed-loop behavior". It is shown that, if (and only if) the plant is asymptotically stable, plant-order linear antiwindup compensation is always feasible for large enough L_2 gain and that static antiwindup compensation is feasible provided a quasi-common Lyapunov function, between the open-loop and unconstrained closed-loop, exists.

In the paper [10], based on [11], a linear matrix ineqaulity (LMI) formulation of high-performance AW design for control systems with linear asymptotically stable plants is provided. A linear quadratic-based formulation of linear AW compensation, in terms of the solution of a set of (always feasible) LMI constraints is given.

The authors of [12] demonstrated the application of an AW technique for systems with rate-saturated actuators to a realistic flight control example. An approach to tuning the anti-windup compensator was devised, allowing a transparent trade-off between performance and the size of an estimate of the region of attraction. The AW algorithm was applied to a nonlinear simulation model of the longitudinal and lateral dynamics of an experimental aircraft showing the potential of AW to lessen an aircraft's susceptibility to PIOs. Design, flight testing and accompanying analysis of two AW compensators for an experimental aircraft – the German Aerospace Center's (DLR) advanced technologies testing aircraft (ATTAS) have been presented in [13]. The AW compensators are aimed to reduce the deleterious effects of rate-saturation of the aircraft's actuators on handling qualities. The further results were presented in [14], where a variety of low-order AW compensators were compared to determine the importance of different design parameters. The problem of static AW strategy for flight control system of linear unstable aircraft with saturated actuator was considered in [15]. The quadratic Lyapunov functions, *S*-procedure and a sector nonlinearity description were used. The AW design was investigated to increase both a domain of admissible references to track and a safety region over which the stability of the resulting closed-loop saturated system was ensured. The problem of multi-variable AW controller synthesis that incorporates trade-offs between unconstrained linear performance and constrained AW performance was studied by [16]. Results were applied to the multivariable model for the longitudinal dynamics of an *F8* aircraft. In [17] the AW problem was formulated in discrete time using a configuration which effectively decouples the nominal linear and nonlinear parts of a closed loop system with constrained plant inputs. The results were applied to control of a high-performance fighter aircraft model, presented in [18]. The AW design problem for linear control systems with strictly proper controllers in the presence of input magnitude and rate saturation is addressed in [19]. Using generalized sector condition, an LMI-based procedure for the construction of a linear AW gain acting was provided, ensuring regional closed-loop stability. The approach was illustrated by the example of *F8* aircraft longitudinal flight control.

It should be noticed that in the majority of papers, the AW control problem is understood as the problem of ensuring the global asymptotical stability of equilibrium in absence of the exogenous signals. Such an approach may lead to erroneous and impermissible for practice conclusions. As an example, the second order system with a saturation nonlinearity is considered in [20]. The system satisfies the Popov criterion and, therefore, its free motion is globally asymptotically stable. At the same time, if the system is subjected to the harmonic excitation with a relatively large amplitude, the forced oscillations stability may depend on the reference input and, also, different periodic solutions may co-exist for the same reference input. The similar results are provided in [21] for an aircraft control problem. This observation demonstrates necessity of the rigorous mathematical tools for designing the AW compensators.

The paper is organized as follows. Convergence-based AW control strategy of [20], [22]–[24] is briefly described in Sec. II. An application example for aircraft yaw control is presented in Sec. III. Concluding remarks and the future works intensions are given in IV.

II. Convergent systems and anti-windup correction

Following [20], let us present some basic results on the convergence-based AW control strategy.

A. Uniformly Convergent Systems

In this section a basic definition and some properties of uniformly convergent systems are given that will be used in the remainder of this paper. For definitions and properties of quadratic or exponential convergency, the interested reader is referred to e.g. [23].

Consider the following class of systems,

$$\dot{x}(t) = f(x, w(t)) \tag{1}$$

with state $x \in \mathbb{R}^n$ and input $w \in \overline{\mathbb{PC}}_m$. Here, $\overline{\mathbb{PC}}_m$ is the class of bounded piecewise continuous inputs $w(t) : \mathbb{R} \to \mathbb{R}^m$. Furthermore, assume that $f(x,w)$ satisfies some regularity conditions to guarantee the existence of local solutions $x(t,t_0,x_0)$ of system (1) for any input $w \in \overline{\mathbb{PC}}_m$.

Definition 1. System (1) is said to be *uniformly convergent* for a class of inputs $\mathscr{W} \subset \overline{\mathbb{PC}}_m$ if for every input $w(t) \in \mathscr{W}$ there is a solution $\bar{x}(t) = x(t,t_0,\bar{x}_0)$ satisfying the following conditions:

1) $\bar{x}(t)$ is defined and bounded for all $t \in (-\infty, +\infty)$,
2) $\bar{x}(t)$ is globally uniformly asymptotically stable for every input $w(t) \in \mathscr{W}$.

Note that *uniformly* in the above definition, refers to uniformity with respect to time, i.e. if a system is uniformly convergent for a class of inputs $\mathscr{W} \subset \overline{\mathbb{PC}}_m$, this implies that for each arbitrary input $w(t) \in \mathscr{W}$ there exists a unique solution $\bar{x}(t)$ which is globally uniformly asymptotically stable (uniformly with respect to time).

The solution $\bar{x}(t)$ is called a *limit solution*. As follows from the above definition, any solution of an uniformly convergent system "forgets" its initial condition and converges to a limit solution which is independent of the initial conditions.

An important advantage of convergent nonlinear systems over general nonlinear system is that for convergent systems performance can be evaluated in almost the same way as for linear systems. Whereas performance evaluation for general nonlinear systems can be difficult due to the possibility of multiple steady-state solutions, convergent systems have a unique limit solution and therefore performance can also be defined in a unique way.

Furthermore, due to the fact that the limit solution of a convergent system only depends on the input and is independent of the initial conditions, *simulation* can be used to determine the limit solution of the system. That is, evaluation of one solution (one arbitrary initial state) suffices, whereas for general nonlinear systems all (i.e. an infinite number of) initial conditions need to be evaluated to obtain a reliable analysis. This means that for convergent systems simulation is a reliable analysis tool.

B. Uniform Convergency for Marginally Stable Lur'e Systems with Saturation Nonlinearity

Consider a Lur'e system with saturation nonlinearity as given by the following equations

$$\begin{aligned} \dot{x} &= Ax + B\text{sat}(u) + Fw \\ u &= Cx + Dw \\ y &= Hx \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, $w \in \mathbb{R}^m$ is the external input (e.g. reference, disturbance), $y \in \mathbb{R}^p$ is the output, and the saturation function is defined as $\text{sat}(u) = \text{sign}(u)\min(1,|u|)$. Matrix A is marginally stable, i.e. there exists a $P = P^{\mathrm{T}} > 0$ such that $PA + A^{\mathrm{T}}P \le 0$.

Definition 2. A continuous function $t \mapsto w(t)$, $w(t) \in \mathbb{R}^m$ is said to belong to the class $\mathscr{W}$ if $w(t)$ is bounded and if it satisfies the following conditions

1. $\forall t \in \mathbb{R}$, $Dw(t)$ is uniformly continuous,
2. $\forall t \in \mathbb{R}$, $|F_1 w(t)| \le \alpha_1 |B_1|$ for some constant $\alpha_1 < 1$.

The following theorem is valid [25].

Theorem 1. If there exists a Lyapunov matrix $P = P^{\mathrm{T}} > 0$ such that

$$PA + A^{\mathrm{T}}P \le 0 \tag{3}$$

and

$$P(A+BC)+(A+BC)^{\mathrm{T}}P<0, \tag{4}$$

then for all $w \in \mathscr{W}$ system (2) is uniformly convergent.

Remark 1. Note that if there exists a Lyapunov matrix $P=P^{\mathrm{T}}>0$ such that $PA+A^{\mathrm{T}}P<0$ (instead of condition (3)) and $P(A+BC)+(A+BC)^{\mathrm{T}}P<0$ hold, then the corresponding system can be proven to be quadratically convergent. However, the system we consider is marginally stable thus $PA+A^{\mathrm{T}}P<0$ can not be satisfied.

Remark 2. As follows from the Kalman-Yakubovich-Popov lemma [26], [27], there exists a positive definite P that satisfies conditions (3), (4) if the following frequency domain inequality

$$\operatorname{Re} W(i\omega)<1 \tag{5}$$

holds for all $\omega \neq 0$ with $W(s)=C(sI-A)^{-1}B$.

C. Uniform convergency of anti-windup systems with a marginally stable plant

Consider the system with plant dynamics

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p\left(\operatorname{sat}(u)+w_1\right) \\ y_p &= C_p x_p \end{aligned} \tag{6}$$

where A_p is marginally stable. The controller dynamics are given by

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c(w_2-y_p) + k_A\left(\operatorname{sat}(u)-u\right) \\ u &= C_c x_c + D_c(w_2-y_p) \end{aligned} \tag{7}$$

in which k_A is a *static* AW gain.

For the given system the closed-loop dynamics can be written in Lur'e form (2) with $x=[\ x_p,\ x_c\ ]^{\mathrm{T}} \in \mathbb{R}^n$, $w=[\ w_1,\ w_2\ ]^{\mathrm{T}} \in \mathbb{R}^m$, and

$$A=\begin{bmatrix} A_p & 0 \\ k_A D_c C_p - B_c C_p & A_c - k_A C_c \end{bmatrix},$$

$$B=\begin{bmatrix} B_p \\ k_A \end{bmatrix}, \quad F=\begin{bmatrix} B_p & 0 \\ 0 & B_c - k_A D_c \end{bmatrix},$$

$$C=\begin{bmatrix} -D_c C_p & C_c \end{bmatrix}, \quad D=\begin{bmatrix} 0 & D_c \end{bmatrix}, \quad H=\begin{bmatrix} C_p & 0 \end{bmatrix}.$$

III. Application to the aircraft yaw control problem

Let us apply Theorem 1 for designing the aircarft yaw control system.

A. Aircraft yaw control with static anti-windup correction

Let us consider the aircraft yaw $\psi(t)$ tracking problem with respect to the desired reference signal $\psi^*(t)$.Let the astatism of order one should be ensured. Recall that this means an absence of the steady-state error with respect to linear on time component of the reference signal $\psi^*(t)$. For doing this the following PID-control law may be employed:

$$u_r(t)=k_D\omega_y(t)-k_P\Delta\psi(t)-k_I\int\limits_0^t \Delta\psi(t)\,dt, \tag{8}$$

where $u(t)$ is the control signal, applied to the rudder servo system, $\Delta\psi=\psi^*(t)-\psi(t)$ is the tracking error, $\omega_y(t)$, $\psi(t)$ stand for the yaw angular rate and the yaw angle (respectively). The design parameters k_D, k_P k_I are the *differential*, *propotional* and *integral* gains of the autopilot. The control signal $u_r(t)$, appled to the rudder servo system is restricted due to limitations on the sideslip angle, lateral acceleration and the rudder mechanical properties by the value of $\bar{u}_r$, i.e. the following condition should be accomplished:

$$-\bar{u}_r \le u_r(t) \le \bar{u}_r. \tag{9}$$

To fulfill condition (9) let us introduce the saturation nonlinearity in the rudder deflection δ_r command signal σ_r as follows:

$$\sigma_r=\operatorname{sat}_{\bar{u}}(u), \tag{10}$$

where $\operatorname{sat}_{\bar{u}}(u)=\{\bar{u}\operatorname{sign}(u) \text{ as } |u|>\bar{u} \mid u \text{ as } |u|\le\bar{u}\}$.[1] Equations (8), (10) describe PID-controller with saturation of the rudder servo input signal (to simplify the exposition, the servo static gain is assumed to be equal one without loss of generality).

B. Modeling the aircraft yaw attitude motion

Let us use for an example the linearized motion model of the hypotetical aircraft, presented in [28]. Following [21] let us assume in the sequel that bank angle γ is stabilized by means of the fast autopilot channel (i.e. assume that the aircraft can make a turn in the horizontal plane in a "skid-to-turn" fashion with a certain sideslip angle without banking). Therefore let us employ an isolated yaw motion model, assuming that $\gamma(t)\equiv 0$ in the model of [28].

Neglecting external disturbances let us write down the following model of the aircraft dynamics:

$$\begin{cases} \dot{\beta}(t)=a_z^{\beta}\beta(t)+\cos\alpha^*\cdot\omega_y(t)+a_z^{\delta_r}\delta_r(t), \\ \dot{\omega}_y(t)=a_{m_y}^{\beta}\beta(t)+a_{m_y}^{\omega_y}\omega_y(t)+a_{m_y}^{\delta_r}\delta_r(t), \\ \dot{\psi}(t)=\omega_y(t), \end{cases} \tag{11}$$

where $\beta(t)$ denotes the aircraft sideslip angle; $\psi(t)$, $\omega_y(t)$ are yaw angle and yaw angular rate (respectively); α^* is the trimming angle of attack, which corresponds to the chosen flight path; a_z^{β}, $a_z^{\delta_r}$, $a_{m_y}^{\beta}$, $a_{m_y}^{\omega_y}$, $a_{m_y}^{\delta_r}$ are the linearized model parameters. In the sequel the following parameter values (in SI units of measure) are used [28]: $a_z^{\beta}=-0.152$, $a_z^{\delta_r}=-0.032$, $a_{m_y}^{\beta}=-1.757$, $a_{m_y}^{\omega_y}=-0.136$, $a_{m_y}^{\delta_r}=-1.46$, $\alpha=0.4363$ rad (25^o), which leads to $\cos\alpha^*=0.906$.

With a satisfactory accuracy, the rudder servo drive may be modeled by the following differential equations:

$$\begin{cases} \dot{\delta}_r(t)=\omega_r(t), \\ \dot{\omega}_r(t)=k_\sigma\left(\sigma_r(t)-\delta_r\right)-k_\omega\omega_r(t), \end{cases} \tag{12}$$

[1] Strictly speaking, fulfillment of (10) does not ensure validity of (9) at all time instants for arbitrary control signal due to possible overshoot of the rudder servo drive. This overshoot, however, is usually small and may be neglected in the context of the considered problem. If necessary, the value of $\bar{u}_r$ may be taken with a certain margin.

where $\delta_r(t)$, $\omega_r(t)$ are the rudder displacement angle and its rate (respectively); k_σ, k_ω are the servo drive gains (the design parameters). In sequel the following parameter values are used: $k_\sigma = 67.2\ \mathrm{s}^{-2}$, $k_\omega = 11.5\ \mathrm{s}^{-1}$. This leads to the following servo drive transfer function $W_{\delta_r}^{\sigma_r}(s) = \dfrac{1}{T_r^2 s^2 + 2\xi_r T_r s + 1}$, where the time constant $T_r = k_\sigma^{-\frac{1}{2}} = 0.122$ s, the damping ratio $\xi_r = 0.5 k_\sigma^{-\frac{1}{2}} k_\omega = 0.7$; $s \in \mathbb{C}$ denotes the Laplace transform variable.

Therefore, the "extended plant" (11), (12) transfer function from control signal σ_r to yaw angle ψ has a following form:

$$W_\psi^{\sigma_r}(s) = \frac{\psi(s)}{\sigma_r(s)} = \frac{-98.2(s+0.113)}{s(s^2+0.288s+1.61)(s^2+11.5s+67.2)}. \tag{13}$$

C. Analysis of the aircraft yaw control system

1) Synthesis of the nominal control law: Firstly, let us consider the "nominal" (unsaturated) mode when $\sigma_r^*(t) \equiv u(t)$ and the closed-loop aircraft control system is modeled by linear equations (8), (11) (it should be mentioned that in the nominal mode the AW component of the control signal $u_r(t) - \sigma_r(t)$ in (15) is equal to zero, therefore, as follows from (8), (15), the output signals are equal). Employing the frequency stability criterion for linear system (8), (11) and Nelder–Mead unconstrained nonlinear minimization procedure of package MATLAB [29] one obtains the following set of the gains of PID-controller (8): $k_I = 0.46\ \mathrm{s}^{-1}$, $k_P = 0.37$, $k_D = 1.8$ s. For these gains the open-loop transfer function from servo-drive input $\sigma_r(t)$ to PID-controller (8) output $u(t)$ is as follows:

$$W_u^{\sigma_r}(s) = \frac{-173(s+0.113)(s^2+0.208s+0.261)}{s^2(s^2+0.288s+1.61)(s^2+11.5s+67.2)} \tag{14}$$

which leads to the magnitude stability margin $G_m = 12.8$ dB, phase margin $\varphi_m = 60^o$ and H_∞-gain of the closed-loop system (index of oscillation) $H_\infty = 1.26$. Nyquist chart of $-W_u^{\sigma_r}(i\omega)$ and the step response in the closed-loop system are plotted in Figs. 1, 2.[2]

2) Effect of the saturation: Let us now take into account a saturation nonlinearity in the closed-loop contour. Since the plant is neutrally stable, then the open-loop system, including the PID-controller (8), is unstable due to presence zero poles of order two in (14). Therefore, the frequency inequality (5) can not be fulfilled and system (8), (10), (11) convergence can not be proved based on Theorem II-B. Inequality (5) violation for the examined system is clearly seen from the Nyquist chart of $-W(i\omega)$, plotted in Fig. (1).

As is mentioned above, an input saturation jointly with an integral part of the control signal may lead to degradation of the system performance up to the stability loss. This effect may not be visible in the course of examination of the system free motion, but appears for the particular kinds of the external signal. Let us examine this property for the considered flight control system in more detail.

[2] The frequency response $W_u^{\sigma_r}(i\omega)$ in Fig. 1 is plotted with a reversed sign since for aircrafts of normal aerodynamical scheme the high frequency gain of transfer function (14) is negative.

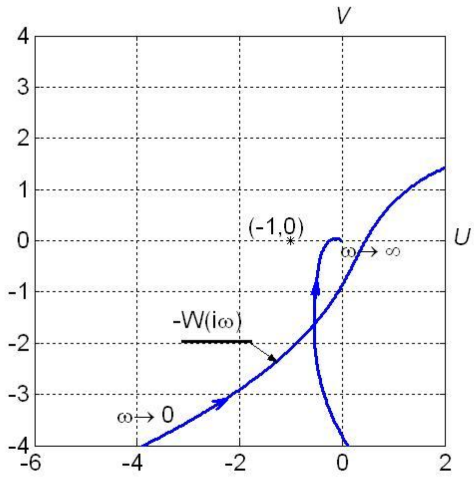

Fig. 1. Nyquist chart for the open-loop aircraft control system. Control law (8).

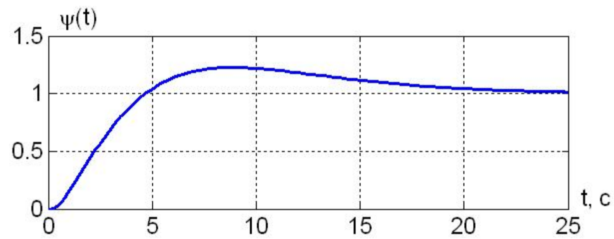

Fig. 2. Step response in the "nominal" closed-loop aircraft control system. Control law (8), $\sigma_r(t) \equiv u_r(t)$.

The step responses in the closed-loop system for different levels of the reference signal $\psi^* = 5^o$ and $\psi^* = 12^o$ are depicted in Fig. 3. As is seen from the plots, no input saturation occurs as $\psi^* = 5^o$ and the step response looks similarly to that of the "nominal" (linear) system demonstarted in Fig. 2. In the case of $\psi^* = 12^o$, due to the saturation effect, the system performance degradates significantly. Despite the equilibrium is still asymptotically stable, the overshoot increases up to 60 %, and the transient time is twice as much in compare with the linear mode. It is also seen from this plot, that the magnitude of the PID-controller (8) output signal $u_r(t)$ exceeds in the several times the servo drive saturation level $\bar{u}$.

Let us consider now the system response to the harmonic input signal of the form $\psi^*(t) = \psi^*_{\max}\sin(\Omega t)$, where $\Omega = 0.025\ \mathrm{s}^{-1}$ is taken. Let us increase the amplitude $\psi^*_{\max}$ from zero to $\psi^*_{max} = 26^o$ and back so slowly that the transient behavior in the system is negligible. The simulation results are shown in Fig. 4 as dependences of the control signal u_r and the reference error $\Delta\psi(t)$ magnitudes on $\psi^*_{\max}$. As is seen from the plots, as ψ^*_{max} increases, the amplitudes u_{max} and $\Delta\psi_{max}$ increases gradually until a jump occurs. Then, when the magnitude $\psi^*_{\max}$ decreases, the magnitudes of u_r and $\Delta\psi(t)$ decrease monotonically until a second jump takes place. Such a hysteresis effect is evident from Fig. 4.

As reported in [30], the similar result was obtained by

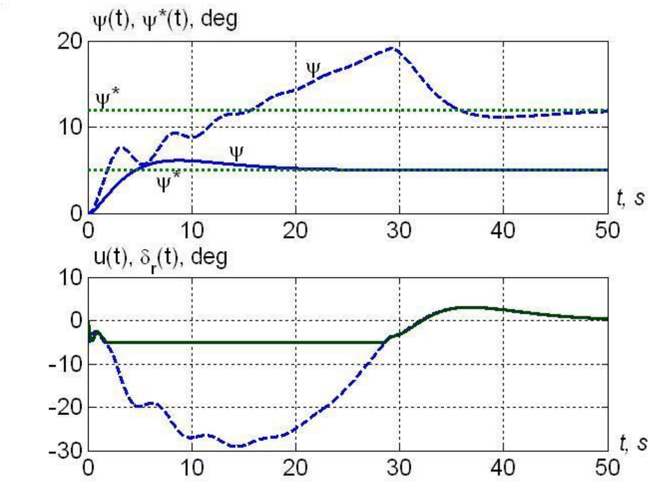

Fig. 3. Step responses in the aircraft control system in the case of reference signal $\psi^* = 5^o$ (solid line) and $\psi^* = 12^o$ (dashed line); the time histories of PID-controller output signal $u_r(t)$ (solid line) and the rudder deflection angnle $\delta_r(t)$ (dashed line) in the case of reference signal $\psi^* = 12^o$.

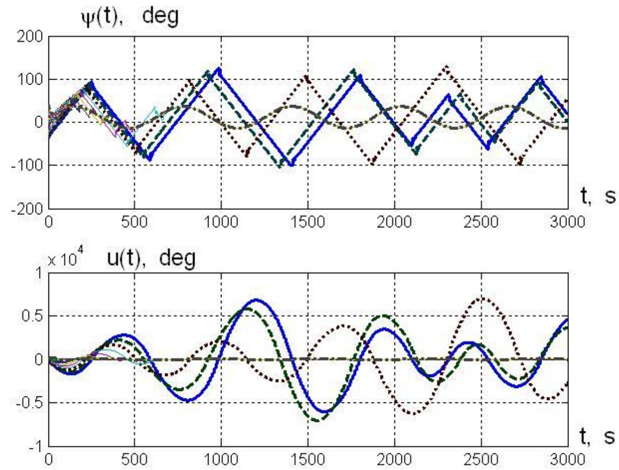

Fig. 5. A set of the time histories $\psi(t)$, $u_r(t)$ in the case of control law (8), (10) for reference signal $\psi^*(t) = \psi_0^* + \psi_{\max}^* \sin(\Omega t)$ and various initial conditions $\psi(0)$.

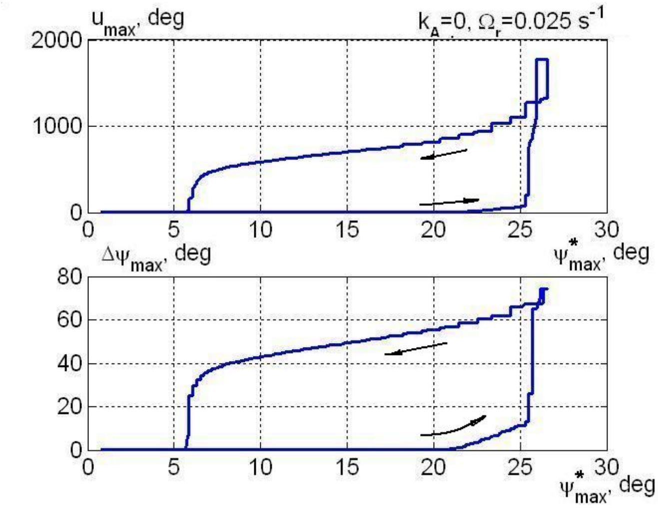

Fig. 4. Plots of the control signal magnitude u_r and the reference error $\Delta\psi(t)$ versus the reference signal magnitude in the case of control law (8), (10).

means of both computer simulation and a real saturated PI-controlled brushless DC motor. These results provides an evidence that stability of the forced oscillations due to windup phenomenon may depend on the reference input: the same input signal may lead to different steady-state processes for various initial conditions. This effect is demonstrated by the plots in Fig. 5, where the time histories of $\psi(t)$, $u(t)$, obtained in the case of $\psi^*(t) = \psi_0^* + \psi_{\max}^* \sin(\Omega t)$, $\psi_0^* = 11^o$, $\psi_{max}^* = 25^o$, $\Omega = 0.01$ s^{-1} and various initial conditions $\psi(0) \in [-40, 40]^o$ are depicted (other initial conditions are zero). As is seen from the plots, that for the same reference signal the different steady-state motions may appear. Moreover, while the majority of the processes converge to the steady-state motion of a small magnitude of the reference error (about part of degree) and no saturation occurs, but there exist certain realizations with the "tracking loss" and unallowable magnitudes (up to hundred degrees). The possibility of such processes is difficult to establish even by means of intensive computer simulation, but they can occur during the system operation.

3) Flight control system with AW-correction: Let us consider now the following control law with AW-correction

$$u_r(t) = k_D \omega_y(t) - k_P \Delta\psi(t) - k_I \int_0^t \big(\Delta\psi(\tau) + k_A\big(u(\tau) - \sigma_r(\tau)\big)\big)\, d\tau, \qquad (15)$$

where k_A denotes the AW-correction gain (the design parameter), the rest variables are the same as in (8), (10).

Let us employ now the control law (10), (15) with an AW-correction instead of (8). As is seen from Fig. 6, inequality (5) is fulfilled as $0.24 \le k_A \le 5.0$. Therefore, as follows from Theorem II-B, system (15), (10), (11) is convergent as $k_A \in [0.24, 5.0]$. The transfer function of the linear part of the system, taking into account the AW-correction with $k_A = 2$, is as follows:

$$W_u^{\sigma_r}(s) = \frac{-111(s+0.90)(s-0.44)(s-0.11)}{s(s+0.92)(s^2+0.29s+1.6)(s^2+11.5s+67.2)}.$$

The corresponding Nyiquist plot is depicted in Fig. 7.

Let us simulate the flight control system with AW-correction for $k_A = 2$, taking the same external input signals and the initial conditions, which have been used above for simulation of the control system without AW-correction. The simulation results are plotted in Figs. 8–10.

The step responses in the flight control system for the case of the constant refrence signal $\psi^* = 12^o$, the time histories of the PID-controller output signal $u_r(t)$ and the rudder deflection angle $\delta_r(t)$ are depicted in Fig. 8. Comparing the results obtained with those plotted in Fig. 3 for the case of $\psi^* = 12^o$, one can see that the system performance is significantly

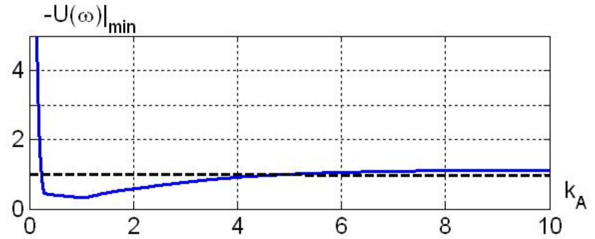

Fig. 6. Plot of $\min_{\omega\in\mathbb{R}}(-U(\omega))$ for the linear part of system (10) – (12), (15) versus k_A gain.

improved. Of course, the saturation of the control signal leads to some reducing the response speed comparing with the "nominal" (linear) control system (cf. Fig. 2), however this degradation of the performance is not as a dramatic one as for the case of the system without the AW-correction.

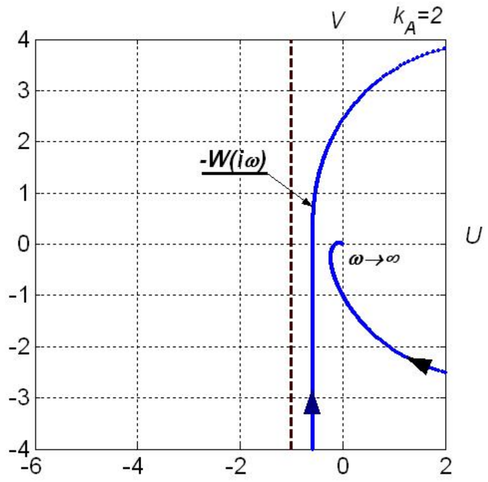

Fig. 7. Nyquist chart for the aircraft control system with AW-correction. Control law (10), (15), $k_A = 2$.

Response of the aircraft control system to the harmonic external excitation of the form $\psi^*(t) = \psi^*_{\max}\sin(\Omega t)$, where the magnitude ψ^*_{max} is varying in the interval $0 \div 30^o$ as the frequency $\Omega = 0.025$ s^{-1} is depicted in Fig. 9 in the form of dependence of the control signal u_r and the reference error $\Delta\psi(t)$ magnitudes on the magnitude of the reference signal. As opposed to the flight control system without the AW correction (cf. Fig. 4), a hysteresis effect is absent in this case and, besides, the magnitudes of the control signal and the yaw error meet the working requirements.

The time histories of $\psi(t)$, $u(t)$ for the case of $\psi^*(t) = \psi^*_0 + \psi^*_{\max}\sin(\Omega t)$, $\psi^*_0 = 11^o$, $\psi^*_{max} = 25^o$, $\Omega = 0.01$ s^{-1} and various initial conditions $\psi(0)$ from the interval $\psi(0) \in [-40, 40]^o$ are plotted in Fig. 10. It is seen from the plots that the flight control system outputs in the steady-state mode concur with each other for different initial conditions and a given reference signal.

Since the aircraft control system with the AW-correction (10), (15) is convergent as $k_A = 2$, it makes possible to

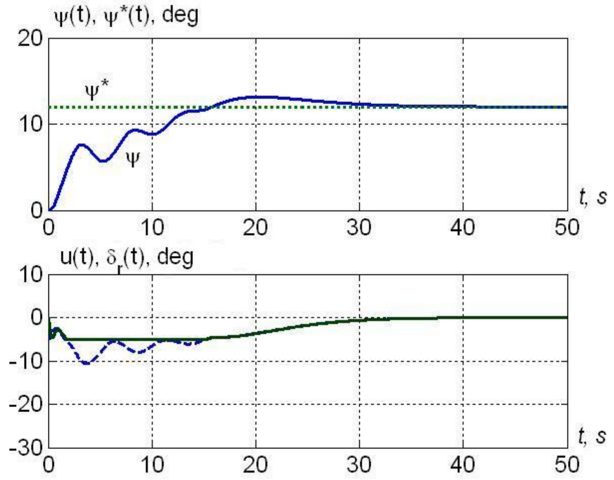

Fig. 8. Step responses in the aircraft control system in the case of reference signal $\psi^* = 12^o$; the time histories of PID-controller output signal $u_r(t)$ (solid line) and the rudder deflection angnle $\delta_r(t)$ (dashed line). Control law (10), (15), $k_A = 2$.

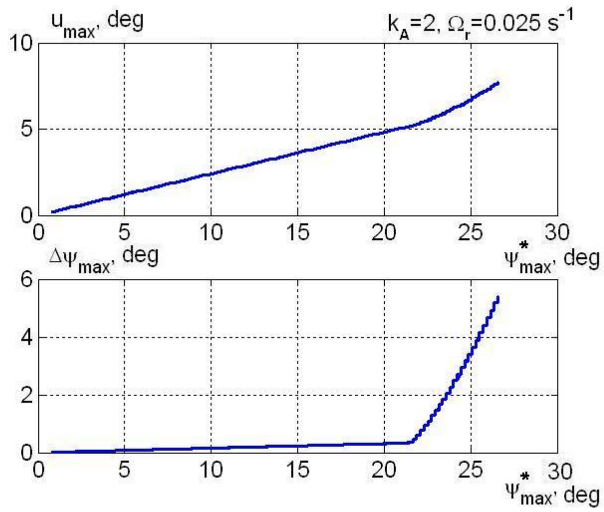

Fig. 9. Plots of the control signal magnitude u_r and the reference error $\Delta\psi(t)$ versus the reference signal magnitude. Control law (10), (15), $k_A = 2$.

calculate the sensitivity function $\mathbb{S}(\psi^*_{\max}, \omega)$ depending on the reference signal frequency and magnitude as a measure of the tracking accuracy of nonlinear system in the case of the harmonic excitation (see [31], [32] for more detail). For the considered numerical example, function $\mathbb{S}(\psi^*_{\max}, \omega)$ is plotted in Fig. 11.

IV. Conclusions

The paper is devoted to the nonlinear control problem caused by limitations of the input signal. The windup phenomenon is described and the brief exposition of the methods of anti-windup correction is given. In more detail the convergence based approach to anti-windup correction is described. The application example for control of the aircraft attitude is presented, demonstrating efficiency of the proposed method.

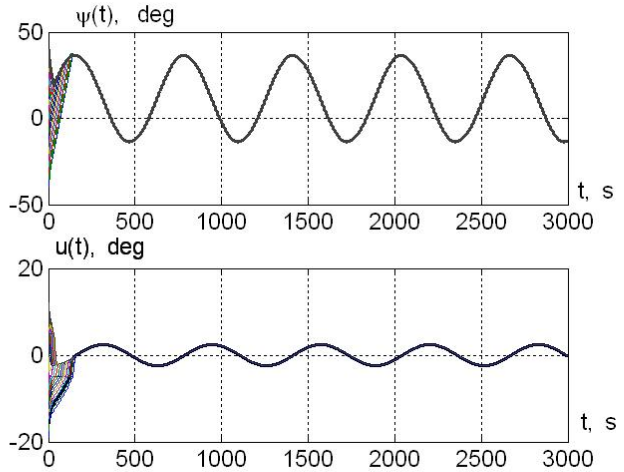

Fig. 10. Time histories $\psi(t)$, $u_r(t)$. Control law (10), (15) ($k_A = 2$) in the case of $\psi^*(t) = \psi_0^* + \psi_{\max}^* \sin(\Omega t)$ and varios initial conditions $\psi(0)$.

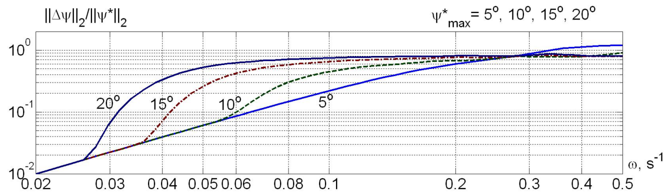

Fig. 11. Sensitivity function $\mathbb{S}(\psi_{\max}^*, \omega) = \|\Delta\psi\|_2 / \|\psi^*\|_2$. Control law (10), (15) ($k_A = 2$) as $\psi_{\max}^* = 5^o \div 25^o$.

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