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Observer-based Production control of Manufacturing Machines ⋆

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Abstract: The paper deals with the problem of controlling manufacturing machine such that an unknown customer demand is tracked with a desired accuracy. To study this problem, a manufacturing machine is approximated by an integrator which is subject to input saturation as a result of the finite capacity of the machine. To solve the problem in case of unknown demand rate, a combination of feedforward-feedback controller with a reduced-order observer is proposed. A steady-state performance of the system with periodic demand fluctuations is studied.

Keywords: Manufacturing systems; Industry automation; Production control; Convergence analysis

1. INTRODUCTION

The production control of manufacturing systems, i.e. how to control the production rates of machines such that the system tracks a certain customer demand while keeping a low inventory level, has been a field of interest for several decades. Early control strategies based on simple push and pull concepts, such as material requirements planning (MRP), enterprise resources planning (ERP), and just-in-time (JIT), see e.g. (Hopp and Spearman, 2000), can provide an adequate solution if the system requirements are not very strict and a fast reaction to possible disturbances/failures is not required (e.g. since such disturbances/failures hardly occur). However, as manufacturing systems become more complex and the system's performance must constantly improve in order to stay competitive in today's global economy, these control strategies become less effective.

A possible way to tackle the problem is to describe the manufacturing systems using so-called flow models, see e.g. (Alvarez-Vargas et al., 1994). These models, which are based on ordinary differential/difference equations (ODEs), or sometimes partial differential equations (PDEs, see e.g. (van den Berg et al., 2008; Lefeber et al., 2005)), form a continuous approximation of the discrete-event manufacturing systems and therefore result in a simpler control problem. Moreover, various (advanced) control theories are already available for ODEs, which makes these models attractive to work with. Most control strategies proposed in literature that use flow models to describe the manufacturing system, are based on the assumption that (an estimate of nominal) the future demand is known, and use some optimization algorithm to find a suitable control signal, see e.g. (Gershwin, 1989; Sharifnia, 1994; Vargas-Villamil et al., 2003; Savkin, 1998; Bauso et al., 2006) and references therein.

In the ODE models, a manufacturing machine is usually interpreted as an integrator, where the cumulative number of finished products is the integral of the production rate. Bounds on the production rate, due to the finite capacity of the machine, are then taken into account in the optimization problem. Disadvantages of these control strategies are that they depend on future demands (which are hard to predict and therefore often inaccurate) and that in general the optimization problem requires much computational effort.

In this paper, a different strategy is employed for the control of manufacturing machines, which does not depend on future demands and requires less computational effort. For this control strategy, the manufacturing machines are still approximated by an integrator, but the bounds on the production rate are interpreted as a saturation function. This approach is based on the previous results (van den Bremer et al., 2008) where a simple PI-controller is proposed to solve the problem. Due to saturation in the control loop, an anti-windup compensator is required in (van den Bremer et al., 2008) to avoid undesirable oscillations in the presence of disturbances. In this paper we implement a simpler approach: under assumptions that the nominal demand rate and is constant (as well as possible disturbances) we design an observer to estimate this rate to utilize in the control algorithm. In case of uncertainty in the demand, caused, for example, by seasonal fluctuations in the market, we study if the closed loop system affected by this fluctuations is convergent. This idea results a simple control algorithm that can be also used in more complex manufacturing lines where additional constraints on buffer content are also imposed.

To analyze performance of the control algorithm we utilize a standard engineering approach based on describing functions method, to make this approach rigorous we estimate the accuracy of this method following results of (van den Berg et al., 2007).

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The paper is organized as follows. Combined feedforward–feedback control law for a manufacturing is described in Section 2. The observer-based feedback controller, implementing the feedforward control strategy without measurements of the demand production rate is presented in Section 3. Section 4 is devoted to the frequency domain analysis of the manufacturing system. The numerical example results are presented in Section 5.

2. COMBINED FEEDFORWARD–FEEDBACK CONTROL OF A MANUFACTURING MACHINE

2.1 Problem statement

Following (van den Berg et al., 2006; van den Berg, 2008), let us use a continuous approximation of the single discrete-event manufacturing machine. Namely, consider a manufacturing machine that produces items with a *production rate* $u_p(t) \in \mathbb{R}$, $t \in \mathbb{R}$. Assume that there is always sufficient raw material to feed the machine. The total amount of items produced by the machine is denoted by $y(t) \in \mathbb{R}$ and is related to production rate $u_p(t)$ by the following equation

$$\dot{y}(t) = u_p(t) + f(t), \tag{1}$$

where $f(t) \in \mathbb{R}$ stands for an unknown *external disturbance*. This term may describe manufacturing losses, or variations of the machine capacity, for example. The production rate u_p can not be negative and has a certain upper bound $u_{p,\max}$, caused by the machine capacity limitation. Therefore the following bounds are valid for the production rate:

$$0 \le u_p \le u_{p,\max}. \tag{2}$$

Inequalities (2) introduce a *saturation* in the control loop. The saturation effect complicates design of the control law and the system performance analysis.

The control aim is tracking the non-decreasing *reference production signal* $y_d(t)$. In what follows we assume that $y_d(t)$ may be modeled as

$$y_d(t) = y_{d,0} + v_d t + \varphi(t), \tag{3}$$

where $y_{d,0}$ denotes the desired production at $t = 0$, v_d is a constant that represents the average desired production rate, $\varphi(t)$ is a bounded function, describing fluctuation of the desired production from the linearly increasing time-varying demand, caused by market (e.g. seasonal) fluctuation. It is natural to suppose that $0 \le v_d \le u_{p,\max}$. It may be also assumed that $\varphi(t)$ has a "zero mean" in a some sense because its averaged value may be considered as a part of $y_{d,0}$.

The PI-controller with an anti-windup control strategy is proposed and thoughtfully studied in (van den Berg et al., 2006; van den Berg, 2008). This controller ensures asymptotically vanishing *tracking error* $e(t) = y_d(t) - y(t)$ for constant $\varphi(t)$, $f(t)$ and independence of the asymptotic system behavior of the initial conditions if fluctuations and disturbances are present (the so called "*convergence property*", see (Pavlov et al., 2004; van den Berg et al., 2006; Pavlov et al., 2005b) for details).

In this paper we propose an alternative control law, utilizing a combined feedforward–feedback control strategy. Since the integral of the tracking error is not used in the proposed controller, the anti-windup compensator in the controller is no longer needed.

2.2 Combined control law

At the beginning assume that the variables $y(t)$, $f(t)$ and the value of v_d may be measured and used to form the control action $u_p(t)$. Let us take the control law in the following feedforward–feedback form:

$$u_p(t) = \mathrm{sat}_{[0,u_{p,\max}]}\big(k_p e(t) + v_d - f(t)\big), \tag{4}$$

where $e(t) = y_d(t) - y(t)$ denotes the tracking error, k_p is the controller parameter (a *proportional gain*), $\mathrm{sat}(\cdot)$ denotes the *saturation function*

$$\mathrm{sat}_{[a,b]}(z) = \min\big(b, \max(a,z)\big). \tag{5}$$

Equations (1), (4) describe the closed-loop manufacturing system model for time-varying demand $y_d(t)$ given by (3).

3. OBSERVER-BASED FEEDBACK CONTROLLER

In the above Sections it was assumed that the average rate v_d and the disturbance $f(t)$ are known (measured) signals. This assumption is rather unpractical. From now on assume that only the error signal $e(t)$ can be measured and used to form the control action. Let us replace the signals v_d, f in the control law (4) by their estimates $\hat{v}_d(t)$, $\hat{f}(t)$ provided by the *observer* (state estimator) which uses only available signals $e(t) = y_d(t) - y(t)$ and $u_p(t)$.

Since the observer design is based on modeling the external signals as outputs of some dynamical system, let us assume at this stage that both v_d and f are constants and that the reference signal is strictly linear: $y_d(t) = y_{d,0} + v_d t$. Differentiating the error $e(t) = y_d(t) - y(t)$ wrt t we obtain from (1) the following error model

$$\dot{e}(t) = -u_p(t) - f(t) + v_d. \tag{6}$$

From (6) it is clear that the signals $f(t)$ and v_d can not be estimated separately based on the measurements of $e(t)$. It is possible to estimate the joint signal $r(t) = -f(t) + v_d$ only. Using the above notation and assumptions, we obtain from (6) the following *extended plant model*:

$$\begin{cases} \dot{e}(t) = -u_p(t) + r(t), \\ \dot{r}(t) = 0. \end{cases} \tag{7}$$

Luenberger's design method (Luenberger, 1971) leads to the following *reduced-order observer*

$$\begin{cases} \dot{\sigma}(t) = -\lambda\sigma(t) - \lambda^2 e(t) + \lambda u_p(t) \\ \hat{r}(t) = \sigma(t) + \lambda e(t), \end{cases} \tag{8}$$

where $\hat{r}(t)$ denotes the estimate of the signal $r(t)$, produced by observer (8), $\lambda > 0$ is the observer parameter (observer gain), setting the transient time for the estimation procedure.

Let us use the estimate $\hat{r}(t)$ in the control law (4) instead of $v_d(t) - f(t) = r(t)$. Then the control action $u_p(t)$ takes the form

$$u_p(t) = \mathrm{sat}_{[0,u_{p,\max}]}\big(k_p e(t) + \hat{r}(t)\big), \tag{9}$$

where $e(t) = y_d(t) - y(t)$, $\hat{r}(t)$ is governed by (8). Equations (8), (9) describe the first-order feedback controller. The control signal $u(t) = k_p e(t) + \hat{r}(t)$ is calculated based on the error $e(t)$ measurement only. The gains $k_p > 0$ and $\lambda > 0$ are the controller parameters.

The closed-loop system (1), (8), (9) performance differs from that of the system with the controller (4), described in Sec. 2.2 due to the estimation error $\varepsilon_r(t) = r(t) - \hat{r}(t)$. This error is caused by the difference in the initial conditions of the external and estimated signals, variations of the estimated variable $r(t)$ (due to fluctuation $\varphi(t)$ of $y_d(t)$ and inconstancy of $f(t)$), and the measurement errors. To find the estimation error let us write down the plant–observer model taking into account representation (3) for the reference signal $y_d(t)$. We get the following equations

$$\begin{cases} \dot{y}(t) = u_p(t) + f(t), \\ e(t) = y_{d,0} + v_d t + \varphi(t) - y(t), \\ \hat{r}(t) = \sigma(t) + \lambda e(t), \\ \dot{\sigma}(t) = -\lambda\sigma(t) - \lambda^2 e(t) + \lambda u_p(t). \end{cases} \tag{10}$$

where $u_p(t)$, $f(t)$, $y_{d,0}$, v_d, $\varphi(t)$ are external inputs. After the simple algebra we obtain form (10) the following equation for the estimation error $\varepsilon_r(t)$:

$$\varepsilon_r(t) = \mu(t) - \xi(t), \tag{11}$$
$$\dot{\mu}(t) + \lambda\mu(t) = \lambda\xi(t), \ \ \mu(0) = \mu_0,$$

where $\xi(t) = f(t) + \lambda\varphi(t)$. We see that the error $\varepsilon_r(t)$ is independent of $u_p(t)$, $y_{d,0}$, v_d in (10).

Taking into account the estimation error $\varepsilon_r(t)$, the control law (4) reads as

$$u_p(t) = \operatorname{sat}_{[0,u_{p,\max}]}\big(k_p e(t) + v_d - f(t) - \varepsilon_r(t)\big), \tag{12}$$

and the closed-loop system dynamics is described by (1), (11), (12).

If there exists a fluctuation of the desired production $\varphi(t) \neq$ const, convergence of the tracking error $e(t)$ to zero can not be attained. In such a case it is advisable, both for simplification of the analysis and for system industrial applicability, to assure convergence of the system trajectories to each other for different initial conditions. This property is represented by the notion of the *incremental stability* (Angeli, 2002). To verify incremental stability of the considered system let us rewrite (1), (4) in the transformed coordinates. Introduce a new variable z as $z(t) = y(t) - y_{d,0} - v_d t$. Differentiating $z(t)$ wrt t and taking in view (1) we obtain that $\dot{z}(t) = u_p(t) - v_d + f(t)$. Then the tracking error $e(t)$ reads as

$$e(t) = y_d(t) - y(t) = y_{d,0} + v_d t + \varphi(t) - y(t) = \varphi(t) - z(t).$$

Let us rewrite expression (4) for the control signal u_p using the standard (symmetric) saturation function $\operatorname{sat}_m(x) = \min\big(m, \max(-m,x)\big)$. Introducing the "unsaturated" control signal as $u(t) = k_p e(t) + v_d - f(t)$ we obtain that $u_p = \operatorname{sat}_{[0,u_{p,\max}]}(u) = \operatorname{sat}_m(k_p e + v_d - f - m) + m$. This leads to the following closed-loop system model in the transformed coordinates:

$$\dot{z}(t) = \bar{u}(t) + \eta(t), \tag{13}$$

$$\begin{cases} \bar{u}(t) = \operatorname{sat}_m\big(k_p e(t) + \hat{r}(t) - m\big), \\ e(t) = \varphi(t) - z(t), \end{cases} \tag{14}$$

$$\begin{cases} \hat{r}(t) = \sigma(t) + \lambda e(t), \\ \dot{\sigma}(t) = -\lambda\sigma(t) - \lambda^2 e(t) + \lambda(\bar{u}(t) + m), \end{cases} \tag{15}$$

where $\eta(t) = f(t) + m - v_d$, $m = u_{p,\max}/2$.

Recall that *convergent systems* being excited by a bounded input have a unique bounded globally asymptotically stable steady-state solution (Pavlov et al., 2004, 2005b). Therefore, for any given input the solutions of the convergent system independently of the initial conditions converge to the uniquely defined limit solution.

Let us check if this property is valid for system (13)–(15), forced by the reference signal $\varphi(t)$. Let us apply the general results presenting in (van den Berg, 2008). For doing that rewrite the closed-loop system model in the following state-space form:

$$\dot{x}(t) = Ax(t) + B\operatorname{sat}_m(u) + Fw(t),$$
$$u(t) = Cx(t) + Dw(t), \ \ z(t) = Hx(t) \tag{16}$$

To this end introduce the variables $\psi(t) = \sigma(t) - m$ and $\rho(t) = \lambda e(t) + \sigma(t) - m$. In new notation (13)–(15) read as

$$\begin{aligned} &\dot{z}(t) = \bar{u}(t) + \eta(t), \\ &\bar{u}(t) = \operatorname{sat}_m\big(k_p e(t) + \rho(t)\big), \\ &e(t) = \varphi(t) - z(t), \\ &\rho(t) = \psi(t) + \lambda e(t), \\ &\dot{\psi}(t) = -\lambda\psi(t) - \lambda^2 e(t) + \lambda\bar{u}(t), \\ &\eta(t) = f(t) + m - v_d. \end{aligned} \tag{17}$$

Introducing the state-space vector $x(t) \in \mathbb{R}^2$ as $x = [x_1, x_2]^{\mathrm{T}}$ where $x_1(t) = z(t)$, $x_2(t) = \psi(t)$ and the external input vector $w(t) \in \mathbb{R}^2$ as $w = [w_1, w_2]^{\mathrm{T}}$, where $w_1(t) = \varphi(t)$, $w_2(t) = \eta(t)$ we obtain the system model in the form (16) with the following matrices:

$$A = \begin{bmatrix} 0 & 0 \\ \lambda^2 & -\lambda \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ \lambda \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 1 \\ -\lambda^2 & 0 \end{bmatrix}, \tag{18}$$
$$C = \big[-(k_p + \lambda), 1\big], \quad D = \big[k_p + \lambda, 0\big], \quad H = [1, 0].$$

Let us also make the following assumptions.

Assumption 1. There exists a number κ, $0 < \kappa < u_{p,\max}/2$, s.t. for all t the following inequalities

$$\kappa \le v_d - f(t) \le u_{p,\max} - \kappa. \tag{19}$$

are valid. This means that the demand product rate jointly with the disturbance lie within the admissible bounds of the machine production rate with some margin κ.

In the terms of the transformed coordinates this assumption leads to the inequality $|\eta(t)| \le m - \kappa$ for some $\kappa > 0$.

In compliance with the notion of $\varphi(t)$ as a fluctuation of the desired production relative to the linear trend, it is naturally to make the following

Assumption 2. Function $\varphi(t)$ is bounded, i.e. there exists $C_\varphi > 0$ s.t. $|\varphi(t)| \le C_\varphi$ for all t, and uniformly continuous on $(-\infty, \infty)$.

The convergent property of system (13)–(15) is given by the following Theorem.

Theorem 1. System (13)–(15), forced by the reference signal (3), satisfying Assumptions 1 and 2 is uniformly convergent for all $k_p > 0$, $\lambda > 0$.

Proof. At first let us show that the considered system is uniformly ultimately bounded. Perform a similarity transformation $\bar{x} = Tx$ with the following nonsingular matrix

$$T = \begin{bmatrix} 1 & 0 \\ -\lambda & 1 \end{bmatrix}.$$

In the transformed coordinates the state-space system representation has the following matrices:

$$\bar{A} = TAT^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & -\lambda \end{bmatrix}, \ \ \bar{B} = TB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \ \bar{F} = TF = \begin{bmatrix} 0 & 1 \\ -\lambda^2 & -\lambda \end{bmatrix},$$
$$\bar{C} = CT^{-1} = [-k_p, 1], \quad \bar{D} = D, \quad \bar{H} = H.$$

Then the state-space system equations in expanded form with respect to the transformed variables are as follows:

$$\begin{cases} \dot{\bar{x}}_1(t) = \mathrm{sat}_m(u) + \eta(t), \\ \dot{\bar{x}}_2(t) = -\lambda \bar{x}_2(t) - \lambda^2 \varphi(t) \end{cases} \quad (20)$$

$$u(t) = -k_p \bar{x}_1(t) + \bar{x}_2(t) + (k_p + \lambda)\varphi(t),$$

where $\eta(t) = f(t) + m - v_d(t)$. The second equation of system (20) represents asymptotically stable LTI system (recall that $\lambda > 0$), forced by the bounded input $\varphi(t)$. Then there exists $\limsup_{t\to\infty} |\bar{x}_2(t)|$. Consider the first equation of (20) and introduce the Lyapunov function $V(\bar{x}_1) = \bar{x}_1^2$. The time derivative of $V(\bar{x}_1)$ along (20) solutions is given by

$$\dot{V} = 2\bar{x}_1 \left(\mathrm{sat}_m\left(-k_p\bar{x}_1 + \bar{x}_2 + (k_p+\lambda)\varphi\right) + \eta(t)\right) \quad (21)$$

Since $|\eta(t)| \le m - \kappa < m$ by Assumption 1, and $\varphi(t)$ is bounded by Assumption 2 it follows that $\mathrm{sat}_m\left(-k_p\bar{x}_1 + \bar{x}_2 + (k_p+\lambda)\varphi = m \cdot \mathrm{sign}(\bar{x}_1)\right)$ if $k_p|\bar{x}_1| > |\bar{x}_2| + (k_p+\lambda)|\varphi| + m$ and therefore

$$\dot{V} \le -2\kappa|\bar{x}_1| < 0 \text{ if } k_p|\bar{x}_1| > |\bar{x}_2| + (k_p+\lambda)|\varphi| + m. \quad (22)$$

which implies that system (20) is uniformly ultimately bounded. Now Theorem 4.3 of (van den Berg, 2008) may be applied proving that system (20) (and, therefore, system (13)–(15)) is uniformly convergent for a given class of input signals.

Remark. Assumption 1 is not necessary for ensuring boundedness of the system error. This assumption may be weaken by permission for $v_d - f(t)$ to leave the interval $(0, u_{p,\max})$ for some "short periods", see (van den Berg et al., 2006) for details.

4. FREQUENCY DOMAIN ANALYSIS OF MANUFACTURING SYSTEM

The performance analysis of linear control systems is essentially based on frequency domain characteristics such as *sensitivity function* $S(i\omega)$ and *complementary sensitivity function* $T(i\omega)$. The generalized versions of these functions are defined in (Pavlov et al., 2007) for nonlinear Lur'e systems, possessing convergence property.

Let the nonlinear system be modeled as

$$\dot{x}(t) = f(x,r), \quad y(t) = h(x,r), \ e(t) = r(t) - y(t), \quad (23)$$

where $x(t) \in \mathbb{R}^n$ is a state vector, $y(t) \in \mathbb{R}$ is the system output, $r(t)$ is the reference signal, $e(t)$ is the reference error. Assume that $r(t) = a\sin(\omega t)$, where a is the amplitude and ω is the frequency of the input (reference) signal, and that the system (23) is uniformly convergent in this class of inputs. Then there exists an unique steady-state $\frac{2\pi}{\omega}$-periodic solution $\bar{x}(t)$ of (23) with the corresponding response $\bar{y}(t)$ and $\bar{e}(t) = r(t) - \bar{y}(t)$ (Pavlov et al., 2005a; van den Berg et al., 2006).

Definition (Pavlov et al., 2007): The functions

$$\mathscr{S}(a,\omega) = \|\bar{e}\|_2/\|r\|_2, \quad \mathscr{T}(a,\omega) = \|\bar{y}\|_2/\|r\|_2$$

where $\|z\|_2 = \left(\frac{\omega}{2\pi}\int_0^{2\pi/\omega} z(\tau)^2\, d\tau\right)^{1/2}$, are called, respectively, the generalized sensitivity and the generalized complementary sensitivity functions of the convergent system (23). For the linear case, the functions $\mathscr{S}(a,\omega)$ and $\mathscr{T}(a,\omega)$ coincide with customary amplification frequency characteristics $|S(i\omega)|$, $|T(i\omega)|$, respectively. For nonlinear systems, the functions $\mathscr{S}(a,\omega)$ and $\mathscr{T}(a,\omega)$ depend not only on the excitation frequency ω, but also on the amplitude a. These generalized sensitivity functions may be evaluated numerically based on system (23) simulation with given harmonic input $r(t)$. Significant reducing the computation costs may be achieved by using the generalization of the *describing functions* (*harmonic linearization*) method on nonautonomous convergent systems given in (van den Berg et al., 2007).

Let us present below the essentials of this method.

Consider behavior of the system (1), (8), (9) (or, equivalently, of the system (13)–(15)), forced by the harmonic fluctuation $\varphi(t) = \varphi_{\max}\sin(\omega t)$ of the magnitude $\varphi_{\max}$ and the frequency ω. To study nonlinear system performance in the frequency domain the well-known harmonic linearization method may be successfully applied. The method is based on an approximation of the nonlinear system by a linear one, for which the properties of the steady state solution can be found analytically or by numerically efficient algorithms. Its generalization for harmonically forced systems is given in (van den Berg et al., 2007), where sensitivity and complementary sensitivity functions for nonlinear convergent Lur'e systems are introduced and a computationally efficient numerical algorithm allowing estimation of these functions is presented.

Let the nonlinear system be presented in the following Lur'e form

$$\dot{x}(t) = Ax(t) + B\psi(y) + Fw(t), \quad (24)$$
$$y(t) = Cx(t) + Dw(t), \quad z(t) = Hx(t),$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $w(t) \in \mathbb{R}$ is a scalar external input signal, $y(t) \in \mathbb{R}$ is the system output, $\psi(\cdot)$ is a continuous scalar function, A, B, C, D, F, H are matrices of corresponding dimensions. It is assumed that the function $\psi(\cdot)$ for some finite $\mu > 0$ satisfies the following *incremental sector condition*

$$0 \le \frac{\psi(y_1) - \psi(y_2)}{y_1 - y_2} \le \mu \quad \forall y_1, y_2, \ y_1 \ne y_2. \quad (25)$$

We are interested in system (25) behavior for the case of harmonic input $w(t) = b\sin(\omega t)$ with amplitude b and frequency ω. In accordance with (Pavlov et al., 2007; Pogromsky et al., 2007), in the steady-state mode the system output may be approximately presented by a harmonic function with the frequency ω and the magnitude a satisfying the following *harmonic balance equation*

$$\left|1 - K(a)G(i\omega)\right|^2 a^2 = |H(i\omega)|^2 b^2, \quad (26)$$

where $G(i\omega) = C(i\omega I_n - A)^{-1}B$, $H(i\omega) = C(i\omega I_n - A)^{-1}F + D$, $K(a)$ is the *describing function* (for nonlinearity $\psi(\cdot)$), I_n is $n \times n$ identity matrix, $i^2 = -1$. For odd nonlinear function $\psi(x)$, the describing function $K(a)$ is defined by

$$K(a) = \frac{2}{\pi a}\int_0^{\pi} \psi(a\sin\theta)\sin\theta\, d\theta. \quad (27)$$

For the saturation nonlinearity $\mathrm{sat}_m(\cdot)$ (5), expression (27) leads to

$$K(a) = \begin{cases} 1, & a \le m, \\ \frac{2}{\pi}\left(\arcsin\left(\frac{m}{a}\right) + \frac{m}{a}\sqrt{1 - \frac{m^2}{a^2}}\right), & a > m. \end{cases} \quad (28)$$

4.1 Sensitivity functions of manufacturing system

Let us apply the method described above for accuracy analysis of the manufacturing system, forced by the harmonic excitation $\varphi(t) = \varphi_{\max}\sin(\omega t)$. Considered manufacturing system is modeled in the state-space form by (16), (18). Since we are interested in sensitivity functions with respect to the reference fluctuation $\varphi(t)$, denoted by $w_1(t)$ in (16), let us omit second columns of matrices D, F in (18). This leads to the following transfer functions $G(s)$ and $H(s)$ in (26):

$$G(s) = C(sI - A)^{-1}B = -k_p/s,$$
$$H(s) = C(sI - A)^{-1}F + D = \frac{(k_p + \lambda)s + k_p\lambda}{s + \lambda}.$$

Then harmonic balance equation (26) in our case reads as

$$\left|1 + \frac{k_p}{i\omega}K(a)^2\right|a^2 = \left|\frac{(k_p + \lambda)i\omega + k_p\lambda}{i\omega + \lambda}\right|^2 b^2, \qquad (29)$$

where b, ω are given parameters of the input signal φ, the describing function $K(a)$ is defined by (28), a is the sought amplitude of the output of linear subsystem (denoted by y in (24)). To solve the harmonic balance equation (29) the standard algebraic equations solvers (such as MATLAB routine *fsolve*) may be used.

It should be mentioned that the variable a denotes the amplitude of the output signal of the linear part of the Lur'e system (24). In our case this signal corresponds to the control signal at the input of the saturation block. To find a complimentary sensitivity function $\mathscr{T}(\omega)$ we need to find the magnitude of the plant output (the variable z in (13)). To this end the matrix $H = [1, 0]$ in (24) should be taken.

Besed on the approach of van den Berg et al. (2007); Pogromsky et al. (2007); van den Berg (2008), the foolowing statement, showing accuracy bounds of the harmonic balance analysis, may be obtained:

Consider system (24) with periodic input $w(t) = b\sin(\omega t)$ and assume the following conditions are met

1. (A, B) is controllable, (A, C) is observable;

2. frequency domain condition $\operatorname{Re} G(ik\omega) > -\mu^{-1}$ is satisfied for $k = \pm 1, \pm 3, \pm 5, \pm 7, \ldots$;

3. $\psi(\cdot)$ is odd function;

Then system (24) has a unique half-wave symmetric $2\pi/\omega$-periodic solution and the $\mathscr{L}_2$-norm of the error between outputs of nonlinear system and its harmonically linearized model is bounded by $\gamma v(a(b, \omega))$, where

$$\gamma = 2\rho_2/(2 - \mu\rho_1),$$
$$\rho_1 = \sup_{k=\pm1,\pm3,\pm5,\ldots} \left|C\left(ik\omega I - A - \tfrac{\mu}{2}BC\right)^{-1}B\right|,$$
$$\rho_2 = \sup_{k=\pm1,\pm3,\pm5,\ldots} \left|H\left(ik\omega I - A - \tfrac{\mu}{2}BC\right)^{-1}B\right|,$$
$$v(a) = \left(\frac{1}{2\pi}\int_0^{2\pi}\left(aK(a)\cdot\sin\vartheta - \psi(a\sin\vartheta)\right)^2 d\vartheta\right)^{1/2}$$

where $\psi(\cdot)$ is a feedback nonlinearity of the system's representation in the Lur'e form, $K(a)$ is a describing function for $\psi(\cdot)$.

To apply the above results for the considered system (13)–(15) let us check fulfillment of the conditions of Theorem 1. Based on the results of van den Berg et al. (2007); van den Berg (2008), it may be shown that for all $\omega \neq 0$ conditions 2, 3 are valid. Evidently, condition 4 for the saturation function is also fulfilled. The controllability condition 1 for the pair (A, B) of the form (18) is not valid. However, this condition is not a necessary one, it appeared in (van den Berg et al., 2007; Pogromsky et al., 2007; van den Berg, 2008) to prove that the matrix $A + BK(a)C$ has no eigenvalues $\pm i\omega$ on the imaginary axis. It may be easily checked that our system has

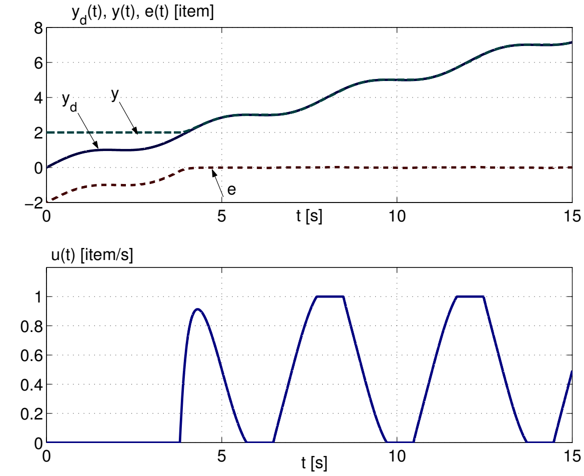

Fig. 1. Time histories. Fluctuation $\varphi(t) = \varphi_{\max}\sin(\omega_\varphi t)$, $\varphi_{\max} = 0.35$ [item], $\omega_\varphi = \pi/2$ [1/s]. $y_d(t)$ – solid line, $y(t)$ – dashed line, $e(t)$ – dash-dot line.

the eigenvalues $\{-\lambda, -K(a)k_p\}$, and the mentioned condition is met as $k_p > 0$, $\lambda > 0$.

5. NUMERICAL EXAMPLE

To evaluate numerically the system (13)–(15) performance, consider the following example.

Let the following parameters of the system (8), (9) and input signal (3) be taken: $u_{p,\max} = 1$ item/s, $k_p = 5.0$ items/s, $\lambda = 25$ 1/s. The gain $k_p$is chosen ensuring the 5%-zone transient time for non-saturated system ≈ 0.6 sec; parameter λ corresponds to five times faster state estimation procedure. The harmonic fluctuation signal $\varphi(t) = \varphi_{\max}\sin(\omega_\varphi t)$ with the magnitude $\varphi_{\max} = 0.35$ item and the frequency $\omega_\varphi = \pi/2$ rad/s is taken.

Time histories of the variables $y(t)$, $y_d(t)$, $e(t)$ and $u(t)$ for initial value $y(0) = 2$ items and the demand signal parameters $y_{d,0} = 0$ items, $v_d = 0.5$ items/s are depicted in Fig. 1. It is seen that the asymptotic error in the steady-state mode is small even if control signal is saturated due to influence of the disturbance $\varphi(t)$.

To evaluate the frequency domain performance of the closed-loop system (13)–(15) the complimentary sensitivity function $\mathscr{T}(\omega)$ and sensitivity functions $\mathscr{S}(\omega)$ were calculated for different magnitude $\varphi_{\max} \in \{0.01, 0.1, 0.35, 0.7\}$ of the fluctuation $\varphi(t)$. The results are presented in Figs. 2, 3. The complimentary sensitivity functions $\mathscr{T}(\omega)$ and sensitivity functions $\mathscr{S}(\omega)$ obtained by the simulation are depicted in Fig. 2. The plots show that at the region for $\omega < 0.7$ rad/s and $\varphi_{\max} < 0.7$ the reference error is practically negligible. Accuracy of the harmonic balance analysis may be evaluated from from Fig. 3, where the calculation results of the complimentary sensitivity functions $\mathscr{T}(\omega)$ by means of the simulation and the harmonic linearization method along with the accuracy bounds are plotted.

6. CONCLUSIONS

In the paper the problem of controlling manufacturing machine such that a customer demand is tracked with a desired accuracy under assumptions that the nominal demand rate and is constant (as well as possible disturbances) is studied. A combination of

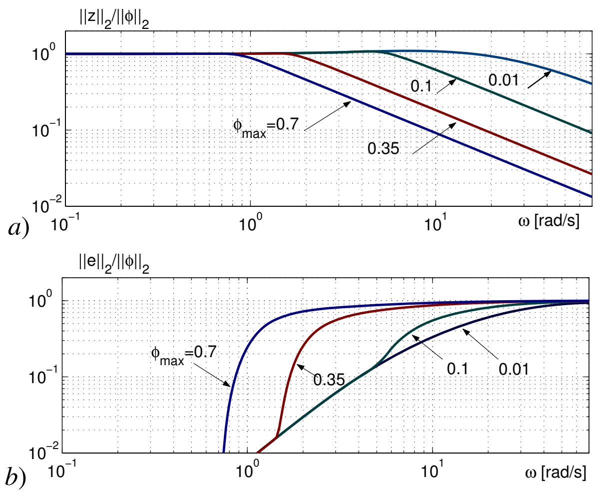

Fig. 2. Nonlinear complimentary sensitivity functions $\mathcal{T}(\omega)$ and nonlinear sensitivity functions $\mathcal{S}(\omega)$ (b) of system (13)–(15), $\varphi_{\max} \in \{0.01, 0.1, 0.35, 0.7\}$

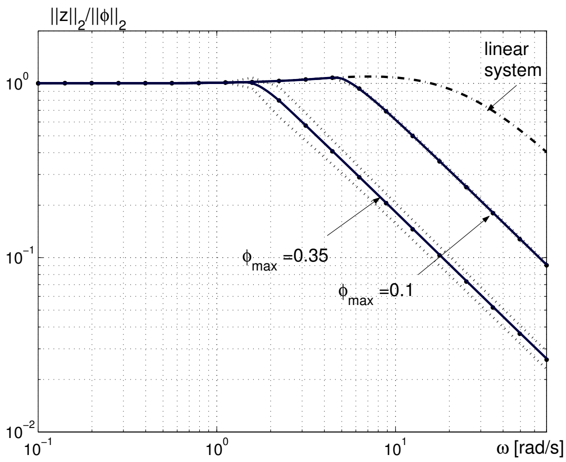

Fig. 3. Nonlinear complimentary sensitivity functions $\mathcal{T}(\omega)$ of system (13)–(15), $\varphi_{\max} \in \{0.1, 0.35\}$. Simulation – solid line, harmonic linearization – points, bounds – dotted lines, linear system – dash-dot line.

feedforward-feedback controller with a reduced-order observer is proposed and the system behavior in time domain and a steady-state performance of the system with periodic demand fluctuations are studied.

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